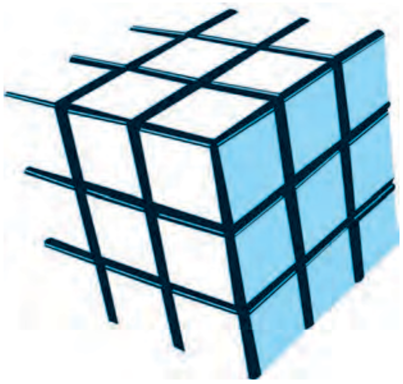
A Gateway to

MATHEMATICS



8

A Gateway to

MATHEMATICS

PART-8

Published by:

____

____

____

Designed & Illustrated by :
EDIT ONE INTERNATIONAL

PREFACE

Mathematics is a demanding, challenging and dynamic subject which is deeply associated with the day to day life activities and experiences to different types of quantities. Every one uses mathematics in his/her daily life in various ways irrespective of their knowledge of mathematical concepts. Study of mathematics introduces to child the basic mathematical concepts and skills needed for the child to face real life problems.

A Gateway to Mathematics is a series of eight books from **Class I to VIII**, based on the latest reviews and guidelines issued by the NCERT and CBSE. Our objective is to empower the students with ideal and quality education. Each chapter is well-illustrated with relevant study material, stepwise solved examples and adequate practice questions are there on each topic. It helps the preceptor to increase the ability of a child to easy understand, analyze and solve the problems with accurate logical sequence.

All the books of this series have enough Diagrams, Clear Explanations, Maths Lab Activities to help children understand the several principles and patterns of mathematics intended for them.

Salient features of the series are:

- Interactive study approach.
- Easy to learn educational methodology.
- Simple and easy language has been used keeping in mind the comprehensive level of the students.
- Each topic has appropriate illustrations which help in visualization of abstract mathematical concepts.
- Examples and word problems to provide a variety of experience to children and to sharpen their observational skills.
- Points to remember is given at the end of each chapter to highlight some important points of the topics.
- Maths Lab Activities to explore and improve the child's memory potential and to utilize the rich and varied opportunities available outside a classroom situation.
- To develop creativity in the children, enough pattern exercises have been introduced.
- Revision exercises including MCQ's are given for self assessment of the learners.

Any constructive suggestions for the improvement of this series are always appreciated.

— Publisher

CONTENTS

1 Let Us Revise

Exercise 1

1. Fill in the blank spaces-

(a) A number of the form $\frac{a}{b}$ where a and b are integers and $b \neq 0$ then the number is called a ....

(b) Fractional numbers whose numerators are smaller than the denominators are called ....

(c) Fractional numbers whose numerators are greater than the denominators are called ....

(d) Numbers having whole numbers as well as fractional numbers are called ....

(e) Numbers like $-3, -2, -1, 0, 1, 2, 3$ are called ....

(f) If the numerator as well as denominator of a rational number is negative or positive the rational number is called .... rational number.

(g) In case the numerator or the denominator of a rational number is negative. The rational number is called .... rational number.

2. Choose rational numbers out of the following.

(a) $\frac{0}{7}$ (b) $\frac{7}{9}$ (c) $\frac{-12}{-5}$ (d) $\frac{0}{0}$ (e) $\frac{-3}{-4}$ (f) -1 (g) ± 17

3. Choose positive rational numbers out of the following.

$\frac{-5}{8}, \frac{-3}{11}, \frac{3}{9}, \frac{0}{9}, \frac{-9}{-11}, \frac{+2}{+5}, \frac{1}{3}, \frac{-6}{-6}$ and $\frac{2}{2}$

4. Change them to rational numbers with positive denominators.

(a) $\frac{9}{-19}$ (b) $\frac{-3}{-7}$ (c) $\frac{1}{-7}$ (d) $\frac{0}{-3}$ (e) $\frac{-11}{-23}$

5. Show that $\frac{6}{18}$ and $\frac{12}{36}$ are equal.

6. Write three equivalent rational numbers of $\frac{2}{5}$.

7. Express a rational number with numerator 35.

8. Find absolute values of rational numbers $\frac{3}{5}$ and $\frac{-3}{5}$.

9. Put <, > or = in the blank spaces.

(a) $\frac{11}{-15}$ ☐ $\frac{-23}{-51}$ (b) $\frac{63}{-71}$ ☐ $\frac{1}{2}$ (c) $\frac{11}{35}$ ☐ $\frac{9}{10}$ (d) $\frac{3}{-6}$ ☐ $\frac{4}{-12}$

10. Arrange in ascending order.

$\frac{-2}{3}, \frac{-4}{5}, \frac{5}{-6}, \frac{-1}{2}$

11. Arrange in descending order.

$\frac{-4}{5}, \frac{9}{-15}$ and $\frac{-2}{3}$

12. Add

(a) $\frac{2}{5}$ and $\frac{7}{3}$ (b) $\frac{-1}{8}$ and $\frac{-2}{9}$

13. Subtract $\frac{-7}{10}$ from $\frac{5}{10}$.

14. Write additive inverse of the following rational numbers.

(a) $\frac{3}{7}$ (b) $\frac{-11}{19}$ (c) $\frac{2}{-15}$

15. Write multiplicative inverse of the following rational numbers.

(a) $\frac{2}{3}$ (b) $\frac{1}{7}$ (c) $\frac{-5}{11}$

16. Without actual division find which of the given rational numbers are terminating.

(a) $\frac{11}{30}$ (b) $\frac{17}{24}$ (c) $\frac{7}{16}$ (d) $\frac{12}{25}$

17. Write fifteen million three hundred seventy four thousand five hundred twelve in numeral.

18. Which of the following numbers are divisible by 8.

(a) 1790184 (b) 136976 (c) 901674 (d) 36792

19. Encircle the numbers which are divisible by 11.

(a) 66311 (b) 137269 (c) 4334 (d) 83721

20. Write 10 rational numbers between $\frac{3}{5}$ and $\frac{3}{4}$

21. Write additive inverse of the following numbers.

(a) –15 (b) 0 (c) 1

22. What must be subtracted from –7 to get 14.

23. 0°C, –2°C. Which of the two given temperatures is warmer.

24. Determine the numbers given below:

(a) Predecessor of 49 (b) Successor of 32

(c) Predecessor of 56 (d) Successor of 67

25. The product of four numbers is 48. if three of them are 2, 4 and 6. Find the fourth number.

26. Find 12 rational numbers between –1 and 2.

27. What are the three preceding consecutive whole numbers of the number 7510001?

28. Find the value of 'n' if –

(a) n – 7 = 5 (b) n + 7 = 17 (c) n + 4 = 9

29. Determine the smallest whole number which can be added to 735 so that the resulting number is exactly divisible by 11.

30. **Find the largest four digit number exactly divisible by 28.**

31. **There are 222 red balls in a basket. A boy takes out six red balls and replaces them with 12 white balls. He continues to do so till all red balls are replaced by white balls. Determine the number of white balls put in the basket.**

32. **Write all the pairs of twin primes between 50 and 100.**

33. **Which of the following numbers are prime?**

(a) 91 (b) 63 (c) 89 (d) 87

34. **Find HCF of the following pairs of numbers.**

(a) 396, 1080 (b) 144 and 198

35. **Write HCF of –**

(a) Two prime numbers (b) Two consecutive numbers

(c) Two coprimes numbers (d) 2 and an even number

36. **A trader has 120 litres of oil of one kind, 180 litres of oil of another kind and 240 litres of oil of third kind. Find the greatest capacity of a measuring tin which would measure each type of oil exactly.**

37. **Reduce each of the following numbers to its lowest term.**

(a) $\frac{296}{481}$ (b) $\frac{517}{799}$

38. **Find LCM by prime factorization method.**

(a) 112, 54, 108, 135, 198 (b) 24, 36, 40

39. **Find LCM of the following set of numbers by division method.**

(a) 22, 54, 108, 135, 198 (b) 12, 15, 20, 27

40. **Which smallest number is exactly divisible by 96 and 240 ?**

41. **The LCM and HCF of two numbers are 180 and 6 respectively. If one number is 30. Find the other number.**

42. **Find the sum of** $\frac{-3}{7}+\frac{2}{9}+\left(\frac{-11}{18}\right)$

43. **Find the additive inverse of :**

(a) $\frac{2}{11}$ (b) $\frac{-13}{-23}$ (C) $\frac{-117}{199}$ (d) $\frac{-91}{-237}$

44. **Verify that** $\left(\frac{-1}{3}+\frac{2}{5}\right)+\frac{6}{5}=\frac{-1}{3}+\left(\frac{2}{5}+\frac{6}{5}\right)$

45. **Sukhbeer bought a television for ₹ 10,000 and a camera for ₹ 7500. He sold the television at a loss of 12% and the camera at a gain of 20%. Find his total loss or gain in percent.**

46. **In an alloy of zinc, nickel and copper. The percentage of zinc and nickel are 35% and 45% respectively. Find the mass of copper in 1 kg of alloy.**

47. **A 130 metre long train moving at a speed of 65 km/hour passes a tree. How long will it take to pass the tree?**

48. Convert 105 m/sec into km/hour.

49. **Study the tables given below and find the table in which x and y vary directly.**

(a)

X	3	5	7	9	13	15
Y	12	20	28	36	52	60

(b)

X	5	6	7	9
Y	25	30	35	50

50. Find the value of 'P' in the given equation if t = 1.

$$\frac{3}{4}(7P - 1) - (2P - \frac{1-t}{2}) = t + \frac{3}{2}$$

51. **Write the following numbers in the form of scientific notations–**

(a) 0.000000000032 (b) 0.000000001234

(c) 6300000000000 (d) 15430000000000

52. **Find the reciprocal –**

(a) $(\frac{1}{4})^3$ (b) $(\frac{3}{4})^2$

53. **Find the quotient without actual division.** 130013 ¸ 13

54. Subtract $2x^3 - 4x^2 - 3x + 5$ from $4x^3 + x^2 - x + 6$

55. One third of the length of a tree is under the ground, one fourth is under water and the remaining 5 metres is over the ground. Find the total length of the tree.

56. Simran bought a pen set for ₹ $75\frac{1}{4}$, a book for ₹ $250\frac{1}{2}$ and and umbrella for ₹ $125\frac{3}{4}$, from a departmental store. How money did she spend in all?

57. The sum of two rational numbers is -2. If one of the numbers is -2. If one of the numbers is $\frac{-14}{5}$, Find the other.

58. The boarding house of a school has enough food for 720 students for 35 days. After 5 days 120 students leave the school. How many days will the food last?

59. **Fill in the blanks to complete the statements.**

(a) The product of a rational number and its reciprocal is always ....

(b) The number which has no reciprocal is ....

(c) The number .... and .... are their own reciprocals.

(d) Zero is .... reciprocal of any number.

60. **Encircle the correct answer.**

(a) If all the three angles of a triangle are equal then each of them is equal to –

(i) 90° (ii) 45° (iii) 60° (iv) 30°

(b) If the two acute angles of a right triangle are equal. Then each acute angle is equal to –

(i) 30° (ii) 45° (iii) 60° (iv) 90°

(c) An exterior angle of a triangle is equal to 105° and two interior opposite angles are equal, then each of these angles is equal to –

(i) 75° (ii) $72\frac{1}{2}°$ (iii) $52\frac{1}{2}°$ (iv) $37\frac{1}{2}°$

(d) A triangle can be constructed by taking its as–

(i) 80°, 60° (ii) 75°, 115° (iii) 135°, 45° (iv) 90°, 90°

(e) The length of a rectangle is doubled and its breadth is halved. Its area will be –

(i) Half times the original area (ii) Same as the original area

(iii) Two times the original area (iv) Four times the original area

2 Rational Numbers

Revision of The Number Systems

Let us recall the number systems, that we have studied in our earlier classes. So far we have studied—

1. Natural Numbers
2. Whole Numbers
3. Fractional Numbers
4. Integers

NATURAL NUMBERS

The numbers other than zero are called natural numbers. Numbers like 1, 2, 3, 4, 5 .... are called natural numbers.

WHOLE NUMBERS

All the numbers used for counting including zero are called whole numbers. 0, 1, 2, 3, 4, 5 .... are whole numbers.

All whole numbers are natural numbers but all natural numbers are not whole numbers.

FRACTIONAL NUMBERS

The numbers of the form of $\frac{p}{q}$, whose p and q are whole numbers and $q \neq 0$ are fractional numbers.

The number 0, 1, $2\frac{1}{2}$, $\frac{2}{3}, \frac{12}{7}$, $2\frac{1}{2}$ are fractional numbers.

INTEGERS

The numbers $-3, -2, -1, 0, 1, 2, 3$.......... n are called integers.

The difference between fractional number and rational number : Fractional numbers include only positive integers whereas rational numbers include positive as well as negative integers.

Numbers like $\frac{3}{5}$ are fractional as well as rational numbers. Whereas $\frac{3}{-5}$ is a rational number but not a fraction.

Similarly all natural numbers are rational numbers also but all rational numbers are not natural numbers. All whole numbers are also rational numbers.

RATIONAL NUMBERS

All the numbers of the form of $\frac{p}{q}$, where *p* and *q* are integers and $q \neq 0$. $-1, -2, -3, 0, 1, 2, 3$, are rational $\frac{2}{3}, \frac{-2}{3}, \frac{2}{-3}, \frac{-2}{-3}, \sqrt{4}, \sqrt{25}$ numbers.

Properties of Rational Numbers

1. Positive rational numbers : The rational numbers whose both the numerator and denominators are either positive or negative are said to be positive rational numbers.

2. Negative rational numbers : Rational numbers whose numerators or denominators are negative are said to be negative rational numbers or simply negative rationals.

3. Equivalent rational numbers : If $\frac{p}{q}$ is a rational number then $\frac{p}{q} = \frac{p \div m}{q \div m}$, where m is a non zero integer.

Example : $\frac{p}{q} = \frac{4}{16}, \frac{p}{q} = \frac{p \div m}{q \div m} = \frac{4 \div 2}{16 \div 2} = \frac{2}{8}$

4. if $\frac{p}{q}$ is a rational number and m is a common divisor of p and q. Then $\frac{p}{q} = \frac{p \div m}{q \div m}$. Where m is a non zero integer.

Example : $\frac{3}{15} = \frac{3 \div 3}{15 \div 3} = \frac{1}{5}$

5. Standard form of rational number : if $\frac{p}{q}$ is a rational number having no common divisor this rational number is said to be in the standard form.

The rational number $\frac{5}{7}$ is in standard form as it has no common divisor. A non standard rational number can be converted into standard form by dividing with a common divisor other than 1.

Example : Express $\frac{25}{45}$ in standard form.

Solution : $\frac{25}{45} = \frac{25 \div 5}{45 \div 5} = \frac{5}{9}$

$\frac{5}{9}$ is a rational number in the standard form as it has no more common divisor other than 1.

Comparison of Rational Numbers (Method – 1)

STEPS OF COMPARISON :

1. A rational number in the standard form must not have a negative denominator. If the denominator is negative convert it to positive.
2. Take LCM of all the denominators.
3. Work out the numerator as we do for addition and subtraction of fractional numbers.
4. Compare the numerators. The rational number having larger numerators are greater.

Example : For any given rational number $\frac{p}{q}$

Solution :

$$\left[\frac{p}{q}\right] = \begin{bmatrix} \frac{p}{q} \text{ if } \frac{p}{q} > 0 \\ 0 \text{ if } \frac{p}{q} = 0 \\ -\frac{p}{q} \text{ if } \frac{p}{q} < 0 \end{bmatrix}$$

for example, $\left|\frac{-3}{11}\right| = \left|\frac{3}{11}\right| \Rightarrow \left|\frac{-7}{-13}\right| = \frac{7}{13}$.

(Method – II)

Property : Let $\frac{a}{b}$ and $\frac{c}{d}$ be rational numbers where b and d are positive integers. Then.

$$\frac{a}{b} \times \frac{c}{d}$$

If a × d > c × b then $\frac{a}{b} > \frac{c}{d}$

If a × d < c × b then $\frac{a}{b} < \frac{c}{d}$

Example : Compare $\frac{-5}{7}$ and $-\frac{3}{4}$

Solution : $\frac{-5}{7} \times -\frac{3}{4}$

$-5 \times 4 = -20$

$-3 \times 7 = -21$

$-20 > -21$

Therefore $\frac{-5}{7} > -\frac{3}{4}$

Example : If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational number, then

Solution : $\frac{a}{b} x \frac{c}{d} = \frac{axc}{bxd} = \frac{\text{Product of numerators}}{\text{Product of denominators}}$

$\frac{2}{3} x \left(\frac{-7}{5}\right) = \frac{2x(-7)}{3x5} = \frac{-14}{15}$

$\frac{a}{b} x \frac{c}{d} x \frac{e}{f} x...... = \frac{axcxe.....}{bxdxf.....}$

Arranging Rational Numbers in Ascending and Descending Order

Example : Arrange $\frac{-2}{3}$, $\frac{-9}{15}$ and $\frac{-4}{5}$ in ascending order.

Solution : The LCM of denominators 3,15 and 5 is 15

$\frac{-2}{3} = \frac{-2 \times 5}{3 \times 5} = \frac{-10}{15}$

$\frac{-4}{5} = \frac{-4 \times 3}{5 \times 3} = \frac{-12}{15}$

$\frac{-9}{15} = \frac{-9 \times 1}{15 \times 1} = \frac{-9}{15}$

$\frac{-12}{15} < \frac{-10}{15} < \frac{-9}{15}$

$\therefore \frac{-4}{5} < \frac{-2}{3} < \frac{-9}{15}$

Exercise 2.1

1. Express in standard form.

(a) $\frac{4}{8}$ (b) $\frac{10}{30}$ (c) $\frac{11}{55}$ (d) $\frac{13}{65}$ (e) $\frac{24}{96}$

2. Write three equivalent rational numbers of $\frac{2}{9}$.

3. Compare each pair of the given rational numbers.

(a) $\frac{11}{25}, \frac{110}{250}$ (b) $\frac{6}{7}, \frac{36}{37}$ (c) $\frac{21}{57}, \frac{42}{114}$ (d) $\frac{5}{9}, \frac{100}{180}$ (e) $\frac{3}{7}, \frac{-3}{7}$

4. Which of the following pairs of rational numbers are equal?

(a) $\frac{-11}{7}, \frac{33}{-21}$ (b) $\frac{3}{-5}, \frac{6}{10}$ (c) $\frac{7}{4}, \frac{-28}{-16}$

(d) $\frac{3}{13}, \frac{-12}{52}$ (e) $\frac{4}{12}, \frac{-1}{3}$ (f) $\frac{2}{5}, \frac{5}{2}$

5. Write each of the mixed fractions in p/q form.

(a) $3\frac{4}{5}$ (b) $6\frac{2}{3}$ (c) $-5\frac{1}{4}$ (d) $-7\frac{2}{3}$

6. Sort out the rational numbers which are not equal to $\frac{3}{5}$.

(a) $\frac{-3}{5}$ (b) $\frac{3}{-5}$ (c) $\frac{3}{5}$ (d) $\frac{6}{10}$ (e) $\frac{30}{50}$

7. Write rational numbers equivalent to $\frac{-3}{5}$ with denominators.

(a) 20 (b) −30 (c) 35 (d) −40

8. Fill in the blank boxes with symbols <, > or = .

(a) $\frac{3}{8}$ ☐ 0 (b) $\frac{-2}{9}$ ☐ 0 (c) $\frac{-3}{4}$ ☐ $\frac{1}{4}$

(d) $\frac{-5}{7}$ ☐ $\frac{-4}{7}$ (e) $-\frac{2}{3}$ ☐ $-\frac{3}{4}$ (f) $\frac{-1}{2}$ ☐ 0

9. Which of the two rational numbers is greater in the given pair.

(a) $\frac{-12}{5}$ or -3 (b) $\frac{4}{-5}$ or $\frac{-7}{10}$ (c) $\frac{9}{-13}$ or $\frac{7}{-12}$

(d) $\frac{-1}{3}$ or $\frac{4}{-5}$ (e) $\frac{7}{-9}$ or $\frac{-5}{8}$ (f) $\frac{-4}{3}$ or $\frac{-8}{7}$

10. Arrange in ascending order.

(a) $\frac{4}{-9}, \frac{-5}{12}, \frac{7}{-18}, \frac{-2}{3}$ (b) $\frac{-3}{4}, \frac{5}{-12}, \frac{-7}{16}, \frac{9}{-24}$

(c) $\frac{3}{-5}, \frac{-7}{10}, \frac{-11}{15}, \frac{-13}{20}$ (d) $\frac{-4}{7}, \frac{-9}{14}, \frac{13}{-28}, \frac{-23}{42}$

11. Arrange the following rational number in descending order.

(a) $-2, \frac{-13}{6}, \frac{8}{-3}, \frac{1}{3}$ (b) $\frac{-3}{10}, \frac{7}{-15}, \frac{-11}{20}, \frac{17}{-30}$

(c) $\frac{-5}{6}, \frac{-7}{12}, \frac{-13}{18}, \frac{23}{-24}$ (d) $\frac{-10}{11}, \frac{-19}{22}, \frac{-23}{33}, \frac{-39}{44}$

12. Find two rational numbers whose absolute value is $\frac{1}{5}$.

13. Fill in the blank space –

(a) Every negative rational number is .... zero.

(b) If x, y, z are rational numbers such that $x > y$ and $y > z$ then ....

(c) Two rational numbers are said to be equal if they are equal in their .... form.

(d) If the integers p and q have no common divisor other than 1 and q is positive then the rational number is said to be in the .... form.

(e) If $\frac{p}{q}$ is a rational number, then q cannot be ....

(f) Between two rational numbers there lie .... number of rational numbers.

(g) The reciprocal of $\frac{1}{a}$, where $a \neq 0$ is ....

(h) The number which cannot be the reciprocal of any number is ....

(i) 1 and –1 are .... of itself.

(j) The product of a rational number and its reciprocal is ....

14. Mark (✓) for true or (×) for False.

(a) If $\frac{a}{b}$ is a rational number and m is an integer then $\frac{a}{b} = \frac{a \div m}{b \div m}$

(b) Every whole number is a rational number but every rational number is not a whole number.

(c) Zero is the smallest rational number.

(d) $\frac{a}{0}$ is rational number where $a \neq 0$.

(e) All integers are rational numbers.

(f) The quotient of two integers is always a rational number.

(g) The quotient of two integers is always an integer.

15. Encircle the correct answers.

(a) The greatest rational number out of the following rational numbers is ?

(i) $\frac{5}{-9}$ (ii) $\frac{5}{4}$ (iii) $\frac{5}{7}$ (iv) $\frac{-5}{6}$

(b) Which one is the smallest rational number?

(i) $\frac{3}{7}$ (ii) $\frac{4}{-7}$ (iii) $\frac{-5}{7}$ (iv) $\frac{2}{7}$

(c) Which of the following is not in standard form?

(i) $\frac{7}{5}$ (ii) $\frac{10}{20}$ (iii) $\frac{13}{33}$ (iv) $\frac{27}{28}$

(d) If $\frac{5}{8} = \frac{20}{x}$ then the value of x is –

(i) 23 (ii) –23 (iii) 32 (iv) 2

(e) If $\frac{1}{4}$ is written with denominator 12. Then its numerator will be –

(i) 48 (ii) 3 (iii) –8 (iv) 8

(f) Which of the following is a positive rational number ?

(i) $\frac{-3}{-4}$ (ii) $\frac{0}{4}$ (iii) $\frac{3}{-4}$ (iv) $\frac{-3}{4}$

Representing Rational Numbers on The Number Line

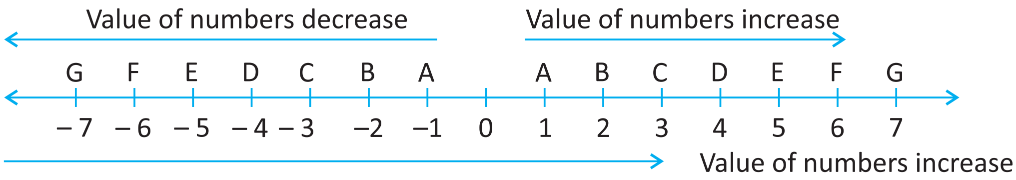
NUMBER LINE

A line divided into equal parts and numbers written on it is said to be a number line. To draw a number line —

(i) Draw a line of any length in your note book.

(ii) Select the middle point on the number line and select it to be equal to '0'.

(iii) On either side of the number line divide the line into unit lengths.



(iv) On the right hand side of the '0' we have positive integers. The numbers or integers corresponding to A, B, C, D, E, F, are positive integers. Whereas the integers corresponding to A, B, C, D, E, F and G on left hand side are negative integers.

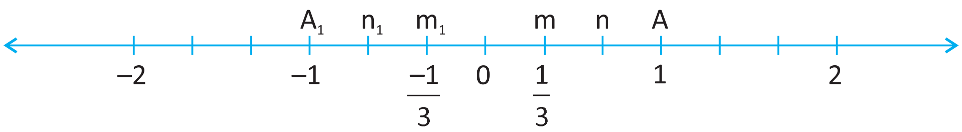
(v) When we move on the right hand side of 0, the values of integers increase. Whereas when we move on the left hand side of 0, the value of numbers decreases.

(vi) In general the value increase towards the right hand side. Therefore $-3<-2<-1<0<1<2<3$......

(vii) Between any two rational numbers of the number line there lies infinite numbers of rational.

Example 1 : Represent $\frac{1}{3}$ and $-\frac{1}{3}$ on a number line.

Solution :



- ❖ Draw a line. Mark a point O on it. This point represent 0 (zero). On either side of O mark A and A_1 respectively of equal lengths.
- ❖ A represents 1 whereas A_1 represents –1
- ❖ Divide OA and OA_1 in three equal parts
- ❖ The point m in OA represents $\frac{1}{3}$
- ❖ The point m_1 on OA_1 represents $\frac{-1}{3}$

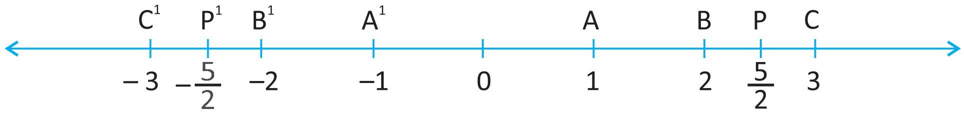
Example 2 : Represent $\frac{5}{2}$ and $-\frac{5}{2}$ on the number line.

Solution : Convert the improper rationals into mixed numbers.

$$\frac{5}{2} = 2\frac{1}{2} \qquad -\frac{5}{2} = -2\frac{1}{2}$$

- ❖ Draw a number line and mark the point ABC and A^1, B^1, C^1 on the right hand side and left hand side and left hand side of O respectively at equal intervals.
- ❖ Divide BC and B^1, C^1 into two equal parts.

❖ Mark the points as p and p^1. These points are the appropriate point for $\frac{5}{2}$ and $-\frac{5}{2}$ respectively.

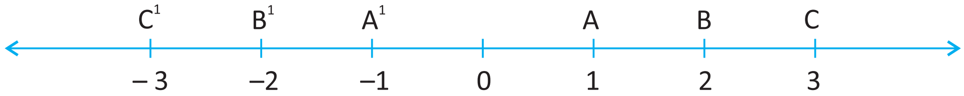


Example 3 : Mark the rational –3, –2, –1, 0, 1, 2, 3 on the number line and arrange them in —

(a) Ascending order

(b) Descending order

Solution :



Draw a number line and mark A, B, C such that OA = AB = BC on the right hand side of O.

Similarly mark the points A1, B1, and C1 of equal unit length on the left hand side of O. Such that OA1= A1B1 = B1C1

a. Ascending order of rationals —

$-3 < -2 < -1 < 0 < 1 < 2 < 3$

b. Descending order of rationals —

$3 > 2 > 1 > 0 > -1 > -2 > -3$

Exercise 2.2

1. Fill in the blanks.

(a) The distance between –2 and $\frac{3}{5}$ on a number line is ....

(b) The distance between 0 and $\frac{3}{5}$ on a number line is ....

(c) While moving left to right we find .... rationals.

(d) $\frac{-2}{3}$ lies to the .... of zero on the number line.

(e) The rational number $\frac{1}{2}$ and $\frac{-5}{3}$ lie on .... sides of 0 on a number line.

(f) The rational number $\frac{-7}{-11}$ lies to the .... of 0 on the number line.

2. Represent the following rational numbers on the number line.

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $1\frac{3}{4}$ (d) $2\frac{2}{5}$

(e) $3\frac{1}{2}$ (f) $6\frac{5}{7}$ (g) $5\frac{2}{3}$ (h) $3\frac{3}{4}$

3. Represent each of the following numbers on the number line.

(a) $-2\frac{5}{8}$ (b) -5 (c) $-2\frac{5}{6}$ (d) $-5\frac{3}{5}$

(e) $-\frac{1}{3}$ (f) $-\frac{3}{4}$ (g) $-1\frac{2}{3}$ (h) $-\frac{1}{3}$

Addition of Rational Numbers

PROPERTIES OF ADDITION OF RATIONAL NUMBERS

1. $\frac{a}{b}+\frac{c}{d}$ = A rational number – Closure property. The sum of two rational numbers is always a rational number.

Example 1 : Add $\frac{4}{9}$ and $\frac{-11}{9}$

Solution :

$$= \frac{4}{9}+\left(\frac{-11}{9}\right) = \frac{4+(-11)}{9}$$

$$= \frac{4-11}{9} = \frac{-7}{9}$$

2. $\frac{a}{b}+\frac{c}{d} = \frac{c}{d}+\frac{a}{b}$ Commutative property.

Example 2 : Add $\frac{1}{3}$ and $\frac{5}{6}$

Solution :

$$\frac{1}{3}+\frac{5}{6} = \frac{5}{6}+\frac{1}{3}$$

$$= \frac{2+5}{6} = \frac{2+5}{6}$$

$$= \frac{7}{6} = \frac{7}{6}$$

3. The, sum of two rational number is -2, if one of the numbers is $\frac{-14}{5}$, find the other.

Example 3 : Sum of two rational number of like -2, $\frac{-14}{5}$.

Solution : Let the number be x.

$$\Rightarrow x+\left(\frac{-14}{5}\right) = -2$$

$$\Rightarrow x = -2+\frac{14}{5}$$

$$= \frac{-10+14}{5} = \frac{4}{5}$$

Hence, the required number is $\frac{4}{5}$.

4. $\left(\frac{a}{b}+0\right) = \left(0+\frac{a}{b}\right) = \frac{a}{b}$ Associative property of zero.

That is when zero is added to any rational number the sum is the rational number itself.

Example 4 : Add $\frac{2}{5}$ and 0.

$$\left(\frac{2}{5}+0\right) = \left(0+\frac{2}{5}\right)$$

$$\left(\frac{2+0}{5}\right) = \left(\frac{0+2}{5}\right)$$
$$\frac{2}{5} = \frac{2}{5}$$

5. $\left(\frac{a}{b}+\frac{-a}{b}\right) = \left(\frac{-a}{b}+\frac{a}{b}\right) = 0$, Additive inverse.

For every rational number $\frac{a}{b}$ there exists a rational number $\frac{-a}{b}$ such that $\frac{a}{b}+\frac{-a}{b}=\frac{a-a}{b}=\frac{0}{b}=0$

Therefore $\frac{-a}{b}$ and $\frac{a}{b}$ are additive inverse of each other.

Example 5 : Find additive inverse of $\frac{3}{7}$.

Solution : The additive inverse of $\frac{3}{7}$ is $\frac{-3}{7}$. It can be proved by adding it.

$$\left(\frac{3}{7}+\frac{-3}{7}\right) = \frac{3}{7}-\frac{3}{7} = \frac{3-3}{7} = \frac{0}{7} = 0$$

6. Subtraction of rational numbers.

Let $\frac{a}{b}$ and $\frac{c}{d}$ be two rational numbers.

Then, $\frac{a}{b}$ + additive inverse of $\frac{c}{d} = \frac{a}{b}-\frac{c}{d}$ = rational number.

Solved Example 1 : Find additive inverse of the following rational numbers.

(a) $\frac{3}{9}$ (b) $\frac{-17}{9}$ (c) $\frac{7}{-9}$ (d) $\frac{-4}{-9}$

Solution :

(a) Additive inverse of $\frac{3}{9}$ is $\frac{-3}{9}$

(b) Additive inverse of $\frac{-17}{9}$ is $\frac{+17}{9}$ or $\frac{17}{9}$

(c) Additive inverse of $\frac{7}{-9}$

$\frac{7\times-1}{-9\times-1} = \frac{-7}{9}$ The additive inverse of $\frac{-7}{9}$ is $\frac{+7}{9}$ or $\frac{7}{9}$

(d) Additive inverse of $\frac{-4}{-9}$

$\frac{-4}{-9} = \frac{-4\times-1}{-9\times-1} = \frac{4}{9}$, The additive inverse of $\frac{4}{9}$ is $\frac{-4}{9}$.

Solved Example 2 : Subtract $\frac{1}{4}$ from $\frac{2}{3}$.

Solution :

$$\left(\frac{2}{3}\right)-\left(\frac{1}{4}\right) = \frac{2}{3} + \text{additive inverse of } \frac{1}{4}$$
$$= \frac{2}{3}+\left(\frac{-1}{4}\right) = \left(\frac{2}{3}\right)-\frac{1}{4}$$
$$= \frac{8-3}{12} = \frac{5}{12}$$

Solved Example 3 : Subtract $\frac{-3}{7}$ from $\frac{-2}{5}$

Solution :

$$\left(\frac{-2}{5}-\frac{-3}{7}\right) = \frac{-2}{5} + \text{additive inverse of } \frac{-3}{7}$$
$$= \frac{-2}{5}+\frac{3}{7}$$

$$= \frac{-14+15}{35} = \frac{1}{35}$$

Solved Example 4 : What should be added to $\frac{-5}{8}$ to get $\frac{3}{9}$.

Solution : Let the number to be added be = x

$$\frac{-5}{8} + x = \frac{3}{9}$$

$$x = \frac{3}{9} + \frac{5}{8}$$

$$= \frac{24+45}{72}$$

$$= \frac{69}{72} \quad \text{Ans.}$$

Solved Example 5 : The sum of two numbers is –7. If one of them is $\frac{-11}{6}$, find the other rational number.

Solution : Let the other number be = x

$$\frac{-11}{6} + x = -7$$

$$x = -7 + \frac{11}{6}$$

$$x = \frac{-42+11}{6} = \frac{-31}{6}$$

The other number is $\frac{-31}{6}$

Solved Example 6 : Evaluate $\left(\frac{3}{5}\right)^3 \times \left(\frac{3}{5}\right)^2$

The commutative property states that rational number can be arranged in desired way. The associative property states that rational number can be grouped in desired manner.

Solution :

$$= \left(\frac{3}{5}\right)^3 \times \left(\frac{3}{5}\right)^2$$

$$= \left(\frac{3}{5}\right)^3 \times \left(\frac{3}{5}\right)^2$$

$$= \left(\frac{3}{5}\right)^{3+2} \times \left(\frac{3}{5}\right)^5$$

$$= \frac{3^5}{5^5} = \frac{243}{3125}$$

Solved Example 7 : Simplify $\left(\frac{2}{3} + \frac{4}{7} + \frac{-8}{9} + \frac{-5}{21}\right)$

Solution :

$$\left[\frac{2}{3} + \frac{4}{7} + \left(\frac{-8}{9}\right) + \left(\frac{-5}{21}\right)\right]$$

$$\left[\frac{2}{3} + \left(\frac{-8}{9}\right)\right] + \left[\frac{4}{7} + \left(\frac{-5}{21}\right)\right]$$ – using commutative and associative identities.

$$= \left(\frac{2}{3} - \frac{8}{9}\right) + \left(\frac{4}{7} - \frac{5}{21}\right)$$

$$= \left(\frac{6-8}{9}\right) + \left(\frac{12-5}{21}\right)$$

$$= \frac{-2}{9} + \frac{7}{21} = \frac{-14+21}{63}$$

$$= \frac{7}{63} = \frac{1}{9}$$

Solved Example 8 : What should be subtracted from $\frac{-3}{7}$ to get '1'.

Solution : Let the number be added be = x

$$\frac{-3}{7} - x = 1$$

$$-x = 1 + \frac{3}{7}$$

$$-x = \frac{7+3}{7}$$

$$-x = \frac{10}{7}$$

$$-x \times (-1) = \frac{10}{7} \times (-1)$$

$$x = \frac{-10}{7} \text{ ans.}$$

Solved Example 9 : Find absolute values of the following rational numbers.

(a) $\frac{2}{7}$ (b) $\frac{-2}{7}$ (c) $\frac{21}{-9}$ (d) $\frac{-23}{27}$ (e) $\frac{-151}{309}$

Solution :

(a) $\left|\frac{2}{7}\right| = \frac{|2|}{|7|} = \frac{2}{7}$

(b) $\left|\frac{-2}{7}\right| = \frac{|-2|}{|7|} = \frac{2}{7}$

(c) $\left|\frac{21}{-9}\right| = \frac{|21|}{|-9|} = \frac{21}{9}$

(d) $\left|\frac{-23}{27}\right| = \frac{|-23|}{|27|} = \frac{23}{27}$

(e) $\left|\frac{-151}{309}\right| = \frac{|-151|}{|309|} = \frac{151}{309}$

Solved Example 10: Add $\left|\frac{-3}{7}\right|$ and $\left|\frac{-9}{21}\right|$

Solution :

$$\left|\frac{-3}{7}\right| + \left|\frac{-9}{21}\right| = \frac{|-3|}{|7|} + \frac{|-9|}{|21|} = \frac{3}{7} + \frac{9}{21}$$

$$= \frac{9+9}{21}$$

$$= \frac{18}{21} = \frac{6}{7}$$

Exercise 2.3

1. Add –

(a) $\frac{4}{5}$ and $\frac{-2}{5}$ (b) $\frac{-4}{11}$ and $\frac{-6}{11}$ (c) $\frac{5}{6}$ and $\frac{-1}{6}$ (d) $\frac{-7}{3}$ and $\frac{1}{3}$ (e) $\frac{-17}{15}$ and $\frac{-1}{5}$

2. Find the sum of the following –

(a) $\frac{-3}{5}, \frac{3}{4}$ (b) $\frac{5}{8}, \frac{-7}{12}$ (c) $\frac{-8}{9}, \frac{11}{6}$

(d) $\frac{7}{24}, \frac{-5}{16}$ (e) $\frac{7}{-18}, \frac{8}{27}$ (f) $\frac{2}{-15}, \frac{1}{-12}$

3. Verify the following –

(a) $\frac{9}{-14} + \frac{17}{-21} = \frac{17}{-21} + \frac{9}{-14}$ (b) $-6 + \frac{-11}{-12} + \frac{-11}{-12} + (-6)$

(c) $-1 + \left(\frac{-2}{3} + \frac{-3}{4}\right) = \left(-1 + \frac{-2}{3}\right) + \frac{-3}{4}$ (d) $\left(\frac{-7}{11} + \frac{2}{-5}\right) + \frac{-13}{22} = \frac{-7}{11} + \left(\frac{2}{-5} + \frac{-13}{22}\right)$

(e) $-20 + \left(\frac{3}{-5} + \frac{-7}{-10}\right) = \left(-20 + \frac{3}{-5}\right) + \frac{-7}{-10}$

4. Find additive inverse of each of the following rational numbers –

(a) $\frac{21}{-40}$ (b) $\frac{-21}{30}$ (c) $\frac{-15}{-11}$ (d) 0 (e) $\frac{8}{-29}$

(f) $\frac{-17}{9}$ (g) $\frac{-23}{1}$ (h) $\frac{17}{9}$ (i) $\frac{2}{3}$

5. Subtract :

(a) $\frac{-4}{5}$ from $\frac{9}{8}$ (b) $\frac{-1}{16}$ from $\frac{-3}{8}$ (c) $\frac{3}{-4}$ from $\frac{4}{5}$

(d) $\frac{-4}{15}$ from $\frac{3}{10}$ (e) $\frac{4}{9}$ from $\frac{-1}{6}$ (f) $\frac{1}{5}$ from $\frac{3}{5}$

6. Find the sum using rearrangement property –

(a) $\frac{-11}{5} + \frac{-2}{3} + \frac{3}{5} + \frac{4}{3}$ (b) $\frac{3}{8} + \frac{-11}{6} + \frac{-1}{4} + \frac{-8}{3}$

(c) $\frac{-13}{20} + \frac{11}{14} + \frac{-5}{7} + \frac{7}{10}$ (d) $\frac{-6}{7} + \frac{-5}{6} + \frac{-4}{9} + \frac{-15}{7}$

7. What rational number should be subtracted from $\frac{-2}{3}$ to get $\frac{-1}{6}$?

8. What rational number should be added to –1 to get $\frac{5}{7}$?

9. Fill in the blanks:

(a) $\left(\frac{-12}{5}\right) + \ldots\ldots\ldots = \left(\frac{-3}{17}\right) + \left(\frac{-12}{5}\right)$ (b) $(-9) + \left(\frac{-31}{8}\right) = \ldots\ldots\ldots + (-9)$

(c) $(\ldots\ldots\ldots) + \frac{3}{7} + \frac{-13}{4} = \left(\frac{-8}{13} + \frac{3}{7}\right) + \left(\frac{-13}{4}\right)$ (d) $-12 + \left[\frac{7}{12} + \left(\frac{-9}{11}\right)\right] = \left[(-12 + \frac{-2}{3}\right] + \ldots\ldots\ldots$

(e) $\frac{19}{-5} + \left[\left(\frac{-3}{11}\right) + \left(\frac{-7}{8}\right)\right] = \left(\frac{19}{-5} + \ldots\ldots\ldots\right) + \frac{-7}{8}$ (f) $\frac{-16}{7} + \ldots\ldots\ldots = \ldots\ldots\ldots + \left(\frac{-16}{7}\right) = \frac{-16}{7}$

10. Verify that – (–a) = a, when a = (a) $\frac{7}{6}$ (b) $\frac{-8}{9}$

11. **Verify that – (a+b) = (–a) + (–b), when –**

(a) $a = \frac{3}{4}, \quad b = \frac{3}{4}$ (b) $a = \frac{-3}{4}, \quad b = \frac{-6}{7},$

12. What should be subtracted from the sum of $\left(\frac{2}{5}+\frac{3}{4}+\frac{1}{3}\right)$ to get $\frac{1}{2}$?

13. **Simplify –**

(a) $\frac{13}{6}+\left(\frac{-2}{3}\right)+\left(\frac{-5}{6}\right)+\frac{11}{9}+\frac{1}{3}+\left(\frac{-2}{9}\right)$ (b) $\frac{-1}{3}+\frac{10}{7}+\left(\frac{-1}{6}\right)+\left(\frac{-5}{7}\right)+\frac{1}{12}+\frac{3}{4}$

14. **Write true or false –**

(a) If $|a| = 0$, then $a = 0$

(b) If $|a| = |b|$, then $a = b$

(c) If $\frac{a}{b} < \frac{c}{d}$, then $\frac{|a|}{|b|} < \frac{|c|}{|d|}$

15. **Fill in the blank space with one of the following symbols. >, < or = :**

(a) If $\frac{-5}{7} < \frac{6}{13}$, then $\frac{|-5|}{|7|}$ .... $\frac{|6|}{|13|}$

(b) If $\frac{-5}{5} < \frac{-5}{6}$, then $\frac{|-5|}{|5|}$ .... $\frac{|-5|}{|6|}$

(c) If $\frac{-7}{8} < \frac{21}{24}$, then $\frac{|-7|}{|8|}$ .... $\frac{|21|}{|24|}$

(d) If $\frac{-9}{-10} > \frac{8}{9}$, then $\frac{|-9|}{|-10|}$ .... $\frac{|8|}{|9|}$

(e) If $\frac{-1}{2}+\frac{-3}{2}=\frac{-4}{2}$, then $\frac{|-1|}{|2|}+\frac{|-3|}{|2|}$ .... $\frac{|-4|}{|2|}$

Multiplication of Rational Numbers

Product of Rational Numbers–

$\frac{a}{b} \times \frac{c}{d}$ = a rational number if $\frac{a}{b}$ and $\frac{c}{d}$ are rational numbers : (closure property)

Example 1 : $\frac{-2}{3} \times \frac{5}{7} = \frac{-10}{21}$ which is a rational number.

If $\frac{a}{b}$ and $\frac{c}{d}$ rational numbers then, $\left(\frac{a}{b} \times \frac{c}{d}\right) = \left(\frac{c}{d} \times \frac{a}{b}\right)$ Commutative property of multiplication.

According to this property, the rational numbers can be multiplied in any order.

Example 2 : $\left(\frac{5}{7}\times\frac{3}{4}\right)=\left(\frac{3}{4}\times\frac{5}{7}\right)=\frac{15}{28}$

$\left(\frac{a}{b}\times\frac{c}{d}\right)\times\frac{e}{f}=\frac{a}{b}\times\left(\frac{c}{d}\times\frac{e}{f}\right)$ – Associative property of multiplication.

This property states that while multiplying three or more rational numbers they can be grouped in any order.

Example 3 : $\left[\left(\frac{-5}{2}\right)\times\left(\frac{-7}{4}\right)\right]\times\frac{1}{3}=\frac{-5}{2}\times\left[\left(\frac{-7}{4}\right)\times\left(\frac{1}{3}\right)\right]=\frac{35}{24}$

$\frac{a}{b}\times 1=1\times\frac{a}{b}=\frac{a}{b}$ – Multiplicative property by 1.

When a rational number is multiplied by 1 the product is the rational number itself.

Example 4 : $\frac{3}{7}\times 1=1\times\frac{3}{7}=\frac{3}{7}$

$\frac{a}{b}\times 0=0\times\frac{a}{b}=0$ – Multiplicative property of O.

This law states that when a rational number is multiplied by 0, the product is 0 .

Example 5 : $\frac{9}{11}\times 0=0\times\frac{9}{11}=0$

$\frac{a}{b}\times\left(\frac{c}{d}+\frac{e}{f}\right)=\left(\frac{a}{b}\times\frac{c}{d}\right)+\left(\frac{a}{b}\times\frac{e}{f}\right)$ – Distributive property of multiplication over addition.

Example 6 : $\frac{-3}{4}\times\left(\frac{2}{3}+\frac{-5}{6}\right)=\left(\frac{-3}{4}\times\frac{2}{3}\right)+\left(\frac{-3}{4}\times\frac{-5}{6}\right)=\frac{3}{24}=\frac{1}{8}$

$\frac{a}{b}\times\left(\frac{c}{d}-\frac{e}{f}\right)=\left(\frac{a}{b}\times\frac{c}{d}\right)-\left(\frac{a}{b}\times\frac{e}{f}\right)$ – Distributive property of multiplication over subtraction.

Example 7 : $\frac{1}{2}\times\left(\frac{5}{9}-\frac{2}{9}\right)=\left(\frac{1}{2}\times\frac{5}{9}\right)-\left(\frac{1}{2}\times\frac{2}{9}\right)$

$=\frac{5}{18}-\frac{2}{18}=\frac{5-2}{18}=\frac{3}{18}=\frac{1}{6}$

$\frac{a}{b}\times\frac{b}{a}=1$ – (existence of multiplicative inverse or reciprocal).

The multiplicative inverse of rational number $\frac{a}{b}$ is $\frac{b}{a}$.

Example 8 : What is then multiplication inverse of $\frac{3}{5}$?

$\frac{\cancel{3}^{1}}{\cancel{5}_{1}}\times\frac{\cancel{5}^{1}}{\cancel{3}_{1}}=\frac{1}{1}\times\frac{1}{1}=1$

Solved Example : Verify the following :

(i) $\left(\frac{8}{15}\times\frac{-3}{16}\right)=\left(\frac{-3}{16}\times\frac{8}{15}\right)$

(ii) $\frac{2}{3}\times\left(\frac{6}{7}\times\frac{-14}{15}\right)=\left(\frac{2}{3}\times\frac{6}{7}\right)\times\frac{-14}{15}$

(iii) $\frac{5}{6}\times\left(\frac{-4}{5}+\frac{-7}{10}\right)=\left(\frac{5}{6}\times\frac{-4}{5}\right)+\left(\frac{5}{6}\times\frac{-7}{10}\right)$

Solution : (i) $\left(\frac{8}{15}\times\frac{-3}{16}\right)=\left(\frac{-3}{16}\times\frac{8}{15}\right)$

$$= \left(\frac{1\times -1}{5\times 2}\right) = \left(\frac{1\times -1}{5\times 2}\right)$$

$$= \frac{-1}{10} = \frac{-1}{10}$$

LHS = RHS

$$\# \quad \left(\frac{8}{15}\times\frac{-3}{16}\right) = \left(\frac{-3}{16}\times\frac{8}{15}\right)$$

(ii) $$\frac{2}{3}\times\left(\frac{6}{7}\times\frac{-14}{15}\right) = \left(\frac{2}{3}\times\frac{6}{7}\right)\times\frac{-14}{15}$$

$$= \frac{2}{3}\times\frac{-12}{15} = \frac{4}{7}\times\frac{-14}{15}$$

$$= \frac{-8}{15} = \frac{-8}{15}$$

LHS = RHS Hence verified.

(iii) $$\frac{5}{6}\times\left(\frac{-4}{5}+\frac{-7}{10}\right) = \left(\frac{5}{6}\times\frac{-4}{5}\right)+\left(\frac{5}{6}\times\frac{-7}{10}\right)$$

$$= \frac{5}{6}\times\left(\frac{-8(-7)}{10}\right) = \frac{-20}{30}+\frac{-35}{60}$$

$$= \frac{5}{6}\times\frac{-15}{10} = \frac{-40+(-35)}{60}$$

$$= \frac{-5}{4} = \frac{-75}{60}$$

$$= \frac{-5}{4} = \frac{-5}{4}$$

LHS = RHS. Hence verified.

Exercise 2.4

1. Verify the following and state the laws used.

(a) $\frac{-17}{8}\times\frac{-11}{7} = \frac{-11}{7}\times\frac{-17}{8}$

(b) $\left(\frac{-2}{5}\times\frac{7}{11}\right)\times\frac{-11}{5} = \frac{-2}{5}\times\left(\frac{7}{11}\times\frac{-11}{5}\right)$

(c) $\frac{-1}{2}\times\left(\frac{-5}{6}\times\frac{7}{8}\right) = \left(\frac{-1}{2}\times\frac{-5}{6}\right)\times\frac{7}{8}$

(d) $\frac{-16}{9}\times 1 = 1\times\frac{-16}{9} = \frac{-16}{9}$

(e) $\frac{-11}{19}\times\frac{19}{-11} = \frac{19}{-11}\times\frac{-11}{19} = 1$

(f) $\frac{7}{5}\times 0 = 0$

2. Answer then following question in short –

(a) What is the product of a rational number and its reciprocal?

(b) Does '0' have a reciprocal?

(c) What are the reciprocal of 1 and –1 respectively?

(d) Can zero be a reciprocal y/x , where x=0?

(e) What is the multiplicative reciprocal of a positive rational number ‘a’?

(f) What is the multiplicative reciprocal of a negative rational number ‘–a’?

3. Find the products –

(a) $\frac{5}{-18} \times \frac{-9}{20}$ (b) $\frac{-13}{15} \times \frac{-25}{26}$ (c) $\frac{16}{-21} \times \frac{14}{5}$ (d) $\frac{-7}{6} \times 24$

(e) $\frac{7}{24} \times (-48)$ (f) $\frac{-13}{5} \times (-10)$ (g) $\frac{3}{-5} \times \frac{-7}{8}$ (h) $\frac{-9}{2} \times \frac{5}{4}$

4. Fill in the blanks –

(a) $\frac{-21}{17} \times \frac{18}{35} = \frac{18}{35} \times$............. (b) $28 \times \frac{-7}{19} = \frac{-7}{19} \times$.............

(c) $\left(\frac{15}{7} \times \frac{-21}{10}\right) \times \frac{-5}{6} =$...............$\times \left(\frac{-21}{10} \times \frac{-5}{6}\right)$ (d) $\frac{-12}{7} \times \left(\frac{4}{15} \times \frac{25}{-19}\right) = \left(\frac{-12}{7} \times \frac{4}{15}\right) \times$.............

5. Verify the following –

(a) $\left(\frac{3}{4} \times \frac{1}{2}\right) \times \frac{3}{7} = \frac{3}{4} \times \left(\frac{1}{2} \times \frac{3}{7}\right)$ (b) $\left(\frac{-5}{6} \times \frac{-2}{5}\right) \times \frac{3}{7} = \frac{-5}{6} \times \left(\frac{-2}{5} \times \frac{3}{7}\right)$

(c) $\frac{7}{8} \times \left(\frac{2}{4} + \frac{4}{5}\right) = \left(\frac{7}{8} \times \frac{2}{4}\right) + \left(\frac{7}{8} \times \frac{4}{5}\right)$ (d) $\frac{-3}{7} \times \left(\frac{7}{8} + \frac{-5}{12}\right) = \left(\frac{-3}{7} \times \frac{7}{8}\right) + \left(\frac{-3}{7} \times \frac{-5}{12}\right)$

6. Simplify using the properties of multiplication over addition and multiplication over subtraction of rational numbers.

(a) $\frac{-3}{8} \times \left(\frac{4}{7} + \frac{-11}{7}\right)$ (b) $\frac{-2}{5} \times \left(\frac{3}{8} - 25\right)$ (c) $\frac{7}{4} \times \left(\frac{5}{8} + \frac{1}{2}\right)$

7. Let a, b and c be three rational numbers having the values $a = \frac{-1}{3}$, $b = \frac{-3}{5}$ $c = \frac{-4}{9}$. Verify the following using the given values of a, b and c.

(a) $a \times b = b \times a$ (b) $a \times (b \times c) = (a \times b) \times c$

(c) $a \times (b + c) = a \times b + a \times c$ (d) $(a - b)^{-1} = a^{-1} - b^{-1}$ is false

(e) $(a \times b)^{-1} = a + b^{-1}$ is false (f) $|a^{-1}| = |a|^{-1}$

(g) $|b^{-1}| = |b|^{-1}$ (h) $|c^{-1}| = |c|^{-1}$

(power of –1 is a sign of reciprocal, | | is a sign for finding absolute value)

8. What are the properties of multiplication involved in the equation $7 \times \frac{1}{7} x = x$?

9. Name the properties involved in the following –

$42 \times \frac{1}{3} = (14 \times 3) \times \frac{1}{3} = (3 \times \frac{1}{3}) \times 14 = 1 \times 14 = 14$

10. Find x if x is a rational number and $x \times x = x$

11. What are the two rational numbers, which are reciprocals of themselves ?

12. What is the reciprocal of x if $x \neq 0$?

13. Simplify – $\left[\left(\frac{2}{9}\right)^{-1}\right]^{-1}$

Division of Rational Numbers

(a) $\frac{a}{b} \div \frac{c}{d} \neq \frac{c}{d} \div \frac{a}{b}$

(b) $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$

(c) $\frac{a}{b} \div \frac{c}{d}$, $\frac{a}{b}$ =dividend $\frac{c}{d}$ = divisor, result quotient

Solved Example 1: Divide $\frac{36}{16}$ by $\frac{9}{8}$

Solution : $\frac{36}{16} \div \frac{9}{8} = \frac{\overset{4\;2}{\cancel{36}}}{\underset{1\;2}{\cancel{16}}} \times \frac{\overset{1}{\cancel{8}}}{\underset{1}{\cancel{9}}} = 2$

Solved Example 2 : Divide $\frac{8}{23}$ by 1.

Solution : $\frac{8}{23} \div 1 = \frac{8}{23} \times \frac{1}{1} = \frac{8}{23}$

Solved Example 3 : Divide $\frac{5}{9}$ by 0.

Solution $\frac{5}{9} \div 0$, not defined.

Facts to Know

- Babylonians developed tales of reciprocals. To divide a by b, they wrote a÷b = a: (1/b) (:=ratio =x)

Exercise 2.5

1. Divide –

(a) -18 by $\frac{-36}{37}$ (b) $\frac{-24}{50}$ by $\frac{-4}{75}$ (c) $\frac{-3}{16}$ by $\frac{-15}{18}$

(d) $\frac{10}{33}$ by $\frac{-2}{11}$ (e) $\frac{7}{18}$ by $\frac{-14}{51}$ (f) $\frac{5}{12}$ by 15

2. State whether the following are true or false –

(a) $\frac{-7}{24} \div \frac{3}{-16} = \frac{3}{-16} \div \frac{-7}{24}$

(b) $\frac{-4}{3} \div \frac{-8}{9} = \frac{-8}{9} \div \frac{-4}{3}$

(c) $-12 \div \frac{3}{4} = \frac{3}{4} \div -12$

(d) $\frac{-22}{7} \div \left(\frac{9}{14} - \frac{5}{21}\right) = \left(\frac{-22}{7} \div \frac{9}{14}\right) - \left(\frac{-22}{7} \div \frac{5}{21}\right)$

(e) $\left(\frac{9}{5} + \frac{4}{25}\right) \div \left(\frac{-5}{7}\right) = \frac{9}{5} \div \left(\frac{-5}{7}\right) + \frac{4}{25} \div \left(\frac{-5}{7}\right)$

(f) $\left(\frac{9}{20} - \frac{17}{40}\right) \div \frac{10}{3} = \left(\frac{9}{20} \div \frac{10}{3}\right) - \left(\frac{17}{40} \div \frac{10}{3}\right)$

3. Fill in the blanks –

(a) $\frac{-2}{9} \div \frac{-2}{9} =$............ (b) $\frac{-4}{15} \div (-1) =$.............. (c) $\frac{12}{13} \div$ .... $= -1$

(d) $\frac{6}{7} \div$ .... $= \frac{6}{7}$ (e) ....$\div 1 = \frac{-9}{17}$ (f) $\frac{-11}{25} \div$ .... $= 1$

4. Simplify –

(a) $\frac{4}{1} \div \frac{-5}{12}$ (b) $-9 \div \frac{-7}{18}$ (c) $\frac{-12}{7} \div (-18)$

(d) $\frac{-1}{10} \div \frac{-8}{5}$ (e) $\frac{-16}{35} \div \frac{15}{14}$ (f) $\frac{-65}{14} \div \frac{13}{7}$

5. The product of two numbers is 6. If one number is 12, find the other number.

6. The product of two numbers is $\frac{-20}{9}$. If one number is $\frac{-4}{3}$, find the other number.

7. By what number should we multiply $\frac{-20}{63}$ to get $\frac{-5}{7}$?

8. By what number should $\frac{-8}{39}$ be multiplied to obtain $\frac{1}{26}$?

9. Divide the sum of $\frac{-12}{7}$ and $\frac{13}{5}$ by the product of $\frac{1}{-2}$ and $\frac{-31}{7}$.

10. Divide the sum of $\frac{8}{3}$ and $\frac{65}{12}$ by their difference.

11. Write true or false –

(a) We can divide 11 by 0.

(b) Rational numbers are always associative under division.

(c) Rational numbers are always commutative under division.

(d) Rational numbers are closed under division.

Finding Rational Numbers Between Two Rational Numbers

Method -1

Let p and q be two rational numbers such that $p < q$. If we wish to find out rational numbers between p and q, which are q^1, q^2, q^3 and q^4, then –

(i) $q^1 = ½(p + q)$

(ii) $q^2 = ½(q^1 + p)$

(iii) $q^3 = ½(q^2 + p)$

(iv) $q^4 = ½(q^3 + p)$

Solved Example : For any rational number $\frac{p}{q}$.

Solution : $\frac{p}{q} \div \frac{p}{q} = 1;$

$$\frac{p}{q} \div \left(-\frac{p}{q}\right) = -1;$$

$$\frac{-p}{q} \div \frac{p}{q} = -1$$

Method -2

Solve Example : Find four rational numbers between $\frac{1}{6}$ and $\frac{1}{3}$.

Solution : Since we have to find four rational numbers. Therefore, multiply both the numbers in such a way that their denominators are equal.

$$\frac{1}{6}\times\frac{5}{5}=\frac{5}{30}$$

$$\frac{1}{3}\times\frac{10}{10}=\frac{10}{30}$$

$$\frac{1}{6}<\frac{6}{30}<\frac{7}{30}<\frac{8}{30}<\frac{9}{30}<\frac{1}{3}$$

Solve Example : Find (a) 4 rational numbers between $\frac{1}{3}$ and $\frac{1}{2}$

(b) 40 rational numbers between $\frac{1}{3}$ and $\frac{1}{2}$

(c) 100 rational number between $\frac{1}{3}$ and $\frac{1}{2}$

Solution : (a) $\frac{1}{3}<\frac{1}{2}$, there are infinite rational numbers between $\frac{1}{3}$ and $\frac{1}{2}$.

$$\frac{1}{3}\times\frac{10}{10}=\frac{10}{30},\qquad \frac{1}{2}\times\frac{15}{15}=\frac{15}{30}$$

$$\frac{10}{30}<\frac{11}{30}<\frac{12}{30}<\frac{13}{30}<\frac{14}{30}<\frac{15}{30}$$

Solution : (b) Find 40 rational numbers between $\frac{1}{3}$ and $\frac{1}{2}$.

:

$$\frac{1}{3}\times\frac{100}{100}=\frac{100}{300},\quad \frac{1}{2}\times\frac{150}{150}=\frac{150}{300}$$

$\frac{1}{3}=\frac{100}{300}$ and $\frac{1}{2}=\frac{150}{300}$, there is difference of 50 rational numbers between $\frac{100}{300}$ or $\frac{1}{3}$ and $\frac{1}{2}$ or $\frac{150}{300}$. 150 the numbers 101,102,103, ... 149 lie, between 100 and 150 therefore

$$\frac{1}{2}<\frac{101}{300}<\frac{102}{300}<\frac{103}{300}\text{..........}<\frac{149}{300}<\frac{1}{3}$$

$$\frac{1}{3}\times\frac{500}{500}=\frac{500}{1500},\quad \frac{1}{2}\times\frac{750}{750}=\frac{750}{1500}$$

(c) The 100 rational numbers between $\frac{1}{3}$ and $\frac{1}{2}$ are

$$\frac{1}{3}<\frac{501}{1500}<\frac{502}{1500}<\frac{503}{1500}\text{..........}<\frac{749}{1500}<\frac{1}{2}$$

Exercise 2.6

1. Insert 10 rational numbers between $\frac{-5}{13}$ and $\frac{6}{13}$.
2. Find four rational numbers between –1 and –½.
3. Find three rational number between –3 and 3.

4. Find a rational number between p and q if :

(a) $p = \frac{-4}{9}$, $q = \frac{11}{6}$ (b) $p = \frac{-5}{6}$, $q = \frac{-2}{5}$

(c) $p = \frac{1}{8}$, $q = \frac{7}{12}$ (d) $p = \frac{1}{5}$, $q = \frac{1}{4}$

Exercise 2.7

1. Two pieces of lengths $4\frac{3}{5}$ and $2\frac{3}{10}$ have been cut off from a rope of 11m. Find the length of the remaining rope.
2. A container of sugar weighs $40\frac{1}{6}$ kg. If the weight of the container is $13\frac{3}{4}$ kg. Find the weight of sugar in it.
3. Find the cost of 35 kg of oranges if one kg of orange costs Rs. $46\frac{3}{4}$.
4. Find the area of a rectangular park which is $30\frac{3}{5}$ m long and $20\frac{2}{3}$ m wide.
5. A rope has been cut into 26 pieces. The total length of the rope is $71\frac{1}{2}$ m. Find the length of one piece of rope.
6. A rectangular room is $5\frac{7}{10}$ m. wide. Its area is $68\frac{2}{5}$ m^2. Find the length of the room.
7. The product of two fractions is $7\frac{3}{5}$. If one fraction is $4\frac{3}{7}$. Find the other fraction.
8. In a factory $\frac{5}{8}$ of the workers are women. There are 240 men. Find the number of people working in the factory.
9. How much distance will a bus cover in $7\frac{1}{2}$ hours if it is moving at a speed of $40\frac{2}{5}$ km/hr ?
10. Mr. Kohli sets out for his office with ₹ 80. He spend ₹ $5\frac{1}{2}$ as bus fare. ₹$13\frac{3}{5}$ on snacks and $4\frac{2}{5}$ on repair of his shoes. How much money was left with him when he returned back home ?
11. Japneet gave 9 of grapes to the guests, 40 grapes were left in the bowl. How many grapes did the bowl contain ?
12. On the Independence day celebrations $\frac{2}{7}$ of the audience were seated. While 15000 were standing. Find the total number of the audience.
13. If jane earns ₹ 16000 per month. She spends $\frac{1}{4}$ of her salary on food, $\frac{1}{10}$ of her salary is sent to her parents, she spends $\frac{1}{4}$ of her salary on conveyance. How much is she able to save each month?
14. Aman gets ₹ 300 as pocket money each month he spends $\frac{1}{3}$ of his pocket money to eat fast foods. ¼ of the money is spent on chocolates. How much money is left with him.

Summary of Facts Discussed

1. A number of the form of $\frac{p}{q}$, where p and q are integers and $q \neq o$ is called a rational number.

2. Properties of rational numbers can be discussed on the four basic operations of mathematics. They are :
   (i) Addition (+) (ii) Subtraction (–)
   (iii) Multiplication (×) (iv) Division (÷)

3. The absolute value of a rational number is equal to its numerical value, which symbolically expressed as $\frac{a}{b}$ if a and b are integers.

4. **Closure properties of rational numbers :**
   The rational properties are closed under all the basic properties of operations. That is if $\frac{a}{b}$ and $\frac{c}{d}$ are rational numbers then.
   a. $\frac{a}{b}+\frac{c}{d}$ is a rational number.
   b. $\frac{a}{b}-\frac{c}{d}$ is a rational number.
   c. $\frac{a}{b}\times\frac{c}{d}$ is a rational number.
   d. $\frac{a}{b}\div\frac{c}{d}$ is a rational number if $(\frac{c}{d}\neq 0)$

5. **Commutative properties :**
   a. $\left(\frac{a}{b}+\frac{c}{d}\right)=\left(\frac{c}{d}+\frac{a}{b}\right)$ Commutative law of addition.
   b. $\left(\frac{a}{b}\times\frac{c}{d}\right)=\left(\frac{c}{d}\times\frac{a}{b}\right)$ Commutative law of multiplication.
   c $\left(\frac{a}{b}-\frac{c}{d}\right)\neq\left(\frac{c}{d}-\frac{a}{b}\right)$
   d. $\left(\frac{a}{b}\div\frac{c}{d}\right)\neq\left(\frac{c}{d}\div\frac{a}{b}\right)$

   Under operations of subtraction and division the rational numbers are not commutative.

6. **Associative properties :**
   Associative law states that rational numbers can be grouped in the desired way under the operations of multiplications and additions.
   a. $\left(\frac{a}{b}+\frac{c}{d}\right)+\frac{e}{f}=\frac{a}{b}+\left(\frac{c}{d}+\frac{e}{f}\right)$ – Associative law of addition
   b. $\left(\frac{a}{b}\times\frac{c}{d}\right)\times\frac{e}{f}=\frac{a}{b}\times\left(\frac{c}{d}\times\frac{e}{f}\right)$ – Associative law of multiplication
   c. $\left(\frac{a}{b}-\frac{c}{d}\right)-\frac{e}{f}=\frac{a}{b}-\left(\frac{c}{d}-\frac{e}{f}\right)$
   d. $\left(\frac{a}{b}\div\frac{c}{d}\right)\div\frac{e}{f}=\frac{a}{b}\div\left(\frac{c}{d}\div\frac{e}{f}\right)$

7. Distributive Properties :

$\frac{a}{b}\times\left(\frac{c}{d}+\frac{e}{f}\right)=\frac{a}{b}\times\frac{c}{d}+\frac{a}{b}\times\frac{e}{f}$ – Distributive property of multiplication over addition.

$\frac{a}{b}\times\left(\frac{c}{d}-\frac{e}{f}\right)=\frac{a}{b}\times\frac{c}{d}-\frac{a}{b}\times\frac{e}{f}$ – Distributive property of multiplication over subtraction.

8. Identity properties :

a. $\frac{a}{b}+0=0+\frac{a}{b}=\frac{a}{b}$ – zero additive identity.

b. $\frac{a}{b}-0=\frac{a}{b}$ – zero subtractive identity.

c. $\frac{a}{b}\times 0=0\times\frac{a}{b}=0$ – zero multiplicative identity.

d. $\frac{a}{b}\times 1=1\times\frac{a}{b}=\frac{a}{b}$ – multiplication identity.

e. $\frac{a}{b}\div 0$ – not defined

9. Inverse Identities :

a. $\frac{a}{b}+\frac{-a}{b}=\left(\frac{a}{b}\right)-\frac{a}{b}=0$ – Additive inverse.

b. $\frac{a}{b}\times\frac{a}{b}=1$ or $\frac{a}{b}\times\left(\frac{b}{a}\right)^{-1}\times\frac{a}{b}=1\times\frac{1}{\frac{a}{b}}$ – Multiplicative inverse or reciprocal of $\frac{a}{b}$.

c. $\left[\left(\frac{a}{b}\right)^{-1}\right]^{-1}=\frac{a}{b}$ – Reciprocal of the reciprocal of any number is the number itself.

10. If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers then, $\frac{1}{2}\left(\frac{a}{b}+\frac{c}{d}\right)$ is a rational number lying between $\frac{a}{b}$ and $\frac{c}{d}$.

11. There are infinite numbers of rational numbers between $\frac{a}{b}$ and $\frac{c}{d}$.

Points to Remember :

- The integer p in the rational number $\frac{p}{q}$ is called its numerator and q is called its denominator.
- A rational number is said to be positive if its numerator and denominator are either both positive integers or both negative integers.
- Rational numbers are closed under addition, subtraction, multiplication, and division.
- Rational numbers are commutative and associative under addition and multiplication.
- Zero is the additive identity and 1 is the multiplicative identity for rational numbers.

- For a given rational number $\left(\frac{-p}{q}\right)$, there exists on additive inverse $\frac{p}{q} + \left(\frac{-p}{q}\right) = 0$ such that $\frac{p}{q} + \left(\frac{-p}{q}\right) = 0$
- For a given rational number $\frac{p}{q} \times \frac{q}{p} = 1$, such that
- In rational numbers, multiplication distributes over addition and subtraction.
- Every rational number can be represented on a number line.
- On a number line, a rational number to the right is always greater than the number to its left.
- There exist infinite rational numbers between two given numbers.

EXERCISE

1. MULTIPLE CHOICE QUESTIONS (MCQs) :

Tick (✓) the correct options.

(a) Which of the following is the additive inverse of $\frac{5}{9}$?

(i) $\frac{-5}{9}$ ☐ (ii) $\frac{9}{5}$ ☐ (iii) $-\frac{9}{5}$ ☐ (iv) $\frac{1}{8}$ ☐

(b) The sum of a rational number and its additive inverse is always –

(i) 1 ☐ (ii) 0 ☐ (iii) greater than 1 ☐ (iv) less than 1 ☐

(c) A rational number divided by zero is –

(i) 0 ☐ (ii) 1 ☐ (iii) not defined ☐ (iv) None of these ☐

(d) The difference of $\frac{2}{3} - \frac{1}{7}$ is equal to –

(i) $\frac{-11}{21}$ ☐ (ii) $\frac{-21}{11}$ ☐ (iii) $\frac{11}{21}$ ☐ (iv) $\frac{1}{7}$ ☐

(e) The product of $\frac{3}{19}$ and its multiplication inverse is –

(i) 2 ☐ (ii) $\frac{1}{19}$ ☐ (iii) $\frac{19}{3}$ ☐ (iv) 1 ☐

(f) The sum of $\frac{5}{9} + \frac{1}{5}$ is equal to –

(i) $\frac{6}{14}$ ☐ (ii) $\frac{34}{45}$ ☐ (iii) $\frac{14}{6}$ ☐ (iv) $\frac{45}{34}$ ☐

(g) Which one is the commutative properties of rational number?

(i) $\frac{p}{q} + \frac{q}{p} = \frac{p}{q} + \frac{p}{q}$ ☐ (ii) $\frac{p}{q} + 0 = \frac{p}{q} - \frac{p}{q}$ ☐

(iii) $\frac{p}{q} + \frac{r}{s} = \frac{r}{s} + \frac{p}{q}$ ☐ (iv) $\frac{p}{q} + -1 = 1 - \frac{p}{q}$ ☐

(h) Multiplicative inverse of $\frac{-15}{29}$ is –

(i) $\frac{15}{19}$ ☐ (ii) $\frac{29}{15}$ ☐ (iii) $\frac{-15}{-19}$ ☐ (iv) $\frac{29}{-15}$ ☐

2. **Name the property of a addition used in each of the following :**

(a) $\left(\frac{-6}{7}\right)+\left(\frac{6}{7}\right)=0=\left(\frac{6}{7}\right)+\left(\frac{-6}{7}\right)$

(b) $\left(\frac{2}{9}+\frac{3}{5}\right)$ is a rational number

(c) $\frac{22}{39}+\left(\frac{-22}{39}\right)=0$

(d) $\frac{5}{7}+\left(\frac{-9}{19}\right)=\left(\frac{-9}{19}\right)+\frac{5}{7}$

(e) $\frac{1}{6}+\left(\frac{15}{39}+\frac{2}{11}\right)=\left(\frac{1}{6}+\frac{15}{39}\right)+\frac{2}{11}$

(f) $\frac{1}{18}+0=0+\frac{1}{18}=\frac{1}{18}$

3. **Write the additive inverse of each of the following :**

(a) $\frac{3}{7}$ (b) $\frac{-5}{11}$ (c) $\frac{15}{-7}$ (d) $\frac{-7}{-3}$

(e) $2\frac{1}{5}$ (f) $\frac{18}{-21}$ (g) $\frac{5}{19}$ (h) $\frac{-18}{-23}$

4. **Write the multiplication inverse of each of the following :**

(a) $\frac{3}{5}$ (b) $\frac{-3}{5}$ (c) -7 (d) $-3\times\frac{-2}{7}$

(e) $\frac{-99}{101}$ (f) $2\frac{1}{9}$ (g) $\frac{-1}{91}$ (h) $\frac{20}{-27}$

5. **Name the property of multiplication used in each of the following :**

(a) $\frac{2}{9}\times\left(\frac{1}{7}+\frac{2}{5}\right)=\frac{2}{9}\times\frac{1}{7}+\frac{2}{9}\times\frac{2}{5}$

(b) $\frac{17}{21}\times\left(\frac{23}{45}\times\frac{18}{51}\right)=\left(\frac{17}{21}\times\frac{23}{45}\right)\times\frac{18}{51}$

(c) $\frac{3}{4}\times\left(\frac{2}{7}-\frac{3}{5}\right)=\frac{3}{4}\times\frac{2}{7}-\frac{3}{4}\times\frac{3}{5}$

(d) $\frac{78}{103}\times1=1\times\frac{78}{103}=\frac{78}{103}$

(e) $\left(\frac{-41}{67}\right)\times\frac{8}{21}=\frac{8}{21}\times\left(\frac{-41}{67}\right)$

(f) $\frac{-15}{6}\times\frac{6}{-15}=1$

6. **Simplify the following :**

(a) $\left(\frac{-8}{7}\right)\times\frac{2}{5}\times\frac{7}{15}\times\frac{1}{32}$

(b) $\frac{3}{5}\times\frac{-2}{7}+\frac{4}{35}-\frac{3}{10}\times\frac{2}{7}$

7. **Represent the following rational numbers on the number line.**

(a) -3 (b) $\frac{-3}{5}$ (c) $\frac{16}{11}$ (d) $\frac{5}{9}$ (e) $\frac{11}{15}$

HOTS

1. Nine times the reciprocal of a rational number equals 6 times the reciprocal of 17. Find the rational number.

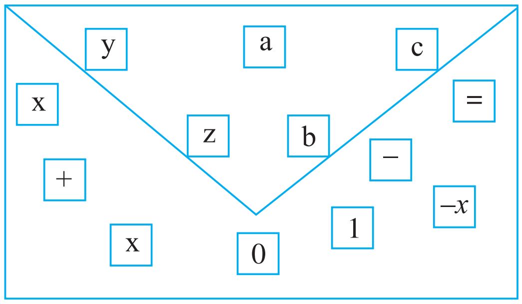
2. Which rational numbers have absolute value less than 6 ?

Lab Activity

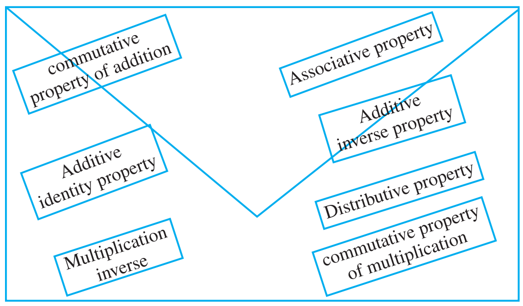
Objective : To understand the properties of rational numbers through activity.

Materials Required : Two envelopes : one envelope containing cards on which rational numbers and symbols are written and the other envelop containing strips on which properties of rational numbers are written.

Envelope 1



Envelope 2



Procedure : This game is played between two students. (Student A and Student B)

Step 1 : Student A is asked to take out a strip from envelope 2 randomly.

Step 2 : Student B is asked to choose number cards and symbol cards from Envelope 1 and demonstrate the property shown on the strip.

Step 3 : Each correct answer gets 2 marks and each wrong answer gets 1 negative mark.

Step 4 : The student who gets more marks will be judged the winner.

For example : Student A chooses the strip commutative property of addition.

Student B demonstrates the property :

| x | + | y | = | y | + | x |

3 Playing with Numbers

Introduction

Puzzles have been part of our lives since ancient days. In the ancient days puzzles were more popular. Philosophers in the ancient days were very patient thinkers and this helped them to invent new laws and identities of mathematics.

The number puzzles are based on algebraic identities. We all know the popular ancient mathematicians like Pythagoras, Pascal, Aryabhatta, Ibne Masa Alkhwarizmi. Pythagoras introduced his theorem about the right angled triangle, Pascal introduced algebra, the bases of modern mathematics. The word algebra derives its name from an Arabic word aljabr.

Ramanujam was also an Indian mathematical genius in the recent past. He introduced many numerical facts.

Example :

1. The cube of a number that ends in 0 also ends in 0. What about the cube of a number that ends in 8?
2. The cube of a number that ends in 6 also ends in 6. What about the cube of a number that ends in 3?
3. The cubes of all numbers that end in 5 also end in 5. What about the cubes of numbers that ends in 2?

The answers to questions 1, 2 and 3 are 2, 7 and 8 respectively.

4. Are the following groups of number equal?

$8 = 2^3 = (1^3 - 0^3) + (2^3 - 1^3) = (1 + 0 \times 6) + (1 + 1 \times 6) = (1 + 1 \times 0 \times 0) + (1 + 2 \times 1 \times 3)$

If you observe it carefully you will find that they are all equal. These are the way how the ancient people used to find cubes of numbers.

5. Are the following numbers divisible by 11?

(a) 345693 (b) 7204252 (c) 3240237

Solution :

(a) 345693

$(3 + 5 + 9) - (4 + 6 + 3)$

$= 17 - 13 = 4.$

The number 345693 is not divisible by 11 because the difference of sum of alternate numbers is not 0 as well as it is not a multiple of 11. Is it not a mathematical fact?

(b) 7204252

$(7 + 0 + 2 + 2) - (2 + 4 + 5)$

$= 11 - 11 = 0$

The number 7204250 is divisible by 11.

(c) 3240237

$(3+4+2+7)-(2+0+3)$

$=16-5$

$=11$

The difference of the sum of alternate numbers is divisible by 11.

Therefore the number 3240237 is also divisible by 11.

Try these

5. Write Yes or No.

(a) 399870 is divisible by 5.

(b) 999778 is not divisible 2.

(c) 46551 is divisible by 3.

(d) 357114 is divisible by 6.

(e) 7534116 is divisible by 9.

Let us now discuss the divisibility tests in detail.

If one third of one fourth of number is 15, then what is two tenth of that number?
Solution: let the number be x.
then, $\frac{1}{3}$ of $\frac{1}{4}$ of x = 15
= x = 15 x 12 = 180
So, required number = $\frac{2}{10}$ x 180 = 36

Divisibility Test For Numbers Written In Generalized From

1. Divisiblity Test By 2

Let the number be *a* if *a* is single digit number, *ab* if it is two digit number and *abc* if it is a three digit number.

In case of *a* it is at the units place...

In case of *ab*, *b* is at the units place and *a* is at the tens place. We can also write it as $10a + b$.

In case of *abc*, *c* is at the unit place, *b* at the tens place and a is at the hundreds place. We can express it as $100a + 10b + c$.

In all the three cases the digits at *a*, *b* and *c* are variables. While place values are constant.

The divisibility fact of 2 states that, a number having the digits 0, 2, 4, 6 and 8 at its units place is divisible by 2.

Therefore the numbers : 6925230, 925876, 69992, 99774 and 25558 are all divisible by 2.

2. Divisiblity Test By 3

This divisibility fact of 3 states that if the sum of all digits is divisible by 3 without a remainder then the number is also divisible by 3.

Let the number be a three digit number then $\frac{a+b+c}{3} = x$

Example 1 : A number which is divisible by 3, when divides a two digit number and gives a quotient of 12. The digit at the units place is 6. Find the digit at the tens place of its place value.

Solution : Let the number be *ab*, where $b = 6$ and $x = 12$, then

$$\frac{a+b}{3} = 12 = \frac{a+6}{3} = 12$$

$= \quad a + 6 = 12 \times 3$

$= \quad a = (12 \times 3) - 6$

$= \quad a = 36 - 6 = 30 \quad$ Hence $\frac{30+6}{3} = 12$

The number at the tens place is 3, and its place value is 30. The number $ab = 36$.

3. Divisiblity Test By 4

A natural number is divisible by 4 if the number at its units and tens place is divisible by 4.

For example, the numbers 39864, 789312, 56308, 659316, 2225920, 63928, 559732, 33536 are divisible by 4.

4. Divisiblity Test By 5

Natural numbers having 0 or 5 at it units place are divisible by 5.

Example : 1000, 205, 3930, 9995 are divisible by 5.

5. Divisiblity Test By 6

Natural numbers which are divisible 2 as well as 3 are also divisible by 6.

All those numbers which have 0, 2,4, 6 or 8 at its units place and their sum is divisible by 3 are also divisible by 6.

Example : 376122, 712632, 127236 are divisible by 3.

Solved Example : Which of the following numbers are divisible by 6.

(a) 35941 (b) 635112 (c) 73081

Solution :

(a) 35941 — Not divisible by 6 because it has 1 at its units place.

(b) 635112 — The sum of numbers = 6 + 3 + 5 + 1 + 1 + 2 = 18

The number is divisible by 6 as its sum of digits is divisible by 3. At the same time it is also divisible by 2.

(c) 73081 — Not divisible 6 as it is not divisible by 2 as well as 3.

6. Divisiblity Test By 8

A natural number is divisible by 8 if the number formed at hundreds, tens and units place is divisible by 8.

Solved Example : The numbers 34320, 2600 are divisible by 8.

7. Divisiblity Test By 9

All those natural numbers whose sum is divisible by 9 are also divisible by 9.

Solved Example : 35295, 29535 and 53259 are not divisible by 9. As their sums are not divisible by 9.

3 + 5 + 2 + 9 + 5 = 24

2 + 9 + 5 + 3 + 5 = 24

5 + 3 + 2 + 5 + 9 = 24

Example : The numbers, 32868, 35883, 87183 are divisible by 9. As their sums are 27.

3 + 2 + 8 + 6 + 8 = 27

3 + 5 + 8 + 8 + 3 = 27

8 + 7 + 1 + 8 + 3 = 27

8. Divisiblity Test By 10

The numbers which have 0 at its units place, are divisible by 10.

Example : 1000, 35930, 8925000 are divisible by 10.

Exercise 3.1

1. **Find the numbers divisible by 2.**

   67, 21, 3529, 5230, 698, 986, 4354, 5443.

2. **Which of the following numbers are divisible by 3?**

   314, 726, 814, 915, 84, 73, 42, 105

3. **Which of the following numbers are divisible by 4?**

   74, 84, 72, 364, 3536, 6328, 5316, 39014

4. **Which of the following numbers are divisible by 5?**

   395, 930, 33390, 80000, 80009, 11115, 22220, 9682, 8693, 97

5. **Which of the following numbers are divisible by 6?**

   83, 93, 49, 69, 23, 3500, 9000, 498, 558, 294, 414, 138

6. **Which of the following numbers are divisible by 8?**

   280, 552, 525, 4200, 698, 5536

7. **Which of the following numbers are divisible by 9?**

   5536, 525, 280, 698, 6282, 2520, 4200

8. **Find the numbers divisible by 10?**

   2330, 3920, 1000, 100008, 230006, 370, 50008

9. **The sum of a two digit number when divided by 3 gives a quotient of 5. The digit at its units place is 7. Find :**

   (Hints $\frac{a+b}{3} = x$)

   (a) The digit at its tens place

   (b) The place value of the digit at tens place

   (c) The number

10. The sum of the three digits of a number when divided by 3 gives a quotient of 3. The numbers at the hundreds and units places are 2 and 1. Find the number. (Hints $\frac{a+b+c}{3} = x$)

11. The sum of the numbers of a three digit number when divided by 9 gives a gives a quotient of 2. Find the number. The digit *a*,*b*,*c* are in the order of hundreds tens and ones respectively. The digits at the hundreds and tens places are 5 and 8 respectively. Find the number and the digit at its units place.

Patterns of Numbers

Are you aware of the word pattern? The word pattern means that similar numbers of similar design get repeated.

In this sub units we will discuss only number pattern.

Solved Example 1 : Fill in the blanks.

(i) $2^3 = (1^3 - 0^3) + (2^3 - 1^3)$

(ii) $3^3 = (1^3 - 0^3) + (2^3 - 1^3) + (3^3 - 2^3)$

(iii) $4^3 = (1^3 - 0^3) + (2^3 - 1^3) + (3^3 - 2^3) + (\dots\dots\dots\dots)$

Solution : $(4^3 - 3^3)$

Solved Example 2 : Fill in the blanks spaces.

(i) $1^3 = 1 + 0 \times 6$

(ii) $2^3 = (1+0\times6) + (1+1\times6)$

(iii) $3^3 = (1+0+6) + (1+1\times6) + (1+1\times6+2\times6)$

(iv) $4^3 = (1+0\times6) + (1+1\times6) + (1+1\times6+2\times6)+(\ldots\ldots\ldots\ldots)$

Solution : $(1+1\times6+2\times6+3\times6)$

Solved Example 3 : Fill in the blanks.

(i) $1^3 = (1+0\times3)$

(ii) $2^3 = (1+0\times3) + (1+2\times1\times3)$

(iii) $3^3 = (1+0\times3) + (1+2\times1\times3)\ (1+3\times2\times3)$

(iv) $4^3 = (1+0\times3) + (1+2\times1\times3) + (1+1\times2\times3) +(\ldots\ldots\ldots)$

Solution : $(1+4\times3\times3)$

Solved Example 4 : See the pattern and fill the gaps.

(I) $(111111111)^2$ = 1 2 3 4 5 6 7 8 9 8 7 6 5 4 3 2 1

(ii) $(11111111)^2$ = 1 2 3 4 5 6 7 8 7 6 5 4 3 2 1

(iii) $(1111111)^2$ = ...

(iv) $(\ldots\ldots\ldots)^2$ = 1 2 3 4 5 6 5 4 3 2 1

(v) $(11111)^2$ = ...

Solution :

(iii) 1 2 3 4 5 6 7 6 5 4 3 2 1

(iv) $(111111)^2$

(v) 1 2 3 4 5 4 3 2 1

Exercise 3.2

1. (a) Who is Japneet if she is a fraction which is equivalent to $\frac{3}{5}$. If her denominator is 25. Find her numerator.

(b) Who is Jaskaran? If he is fraction with denominator as 9, which is equivalent to $\frac{2}{3}$.

(c) Deviyani asked who is she? She is a fraction in the lowest term equivalent $\frac{77}{121}$.

2. $7^2 = 1+2+3+4+5+6+7+6+5+4+3+2+1$

6^2 = ...

5^2 = ...

3. 1111111 = $123456\times9+6+$ ....

111111 = $12345\times9+$ ....

11111 = $1234\times9+$ ....

4. $123 \times 8 + 3 = 987$

$12 \times 8 + 2 =$ ....

$1 \times 8 + 1 =$ ....

5. $4321 \times 9 - 1 = 38888$

$54321 \times 9 - 1 = 488888$

.... $\times 9 - 1 =$ ....

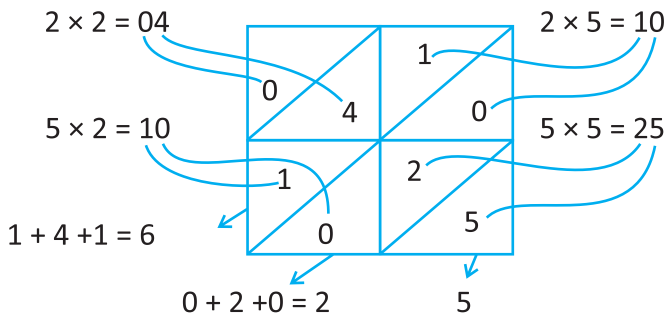
Figure Problems

Solved Example 1 : Evaluate 25^2 with the help of squares.

Solution :

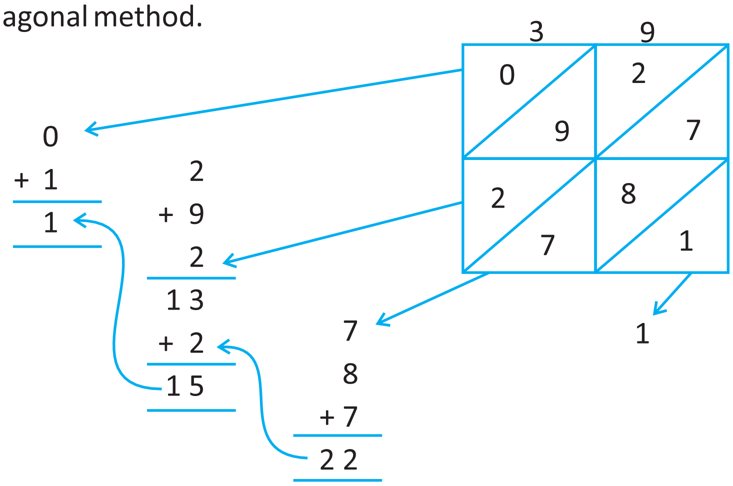
1. If the digits of the number are two. Divide the square into four sub squares. Draw diagonals of the square, as shown in the figure.
2. Write digits of the number horizontally and vertically as shown in the figure.
3. Multiply each digit on the left of the square with each digit on top of the column.
4. Divide each sub square into four more sub square with dotted lines. Write product in the diagonal dotted sub squares. If the number of the product is single, write in the lower square and put a '0' in the upper sub squire.
5. In case the product is a two digit number. Write the digit of the tens place above the diagonal and unit below the diagonal.
6. Starting below the lowest diagonal sum up the digits along the diagonal. Underline the unit digit and take carry if any to the diagonal above.
7. The unit digits which have been underlined forms the number.

$25^2 = 625$



Solved Example 2 : Evaluate 39^2 by the diagonal method.

$39^2 = 1521$



Solved Example 3 : Find square of 486.

$486^2 = 236196$

$4 \times 4 = 16$

$4 \times 8 = 32$

$4 \times 6 = 24$

$8 \times 4 = 32$

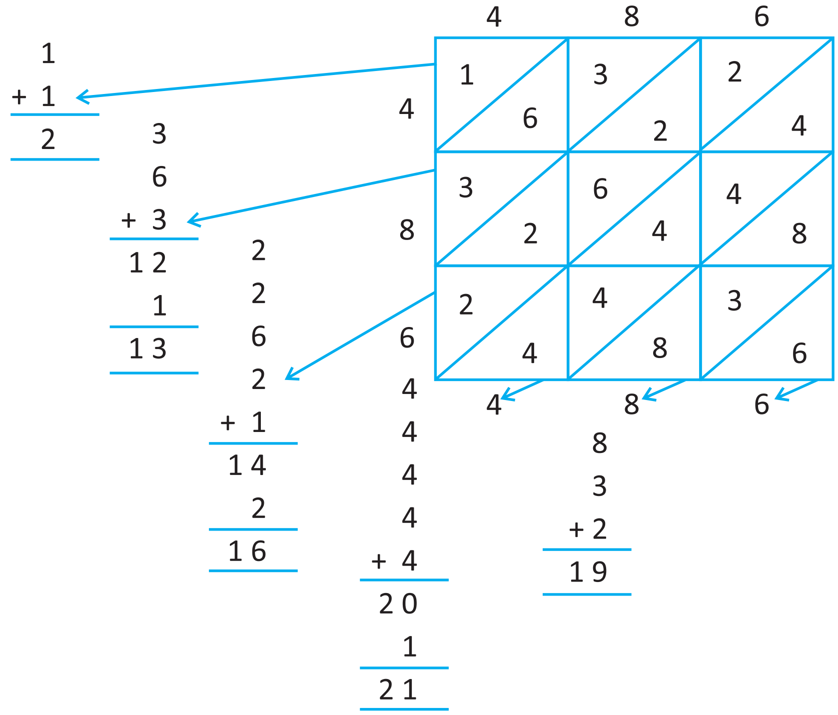
$8 \times 8 = 64$

$8 \times 6 = 48$

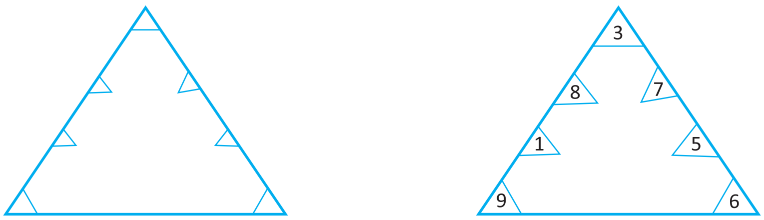
$6 \times 4 = 24$

$6 \times 8 = 48$

$6 \times 6 = 36$

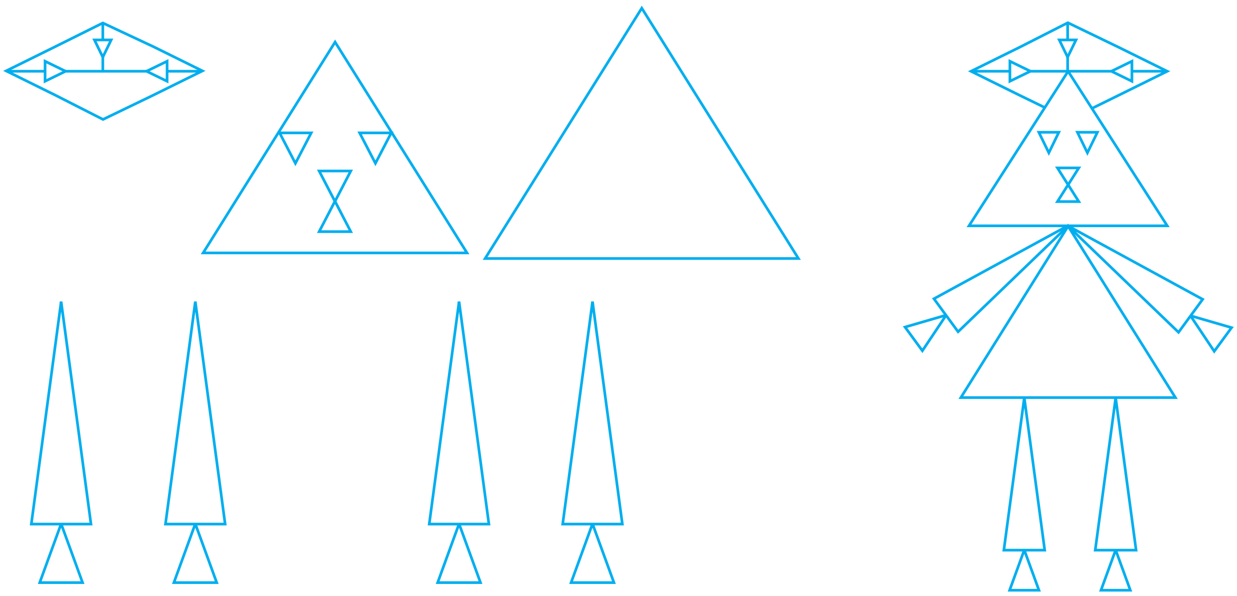


Solved Example 4 : The diagram of a triangle is given below. Fill up the vacant smaller triangles with digits 1 to 9 on each side so that the sum is 21.



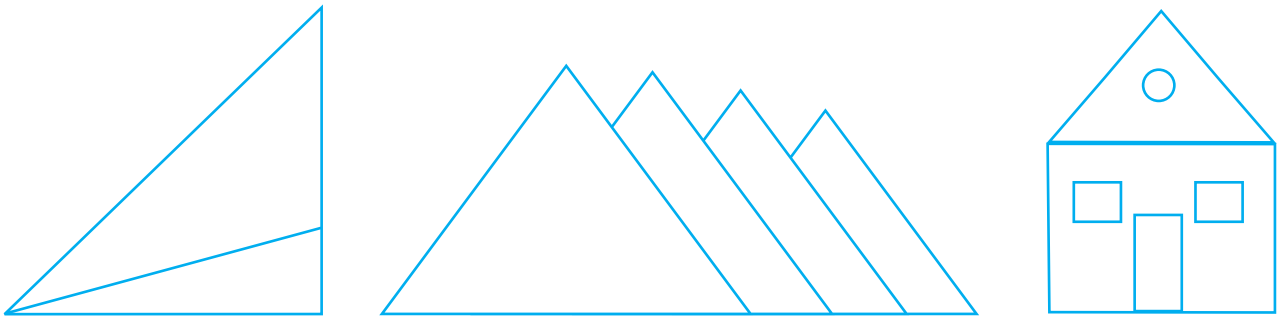
Solved Example 5 : Convert the given figures into a human being.

Solution :



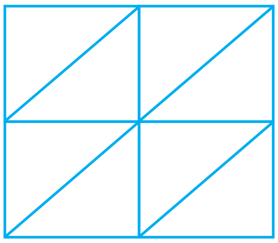
Exercise 3.3

1. Use the given figures to make a scenery.

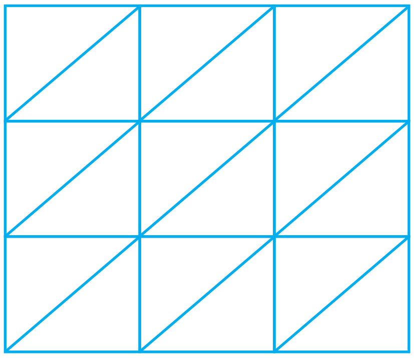


2. Find square of 35 by the diagonal method of squares.

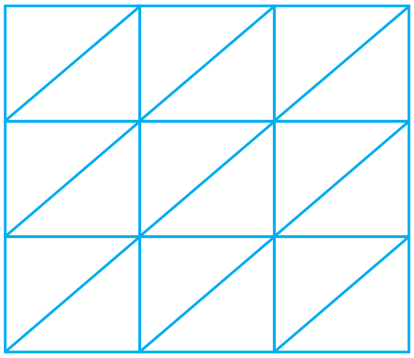
3. A square is given below.
Use this square of find 85.



4. Evaluate 232, using the given square.



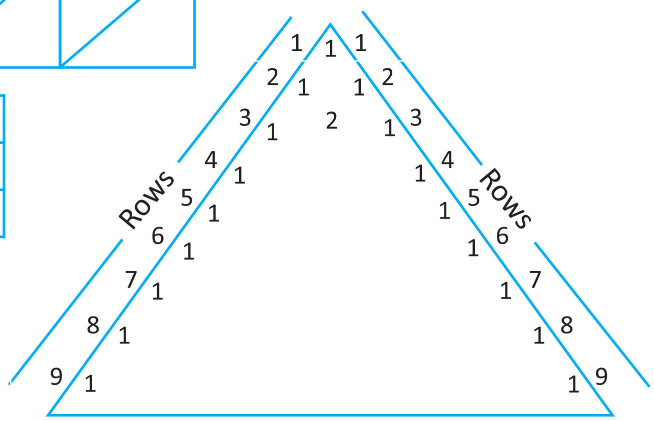
5. Evaluate 692, using the given square.



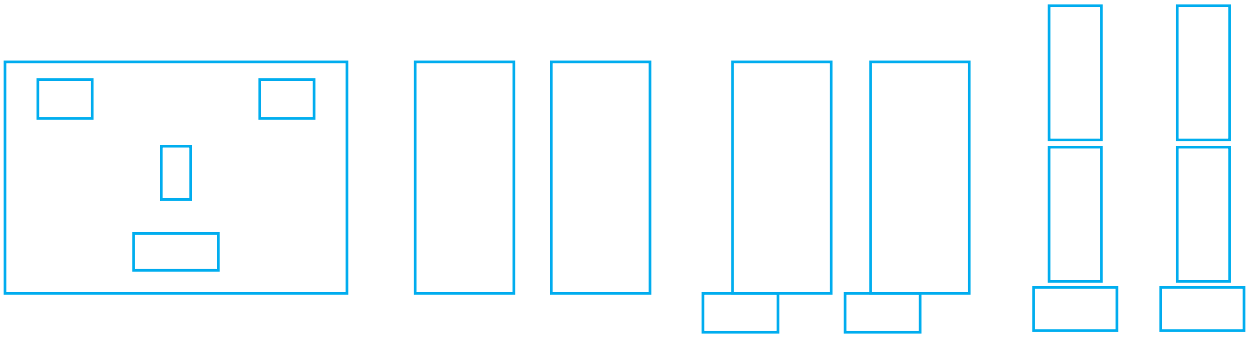
6. Complete the square in such a way that the sum of the numbers in each row, column and along the diagonals is 15.

6		8
	5	
	9	

7. Fill up the missing rows of Pascals triangles.



8. Use these rectangles to make a human being.



9. The figure shown resembles the number eight. Matchsticks have been used to make the figure. Name the sticks that you will remove.

(a) To convert it to nine

(b) To convert it to seven

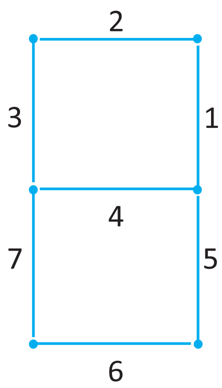
(c) To convert it to six

(d) To convert it to five

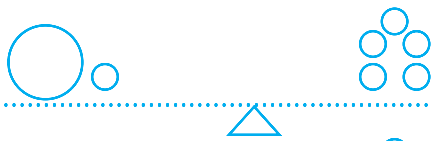
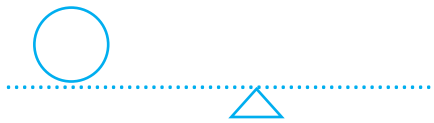
(e) To convert it to four

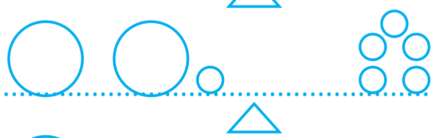
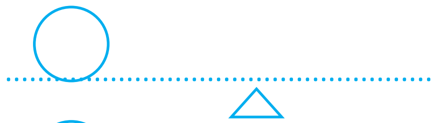
(f) To convert it to three

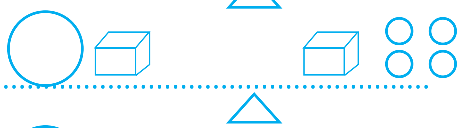
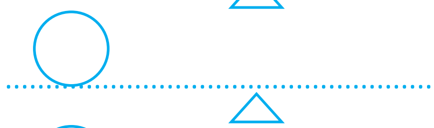
(g) To convert it to two

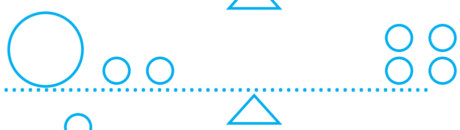
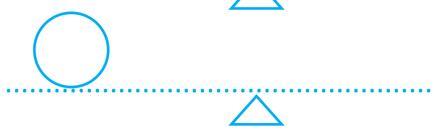


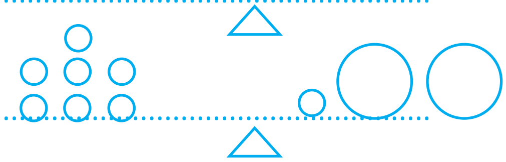
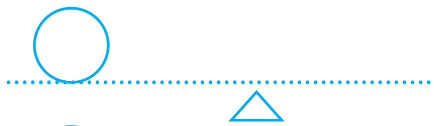
10. Study the figure. Balance the figures of each rows containing balls beads in such a way that figures (i) and (ii) of all rows are in proportion by drawing beads, balls, cubes.

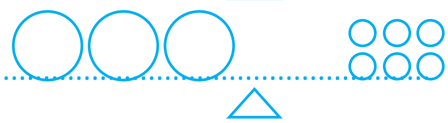
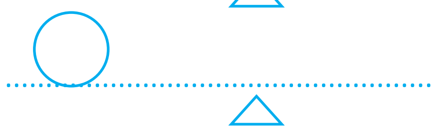
(a) (i)  (ii)  ?

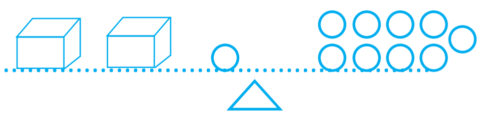
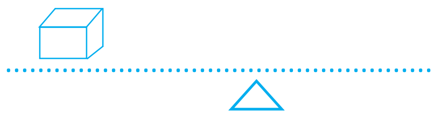
(b) (i)  (ii)  ?

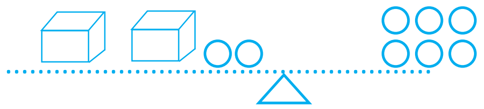
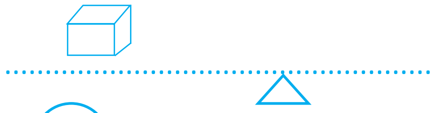
(c) (i)  (ii)  ?

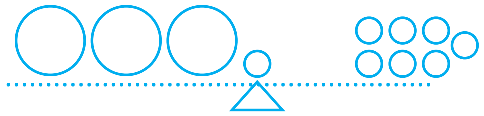
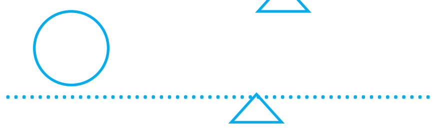
(d) (i)  (ii)  ?

(e) (i)  (ii)  ?

(f) (i)  (ii)  ?

(g) (i)  (ii)  ?

(h) (i)  (ii)  ?

(i) (i)  (ii)  ?

Magic of Numbers

As you know that all the number games and tricks are based on mathematical identical. Let us discuss a few examples.

Example : Write 1 using the digit 9 only in three ways.

Solution : (a) $\frac{\cancel{9}^{1}}{\cancel{9}_{1}} = 1$ (b) $\frac{\cancel{99}^{1}}{\cancel{99}_{1}} = 1$ (c) $\frac{\cancel{999}^{1}}{\cancel{999}_{1}} = 1$

Example : Write 10 using the digit 9 only. ***Solution :*** $9 + \frac{9}{9} = 10$

Solved Example : Reduce the number $\frac{19}{95}$ to its lowest term by cancelling just one number each from the numerator and denominator.

Solution : The usual way is $\frac{\cancel{19}}{\cancel{95}} = \frac{1}{5}$. But we have to cancel only one number. $\frac{\cancel{1}9}{\cancel{9}5} = \frac{1}{5}$

Call all numbers having a digit common between the numerator and denominator be cancelled in such a way.

Finding an Unknown Number

Solved Example : Go through the conversation between Sridhar and Deepali. Also learn how Sridhar found the unknown number.

Sridhar	:	Hello Deepali ! I can find the number that you will imagine.
Deepali	:	O.K. I have selected the number.
Sridhar	:	Add 8 to that number.
Sridhar	:	Multiply the number by 3.
Deepali	:	I have multiplied.
Sridhar	:	Subtract 6 from the multiplied number.
Deepali	:	I have subtracted.
Sridhar	:	Divided the number by 3.
Sridhar	:	Subtract the original number.
Deepali	:	I have completed both division and subtration.
Sridhar	:	You are left with 6.

= Let the number be x

= $x+8$

= $3\times(x+8)=3x+24$

= $3x+24-6=3x+18$

= $\frac{3x+18}{3}=\frac{3x}{3}+\frac{18}{3}$

$=x+6$

$x+6-x=6$

Solved Example : Replace the letters with numbers.

(I)

$$\begin{array}{r} 1\ K\ 7 \\ \times\ 3\ 1\ 2 \\ \hline M\ 5\ L \\ T\ 2\ N\ 0 \\ 3\ P\ Q\ 0\ 0 \\ \hline 3\ R\ 5\ 2\ 4 \\ \hline \end{array}$$

Solution : $2\times7=14,$ $L=4$

$2\times K+1=5$ $2K=5-1=4$ $K=\frac{4}{2}$

$K=2$

$M=2\times1$

$N=1\times7=7$

$T=1\times1=1$

$Q=3\times7=21=1$, carry 2.

$P=3\times2+2=8$

$S=M+2+q=2+2+1=5$

$R=T+P=1+8=9.$

(ii) Replace the letters with numbers.

$$\begin{array}{r} A\ 6\ B\ 4\ C \\ +\ 1\ 7\ 2\ 5\ 5 \\ \hline 9\ D\ 0\ E\ 8 \\ \hline \end{array}$$

C = 8 – 5 = 3

E = 5 + 4 = 9

B + 2 = 10 = 0 carry 1

13 = 10 – 2 8

D = 7 + 6 = 13 = 3 carry 1

A + 1 + 1 = 9

A = 9 – 2 = 7.

Solved Example : Find x in the given sum.

$$\begin{array}{r} 5\ 1\ x \\ +\ \ 1\ x\ 3 \\ \hline 7\ 0\ 1 \\ \hline \end{array}$$

Choose a number which when added to 3 gives 1 at its units place. This number when added to 1 gives zero. As at the tens place to sum we have 0. This number should be 8.

Solution : Therefore x = 8, have the sum = $\begin{array}{r} 5\ 1\ 8 \\ +1\ 8\ 3 \\ \hline 7\ 0\ 1 \\ \hline \end{array}$

Solved Example : Remove the letters of the product with numerals.

$$\begin{array}{r} N \\ \times\ N \\ \hline M\ N \\ \hline \end{array}$$

Solution : Choose a number which when multiplied by itself gives the same digit at its unit place.

Such a number is 6.

6 × 6 = 36, While the other numeral M is different from N.

Product $\begin{array}{r} N \\ \times\ N \\ \hline M\ N \\ \hline \end{array}$ $\quad$ $\begin{array}{r} 6 \\ \times\ 6 \\ \hline 3\ 6 \\ \hline \end{array}$ $\quad$ N = 6, M = 3

Solved Example : Find x and y of the sum.

$x + x + y + x = xy$

Solution : A number which when added three times gives the same numeral as 5.

Therefore 5 + 5 + 5 = 15

$x = 5,\ y = 1$

Solved Example : Express 10 using letters 9 only.

Solution : $9 + \frac{99}{99} = 9 + 1 = 10$

Solved Example : Express 8 as the sum of two numbers. Using the digits 7 only.

Solution : $7 + \frac{77}{77} = 7 + 1 = 8$

Solved Example : An old man tells his friend that he is the grandfather of *A*. But A refuses to be his grandson. Both *A* and the old man are correct. Then who is *A*?

Solution : It is true that old man is correct. But A refuses in view of the facts that she is old man's granddaughter.

Exercise 3.4

1. In the rational number $\frac{26}{65}$ if we cancel 6 to the numerator as well as the denominator. The number is reduced to its lowest form. Make two more such rational numbers in which if the numerator and the denominator are cancelled the rational number reduces to the lowest form?

2. **Replace letters with numbers.**

(a)
$$\begin{array}{r} 6\,1\,A \\ +1\,A\,3 \\ \hline 8\,0\,1 \\ \hline \end{array}$$

(b)
$$\begin{array}{r} N \\ \times N \\ \hline M\,N \\ \hline \end{array}$$

(c)
$$\begin{array}{r} 1\,Z \\ \times Z \\ \hline 7\,Z \\ \hline \end{array}$$

(d)
$$\begin{array}{r} 1\,A \\ \times A \\ \hline 9\,A \\ \hline \end{array}$$

3. **Replace the letters with the numbers.**

(a)
$$\begin{array}{r} 1\,A\,7 \\ \times 3\,1\,2 \\ \hline B\,5\,C \\ D\,2\,E\,0 \\ 3\,F\,G\,0\,0 \\ \hline 3\,H\,K\,2\,4 \\ \hline \end{array}$$

(b)
$$\begin{array}{r|l} 34 & 9\,9\,9\,6 \;(\; K\,L\,M \\ & N\,8\,0\,0 \\ \hline & 3\,1\,P\,6 \\ & Q\,0\,R\,0 \\ \hline & D\,1\,3\,T \\ & X\,Y\,Z \\ \hline & 0 \end{array}$$

4. Select a number which when reversed and added with the selected number, forms a 3-digit number and is divisible by 11.

5. A three digit number was entered into a calculator. The numbers were repeated to form a six digit number. What number will we get if this number is consecutively divided by 13, 11 and 7 ?

Points to Remember :

- A two-digit number can be represented as 10 p + q.
- A three-digit number can be represented as 100 p + 10 q + r
- Magic squares are full of fun. They can be created as 3 × 3 or as 4 × 4 grids.
- We can give codes to numbers and letters. These codes are used in intelligence operations, business and industry.
- There are very interesting patterns hidden in numbers.
- Many series of Mathematics are in vogue. The logic behind them is interesting as well as full of knowledge.
- We must learn to solve puzzles in Mathematics. Puzzles sharpen our brain.

- A number is divisible by 2 if its unit is divisible by 2.
- A number is divisible by 3 if the sum of its digits is divisible by 3.
- A number is divisible by 5 if its ones digit is either 5 or zero.
- A number is divisible by 9 if the sum of its digits is divisible by 9.
- A number is divisible by 10 if its ones digit is 0.
- The rules of divisibility are applicable to all those numbers whose number of digits is three or more.

EXERCISE

1. MULTIPLE CHOICE QUESTIONS (MCQs) :

Tick (✓) the correct options.

(a) Which of the following numbers are divisible by 2?

(i) 162 ☐ (ii) 293 ☐ (iii) 350 ☐ (iv) 467 ☐

(b) Which of the following numbers are divisible by 3 and 9 both?

(i) 81 ☐ (ii) 111 ☐ (iii) 345 ☐ (iv) 567 ☐

(c) Which of these numbers are divisible by 10?

(i) 104 ☐ (ii) 200 ☐ (iii) 650 ☐ (iv) 930 ☐

(d) Number is divisible by 3 if the sum of its digits is divisible by–

(i) 2 ☐ (ii) 4 ☐ (iii) 3 ☐ (iv) 5 ☐

2. Which are the next numbers in these series ?

(a) 2, 5, 10, 17 ?

(b) 0, 7, 26, 63 ?

3. Complete this magic square.

30			
	20	18	
16	12	10	
6			

4. In table 20 eggs to be boiled in 5 minutes. How much time would 40 eggs take to be boiled ?

5. In a certain code language, the word ARMY has been coded as DUPB. How would you be coding the word TIGER in the same code language ?

6. Write the general forms of the following numbers.

(a) 508 (b) 187

7. **Write these numbers in the numerical form.**

(a) $[100 \times 7] + [10 \times 8] + [9]$

(b) $[100 \times 6] + [10 \times 3] + [0]$

8. **Apply the four Arithmetic operators to complete this equation.**

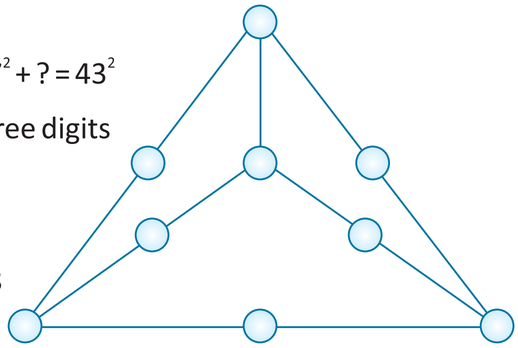
81	9	4	7	19	=	0

9. An old man had 23 buffaloes. He wanted to give $\frac{1}{2}$ of them to his eldest son, $\frac{1}{3}$ of them to the middle son and $\frac{1}{8}$ of them to the youngest son. How many buffaloes did each one of the son get ?

10. **Replace question-marks with appropriate numbers.**

(a) $5^2 + 6^2 + ? = 31^2$ (b) $6^2 + 7^2 + ? = 43^2$

11. Put the digits from 1 to 9 in the circles of the figure given here. The three digits in any lie must add up to make 12.



12. **Check whether the following numbers are divisible by 15.**

(a) 900 (b) 210 (c) 4768

13. **Check whether the following numbers are divisible by 9.**

(a) 695 (b) 5418 (c) 567

14. The number 5 q 5 is divisible by 15. What can be the probable three numbers if q is known ?

15. The number 106 r is divisible by 6. What is the value of r ?

HOTS

Create a magic square (3 × 3). Its central number has been given to you.

	15	

Hint : Make a 3 × 3 magic square for a total of 45.

Lab Activity

Objective : To use the concept of magical squares to solve multiple grids within a grid.

Materials Required : Pencil and eraser.

Procedure : Fill in the grid so that every horizontal row, every vertical column and every 3 × 3 box contains the digits 1 - 9. Without repeating the numbers in the same row, column or box.

The numbers given below are fixed.

			2	3	5			
		9				7		1
		4						6
	1						4	
7	5		1		6		9	3
	6				2		7	
3		1				5		
8		7				9		
			4	7	1			

4 Exponents and Powers

In earlier classes we have studied about how integers can be expressed in the form of power.

Example : $3^4 = 3\times3\times3\times3$ $\quad -2^4 \quad -2\times-2\times-2\times-2 = 2^4$

$-5^3 = -5\times-5\times-5=-5^3 \quad -2^3 \quad -2\times-2\times-2=-2^3$

In the number 3^4, 3 is called the **BASE** and the number 4, is called the **POWER**, or **EXPONENT** or **INDEX** of the number. The number 3^4 is read as " three raised to the power four".

For all positive integers *a* and *n* we have

$$(-a)^n = \begin{cases} a^n, \text{ When n is even} \\ -a^n, \text{ when } n \text{ is odd} \end{cases}$$

The system of writing numbers in this form is called **POWER NOTATION.**

Let *x* be a number then $x^m = x\times x\times x\times x$.....m times. In this class we will extend the system of power notation to rational numbers.

The base as well as the exponents can be positive or negative.

Positive Integral Exponent of a Rational Number

Let $\frac{p}{q}$ be any rational number and n be a positive integer, then.

$$\left(\frac{p}{q}\right)^n = \frac{p}{q}\times\frac{p}{q}\times\frac{p}{q}.... n \text{ times}$$

Thus $\left(\frac{p}{q}\right)^n = \frac{p^n}{q^n}$ for every positive integer '*n*'.

Example 1 : Simplify and evaluate the following.

(a) $\left(\frac{3}{5}\right)^3$ (b) $\left(\frac{-3}{4}\right)^4$ (c) $\left(\frac{-2}{3}\right)^5$

(a) $\left(\frac{3}{5}\right)^3 = \frac{3^3}{5^3} = \frac{3\times3\times3}{5\times5\times5} = \frac{27}{125}$

(b) $\left(\frac{-3}{4}\right)^4 = \frac{-3^4}{4^4} = \frac{-3\times-3\times-3\times-3}{4\times4\times4\times4} = \frac{81}{256}$

(c) $\left(\frac{-2}{3}\right)^5 = \frac{-2^5}{3^5} = \frac{-2\times-2\times-2\times-2\times-2}{3\times3\times3\times3\times3} = \frac{-32}{243}$

Negative Integral Exponent of a Rational Number

$$\left(\frac{p}{q}\right)^{-n} = \left(\frac{q}{p}\right)^n = \frac{q^n}{p^n} = \frac{q\times q\times q....n \text{ times}}{p\times p\times p....n \text{ times}}$$

Example 2 : Evaluate the following.

(a) $\left(\frac{2}{5}\right)^{-1}$ (b) $\left(\frac{3}{5}\right)^{-3}$ (c) 4^{-4} (d) $\left(\frac{-3}{5}\right)^3$

(a) $\left(\frac{2}{5}\right)^{-1} = \left(\frac{5}{2}\right)^{1} = \frac{5}{2}$

(b) $\left(\frac{3}{5}\right)^{-3} = \left(\frac{5}{3}\right)^{3} = \frac{5^3}{3^3} = \frac{125}{27}$

(c) $(4)^{-4} = \left(\frac{1}{4}\right)^{4} = \frac{1^4}{4^4} = \frac{1}{256}$

(d) $\left(\frac{-3}{5}\right)^{-3} = \left(\frac{5}{-3}\right)^{3} = \frac{5^3}{-3^3} = \frac{125}{-27}$

Example 3 : Evaluate —

(a) $\left(\frac{1}{2}\right)^{0}$ (b) $\left(\frac{-3}{5}\right)^{0}$ (c) $\left(\frac{3}{4}\right)^{0}$

Solution : (a) $\left(\frac{1}{2}\right)^{0} = 1$ (b) $\left(\frac{-3}{5}\right)^{0} = 1$ (c) $\left(\frac{3}{4}\right)^{0} = 1$

Expressing Rational Numbers in Exponential form to Standard form and Vice Versa

Rational numbers in exponential form can be converted to standard form and vice versa using laws of exponents.

Example 1 : Express the following rational numbers in standard form.

(a) $\left(\frac{7}{9}\right)^{3}$ (b) $\left(\frac{-5}{11}\right)^{4}$ (c) $\left(\frac{69}{72}\right)^{2}$ (d) $\left(\frac{21}{-25}\right)^{3}$

(a) $\left(\frac{7}{9}\right)^{3} = \frac{7^3}{9^3} = \frac{343}{729}$

(b) $\left(\frac{-5}{11}\right)^{4} = \frac{-5^4}{11^4} = \frac{625}{14641}$

(c) $\left(\frac{69}{72}\right)^{2} = \frac{69^2}{72^2} = \frac{4761}{5184} = \frac{529}{576}$

(d) $\left(\frac{21}{-25}\right)^{3} = \frac{21^3}{-25^3} = \frac{9261}{-15625} = \frac{-9261}{15625}$

To bring a rational number in the standard form it should be reduced to the lowest term if its denominators are negative. It should be changed to negative. It should be changed to positive.

Example 2 : Express the following rational numbers in exponential form.

(a) $\frac{-343}{729}$ (b) $\frac{-49}{64}$ (c) $\frac{81}{625}$ (d) $\frac{-32}{-243}$ (e) $\frac{16}{-49}$ (f) $\frac{8}{125}$

(a) $\frac{-343}{729} = \frac{-7^3}{9^3} = \left(\frac{-7}{9}\right)^{3}$

(d) $\frac{-32 \times -1}{-243 \times -1} = \frac{32}{243} = \frac{2^5}{3^5} = \left(\frac{2}{3}\right)^{5}$

(b) $\frac{-49}{64} = \left(\frac{-7^2}{8^2}\right) = \left(\frac{-7}{8}\right)^{2}$

(e) $\frac{16 \times -1}{-49 \times -1} = \frac{16}{49} = \frac{4^2}{7^2} = \left(\frac{4}{7}\right)^{2}$

(c) $\frac{81}{625} = \frac{3^4}{5^5} = \left(\frac{3}{5}\right)^{4}$

(f) $\frac{8}{125} = \frac{2^3}{5^3} = \left(\frac{2}{5}\right)^{3}$

$$a^{-n} = \frac{1}{a^n}$$

(i) $(5)^{-3} = \frac{1}{5^3} = \frac{1}{5\times5\times5} = \frac{1}{125}$ (ii) $(-3)^{-2} = \frac{1}{(-3)^2} = \frac{1}{-3\times-3} = \frac{1}{9}$

Example 1 : Evaluate each of the following.

(i) $5^2\times5^4$, (ii) $5^8\div5^3$ (iii) $(3^2)^3$ (iv) $\left(\frac{11}{12}\right)^3$ (v) $\left(\frac{3}{4}\right)^{-3}$

Solution : (i) $5^2\times5^4 = 5^{2+4} = 5^6 = 15625.$

(ii) $5^8\div5^3 = \frac{5}{5} = 5^{8-3} = 5^5 = 3125$

(iii) $(3^2)^3 = 3^{2\times3} = 3^6 = 729$

(iv) $\left(\frac{11}{12}\right)^3 = \frac{11^3}{12^3} = \frac{1331}{1728}$

(v) $\left(\frac{3}{4}\right)^{-3} = \frac{1}{\left(\frac{3}{4}\right)^{-3}} = \frac{1}{\frac{(3)^3}{(4)^3}} = \frac{1}{\frac{27}{64}} = \frac{64}{27}$

Example 2 : Using laws of exponent simplify.

Simplify : $(3^{-1}\times5^{-1})^{-1}\div7^{-1}$

$$= \left(\frac{1}{3}\times\frac{1}{5}\right)^{-1}\div\left(\frac{1}{7}\right)$$

$$= \left(\frac{1}{15}\right)^{-1} x7$$

$$= \frac{15}{1} x \frac{7}{1} = 105$$

Example 3 : If $a = 2$ and $b = 3$ then find the values of each of the following.

Solution : (i) $a^b + b^a = 2^2 + 3^3 = 4 + 27 = 31.$

(ii) $a^b + b^a = 2^3 + 3^2 = 8 + 9 = 17.$

(iii) $\left(\frac{1}{a}+\frac{1}{b}\right)^a = \left(\frac{1}{2}+\frac{1}{3}\right)^2 = \left(\frac{3+2}{3\times2}\right)^2 = \left(\frac{5}{6}\right)^2 = \frac{5^2}{6^2} = \frac{25}{36}$

Example 4 : Find the value of x if.

(i) $3^x = 81$ (ii) $2^{x-3} = 1$ (iii) $3^{3x-5} = \frac{1}{9^x}$

Solution : (i) $3^x = 81$ (ii) $2^{x-3} = 1$

$3^x = 3^4$ $= 2^{x-3} = 2^0$

$x = 4$ $= x-3 = 0 = x = 3$

(iii) $3^{3x-5} = \frac{1}{9^x}$

$3^{3x-5} = \frac{1}{3^{2x}} = 3^{3x-5} = 3^{-2x}$

$3x-5 = -2x = 3x+2x = 5$

$5x = 5 = x = 1$

Example 5 : Using prime factorization, express 144×750 as the product of prime factors.

Solution : By prime factorization, we have

2	144
2	72
2	36
2	18
3	9
3	3
	1

2	750
3	375
5	125
5	25
5	5
	1

$\therefore$ 144 = $2\times2\times2\times2\times3\times3 = 2^4\times 3^2$

and, 750 = $2\times3\times5\times5\times5 = 2\times3\times5^3$

So, 144×750 = $2^4\times3^2\times2\times3\times5^3 = (2^5\times3^3\times5^3)$

Example 6 : Solve each of the following exponential equations.

(i) $7^x = 343$ (ii) $2^{x-3} = 1$

Solution : (i) $7^x = 343 \Rightarrow 7^x = 7^3 \Rightarrow x = 3$

(ii) $2^{x-3} = 1 \Rightarrow 2^{x-3} = 2^0$ [$\because 2^0 = 1$] $\Rightarrow x-3 = 0 \Rightarrow x = 3$.

Exercise 4.1

1. Write the base and exponent in each of the following :

(a) 2^5 (b) $(-5)^4$ (c) $\dfrac{1}{3^4}$

2. Write each of the following in exponential form :

(a) $5\times5\times5\times5$ (b) $\dfrac{2}{7}\times\dfrac{2}{7}\times\dfrac{2}{7}\times\dfrac{2}{7}$ (c) $\dfrac{81}{625}$

(d) $(-2)\times(-2)\times(-2)\times(-2)\times(-2)$

3. Simplify and write the answer in the exponential form :

(a) $3^2\times\dfrac{1}{3^5}\times(3^3)^4$ (b) $(-5)^4\times(-5)^3\times(-5)^2$

4. Find the value of x:

(a) $5^{x-2} = 25$ (b) $2^{5x}\div2^x = 2^4$ (c) $(2^2)^x\div(2^3)^4 = 1$

5. (a) Express 729 as a power of 3. (b) Express 343 as a power of 7.

(c) Express –128 as a power of –2.

6. Find the value of *x*, if :

(a) $2^x+2^x+2^x = 192$ (b) $2^3+2^x = 2^4$ (c) $8^{255} = 32^x$

7. Evaluate the following :

(a) $3x^4-(8x^2)^2+8(x^2)^2+(3x^2)^2$ (b) $\left(\dfrac{12}{5}\right)^3\times\dfrac{5^6}{144}$ (c) $\dfrac{9^3\times27\times81^4}{3^{-2}\times3^4\times81^2}$

8. Express each of the following as a product of prime factors only :

(a) 108×192 (b) 363×132

9. Solve the following exponential equations:

(a) $6^{x-2} = 1$ (b) $(\sqrt{2})^x = 2^8$ (c) $3^{3x-5} = \dfrac{1}{9^x}$

Exercise 4.2

1. Express as a rational number in standard form :

(a) $\left(\frac{1}{3}\right)^5$ (b) $-\left(\frac{4}{27}\right)^2$ (c) $-\left(\frac{5}{11}\right)^4$ (d) $\left(\frac{7}{4}\right)^4$

2. Express in the exponential form :

(a) $\frac{9}{49}$ (b) $\frac{243}{1024}$ (c) $\left(\frac{16}{81}\right)$ (d) $\frac{-125}{729}$

3. Express as rational numbers:

(a) $(3^2-2^2)\div\left(\frac{1}{5}\right)^2$ (b) $\left[\left(\frac{1}{2}\right)^2-\left(\frac{1}{4}\right)^3\right]\times 2^3$ (c) $\left(\frac{1}{2}\right)^2\times 2^3\times\left(\frac{3}{4}\right)^2$

(d) $\left(\frac{3}{4}\right)^3\times\left(\frac{2}{3}\right)^2$ (e) $(-2)^5\div\left(\frac{-1}{3}\right)^3$ (f) $\left(\frac{1}{3}\right)^4\div\left(\frac{1}{9}\right)^6$

(g) $\left(\frac{-2}{3}\right)^4\times\left(\frac{-3}{4}\right)^3$ (h) $\left(\frac{3}{5}\right)^4\times\left(\frac{1}{3}\right)^3$

4. Find the reciprocal of the following rational numbers :

(a) $\left(\frac{3}{7}\right)^2$ (b) $\left(\frac{3}{4}\right)^5$ (c) $\left(\frac{-2}{3}\right)^4$ (d) $\left(\frac{-5}{9}\right)^3$

(e) $\left(\frac{1}{3}\right)^5$ (f) $\left(\frac{-7}{-4}\right)^4$ (g) $\left(\frac{5}{11}\right)^4$ (h) $\left(\frac{-4}{27}\right)^2$

5. Find the absolute values :

(a) $\left(\frac{5}{-3}\right)^4$ (b) $\left(\frac{-11}{13}\right)^2$ (c) $\left(\frac{2}{7}\right)^5$ (d) $\left(\frac{-1}{3}\right)^3$

6. State which rational number is greater $\frac{4}{3^2}$ or $\left(\frac{4}{3}\right)^2$

7. Find 12 rational numbers between $\frac{3^2}{4}$ and $\left(\frac{3}{4}\right)^2$

Laws of Exponents

Let $\frac{a}{b}$ be any rational and *m* and *n* be integers. Then we have the following identities.

(1) $\left(\frac{a}{b}\right)^{-n}=\left(\frac{b}{a}\right)^n$

(2) $\left(\frac{a}{b}\right)^0=1$

(3) $\left(\frac{a}{b}\right)^m\times\left(\frac{a}{b}\right)^n=\left(\frac{a}{b}\right)^{m+n}$

(4) $\left(\frac{a}{b}\right)^m\div\left(\frac{a}{b}\right)^n=\left(\frac{a}{b}\right)^{m-n}$ if m>n

(5) $\left(\frac{a}{b}\times\frac{c}{d}\right)^n=\left(\frac{a}{b}\right)^n\times\left(\frac{c}{d}\right)^n$

(6) $\left(\frac{a/b}{c/d}\right)^n=\frac{\left(\frac{a}{b}\right)^n}{\left(\frac{c}{d}\right)^n}$

(7) $\left(\frac{a}{b}\right)^{-1}=\frac{b}{a}$

(8) $\left(\frac{a}{b}\times\frac{c}{d}\right)^{-n}=\frac{1}{\left(\frac{a}{b}\times\frac{c}{d}\right)^n}=\left(\frac{b}{a}\times\frac{d}{c}\right)^n$

Solved Example : Evaluate and name the law of exponent used in evaluating the following rational numbers.

(a) $\left(\frac{2}{3}\right)^{-1}$ (b) $\left(\frac{3}{5}\right)^{-3}$ (c) $\left(\frac{7}{11}\right)^{0}$ (d) $\left(\frac{1}{2}\right)^{4}\times\left(\frac{1}{2}\right)^{3}$

(e) $\left(\frac{2}{5}\right)^{5}\div\left(\frac{2}{5}\right)^{2}$ (f) $\left(\frac{2}{5}\right)^{-5}\div\left(\frac{2}{5}\right)^{-2}$ (g) $\left(\frac{3}{7}\times\frac{5}{11}\right)^{3}$ (h) $\left(\frac{2/3}{5/7}\right)^{3}$ (i) $\left(\frac{7}{13}\times\frac{2}{11}\right)^{-2}$

Solution :

(a) $\left(\frac{2}{3}\right)^{-1}$ using the identity $\left(\frac{a}{b}\right)^{-1}=\frac{b}{a} \Rightarrow \left(\frac{2}{3}\right)^{-1}=\frac{3}{2}$

(b) $\left(\frac{3}{5}\right)^{-3}$ using the identity $\left(\frac{a}{b}\right)^{-m}=\left(\frac{b}{a}\right)^{m} \Rightarrow \left(\frac{3}{5}\right)^{-3}=\left(\frac{5}{3}\right)^{3}=\frac{125}{27}$

(c) $\left(\frac{7}{11}\right)^{0}$ using the identity $\left(\frac{a}{b}\right)^{0}=1 \Rightarrow \left(\frac{7}{11}\right)^{0}=1$

(d) $\left(\frac{1}{2}\right)^{4}\times\left(\frac{1}{2}\right)^{3}$ using the identity $\left(\frac{a}{b}\right)^{m}\times\left(\frac{a}{b}\right)^{n}=\left(\frac{a}{b}\right)^{m+n}$

$$\left(\frac{1}{2}\right)^{4}\times\left(\frac{1}{2}\right)^{3}=\left(\frac{1}{2}\right)^{4+3}=\left(\frac{1}{2}\right)^{7}=\frac{1}{128}$$

(e) $\left(\frac{2}{5}\right)^{5}\div\left(\frac{2}{5}\right)^{2}$ using the identity $\left(\frac{a}{b}\right)^{m}\div\left(\frac{a}{b}\right)^{n}=\left(\frac{a}{b}\right)^{m-n}$ when m > n.

$$\left(\frac{2}{5}\right)^{5}\div\left(\frac{2}{5}\right)^{2}=\left(\frac{2}{5}\right)^{5-2}=\left(\frac{2}{5}\right)^{3}=\frac{2^3}{5^3}=\frac{8}{125}$$

(f) $\left(\frac{2}{5}\right)^{-5}\div\left(\frac{2}{5}\right)^{-2}$ using the identity $\left(\frac{a}{b}\right)^{m}\div\left(\frac{a}{b}\right)^{n}=\left(\frac{a}{b}\right)^{m-n}$ when m > n.

$$\left(\frac{2}{5}\right)^{-5}\div\left(\frac{2}{5}\right)^{-2}=\left(\frac{2}{5}\right)^{-5(-2)}=\left(\frac{2}{5}\right)^{-5+2}=\left(\frac{2}{5}\right)^{-3}$$

$=\dfrac{1}{\left(\frac{2}{5}\right)^{3}}$ using the identity $\left(\frac{a}{b}\right)^{-n}=\dfrac{1}{\left(\frac{a}{b}\right)^{n}}=\left(\frac{5}{2}\right)^{3}=\frac{5^3}{2^3}=\frac{125}{8}$

(g) $\left(\frac{3}{7}\times\frac{5}{11}\right)^{3}$ using the identity $\left(\frac{a}{b}\times\frac{c}{d}\right)^{m}=\left(\frac{a}{b}\right)^{m}\times\left(\frac{c}{d}\right)^{n}$

$$\left(\frac{3}{7}\times\frac{5}{11}\right)^{3}=\left(\frac{3}{7}\right)^{3}=\left(\frac{5}{11}\right)^{3}=\frac{3^3}{7^3}\times\frac{5^3}{11^3}=\frac{27}{343}\times\frac{125}{1331}=\frac{3375}{456533}$$

(h) $\left(\frac{2/3}{5/7}\right)^{3}$ using identity $\left(\frac{a/b}{c/d}\right)^{m}=\dfrac{\left(\frac{a}{b}\right)^{m}}{\left(\frac{c}{d}\right)^{m}}$

$$\left(\frac{2/3}{5/7}\right)^{3}=\frac{\left(\frac{2}{3}\right)^{3}}{\left(\frac{5}{7}\right)^{3}}=\frac{\frac{2^3}{3^3}}{\frac{5^3}{7^3}}=\frac{\frac{8}{27}}{\frac{125}{343}}=\frac{8}{27}\times\frac{343}{125}=\frac{2744}{3375}$$

(i) $\left(\frac{7}{13}\times\frac{2}{11}\right)^{-2}$ using the identity $\left(\frac{a}{b}\times\frac{c}{d}\right)^{-n} = \frac{1}{\left(\frac{a}{b}\times\frac{c}{d}\right)^{n}} = \left(\frac{b}{a}\times\frac{d}{c}\right)^{n}$

$$= \left(\frac{7}{13}\times\frac{2}{11}\right)^{n2} = \frac{1}{\left(\frac{7}{13}\times\frac{2}{11}\right)^{2}} = \left(\frac{13}{7}\times\frac{11}{2}\right)^{2} = \left(\frac{143}{14}\right)^{2} = \frac{20449}{196}$$

Exercise 4.3

1. Write True or False for the following statements :

(a) $\left(\frac{-3}{60}\right)^{50} = \left|\left(\frac{3}{60}\right)^{50}\right|$

(b) The reciprocal of $\left(\frac{3}{7}\right)^{20}$ is $\left(\frac{7}{3}\right)^{20}$

(c) $(100)^{10} = 1000^{10}$

(d) $\left[\left(\frac{1}{3}\right)^{3}\right]$ is reciprocal of 3^3

(e) $(110+110)^{25} = 110^{25} + 110^{25}$

2. Fill in the blanks :

(a) $(-5)^3\times(-5)^2 = (-5)^{\square}$ ....

(b) $(13)^2\times(13)^5 = 13^{\square}$ ....

(c) $\left(\frac{1}{5}\right)^{7}\times\left(\frac{1}{5}\right)^{11} = \left(\frac{1}{5}\right)^{\square}$ ....

(d) $\left(\frac{2}{3}\right)^{8}\div\left(\frac{2}{3}\right)^{5} = \left(\frac{2}{3}\right)^{\square}$ ....

(e) $\left(\frac{-7}{13}\right)^{9}\div\left(\frac{-7}{13}\right)^{5} = \left(\frac{-7}{13}\right)^{\square}$ ....

(f) $2^{11}\div 2^{20} = \left(\frac{1}{2}\right)^{\square}$ ....

(g) $(-79)^3\div(-79)^8 = \left(\frac{1}{-79}\right)^{\square}$ ....

Solved Examples

Example 1 : Express the following in the form of rational numbers.

(a) 2^{-3} (b) $\left(\frac{1}{2}\right)^{-4}$ (c) $\left(\frac{3}{2}\right)^{-3}$ (d) $(-3)^{-2}$ (e) $\left(\frac{-5}{7}\right)^{-4}$

Solution :

(a) $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

(b) $\left(\frac{1}{2}\right)^{-4} = \left(\frac{2}{1}\right)^{4} = \frac{16}{1}$

(c) $\left(\frac{3}{2}\right)^{-3} = \left(\frac{2}{3}\right)^{3} = \frac{2^3}{3^3} = \frac{8}{27}$

(d) $(-3)^{-2} = \frac{1}{-3^2} = \frac{1}{9}$

(e) $\left(\frac{-5}{7}\right)^{-4} = \left(\frac{7}{-5}\right)^{4} = \frac{7^4}{(-5)^4} = \frac{2401}{625}$

Example 2 : Evaluate the following.

(a) $\left(\frac{2}{5}\right)^{3}\times\left(\frac{2}{5}\right)^{2}$ (b) $\left(\frac{3}{7}\right)^{5}\times\left(\frac{3}{7}\right)^{-2}$ (c) $\left(\frac{2}{3}\right)^{-3}\times\left(\frac{2}{3}\right)^{-2}$

Solution :

(a) $\left(\frac{2}{5}\right)^{3}\times\left(\frac{2}{5}\right)^{2} = \left(\frac{2}{5}\right)^{3+2} = \left(\frac{2}{5}\right)^{5} = \frac{2^5}{5^5} = \frac{32}{3125}$

(b) $\left(\frac{3}{7}\right)^{5}\times\left(\frac{3}{7}\right)^{-2} = \left(\frac{3}{7}\right)^{5-2} = \left(\frac{3}{7}\right)^{3} = \frac{3^3}{7^3} = \frac{27}{343}$

(c) $\left(\frac{2}{3}\right)^{-3} \times \left(\frac{2}{3}\right)^{-2} = \left(\frac{2}{3}\right)^{-3+(-2)} = \left(\frac{2}{3}\right)^{-3-2} = \left(\frac{2}{3}\right)^{-5} = \left(\frac{3}{2}\right)^{5} = \frac{3^5}{5^5} = \frac{243}{32}$

Example 3 : Simplify – $\left(\frac{-2}{7}\right)^{-4} \times \left(\frac{-5}{7}\right)^{2}$

Solution : $\left(\frac{-2}{7}\right)^{-4} \times \left(\frac{-5}{7}\right)^{2} = \left(\frac{7}{-2}\right)^{4} \times \left(\frac{-5}{7}\right)^{2}$

$= \frac{-7^4}{2^4} \times \frac{-5^2}{7^2} = \frac{-7^4 \times -5^2}{2^4 \times 7^2} = \frac{-7^2 \times -5^2}{2^4} = \frac{49 \times 25}{16} = \frac{1225}{16}$

Example 4 : Simplify – $\left(\frac{-1}{3}\right)^{-5} \times \left(\frac{-1}{3}\right)^{-4}$

Solution : $\left(\frac{-1}{3}\right)^{-5} \times \left(\frac{-1}{3}\right)^{-4} = \left(\frac{3}{-1}\right)^{5} \times \left(\frac{3}{-1}\right)^{4} = (-3)^5 \times (-3)^4$

$= (-3)^{5+4} = (-3)^9 = -19683$

Example 5 : $(3^{-1} \times 7^{-1}) \div 4^{-1}$

Solution : $\left(\frac{1}{3} \times \frac{1}{7}\right)^{-1} \div 4^{-1} = \left(\frac{1}{21}\right)^{-1} \div \frac{1}{4}$

$21 \div \frac{1}{4} = \frac{\frac{21}{1}}{4} = \frac{21}{1} \div \frac{4}{1} = 21 \times 4 = 84$

Example 6 : $(2^{-1}+8^{-1}) \div \left(\frac{2}{3}\right)^{-1}$

Solution : $(2^{-1}+8^{-1}) \div \left(\frac{2}{3}\right)^{-1} = \left(\frac{1}{2}+\frac{1}{8}\right) \div \frac{3}{2} = \left(\frac{4+1}{8}\right) \times \frac{2}{3} = \frac{5}{\cancel{8}_4} \times \frac{\cancel{2}^1}{3} = \frac{5}{4 \times 3} = \frac{5}{12}$

Example 7 : Simplify – $\left(\frac{1}{5}\right)^{-2} + \left(\frac{1}{3}\right)^{-2} + \left(\frac{1}{2}\right)^{-2}$

Solution : $\left(\frac{1}{5}\right)^{-2} + \left(\frac{1}{3}\right)^{-2} + \left(\frac{1}{2}\right)^{-2}$

$= 5^2 + 3^2 + 2^2 = 25 + 9 + 4 = 38$

Example 8 : By what number should $\left(\frac{1}{2}\right)^{-1}$ be multiply so that the product is $\left(\frac{-5}{4}\right)^{-1}$?

Solution : Let the number be x

$\left(\frac{1}{2}\right)^{-1} \times x = \left(\frac{-5}{4}\right)^{-1}$

$= 2 \times x = \frac{4}{-5}$

$= x = \frac{4}{-5} \div 2$

$= \frac{4}{-5} \times \frac{1}{2} = \frac{2}{-5} \times \frac{-1}{-1} = \frac{-2}{5}$

Example 9 : By what number should $\left(\frac{-2}{3}\right)^{-3}$ be divided so that the quotient is $\left(\frac{4}{27}\right)^{-2}$?

Solution : Let the number be x

$$\left(\frac{-2}{3}\right)^{-3} \div x = \left(\frac{4}{27}\right)^{-2}$$

$$= \frac{\left(\frac{3}{-2}\right)^{3}}{x} = \left(\frac{27}{4}\right)^{2}$$

$$= \frac{27}{-8} \times \frac{1}{x} = \frac{27 \times 27}{4 \times 4}$$

$$= \frac{1}{x} = \frac{27 \times 27}{4 \times 4} \times \frac{-8}{27}$$

$$= \frac{1}{x} = \frac{-27}{2}$$

$$= x \times -27 = 2$$

$$= x = \frac{2}{-27} \times \frac{-1}{-1} = \frac{-2}{27}$$

Example 10 : If $5^{2x+1} \div 25 = 125$, find the value of x.

Solution :

$$5^{2x+1} \div 25 = 125$$

$$= 5^{2x+1} \div 5^{2} = 5^{3}$$

$$= 5^{2x+1} = 5^{3} \times 5^{2}$$

$$= 5^{2x+1} = 5^{3+2}$$

$$= 2x+1 = 5$$

$$= 2x = 5-1 = 2x = 4$$

$$= x = \frac{4}{2} = x = 2$$

Exercise 4.4

1. Simplify the rational numbers and express with positive exponents :

(a) $\left(\frac{4}{25}\right)^{-3} \times \left(\frac{4}{25}\right)^{11} \times \left(\frac{4}{25}\right)^{-10}$ (b) $\left(\frac{-3}{5}\right)^{-6} \div \left(\frac{-3}{5}\right)^{2}$ (c) $\left(\frac{-8}{5}\right)^{-7} \div \left(\frac{-8}{5}\right)^{4}$

(d) $\left[\left(\frac{5}{7}\right)^{-3} \times \left(\frac{5}{7}\right)^{-7}\right] \times \left(\frac{5}{7}\right)^{-3}$ (e) $\left[\left(\frac{9}{11}\right)^{-2}\right]^{-4}$ (f) $\left[\left\{\left(\frac{-7}{11}\right)^{-3}\right\}^{-4}\right]^{-2}$

2. Evaluate : (a) $\left(\frac{1}{20}\right)^{-3} \times (-16)^{-3}$ (b) $\left(\frac{7}{44}\right)^{-4} \div \left(\frac{11}{7}\right)^{4}$ (c) $\left(\frac{-6}{5}\right)^{-2} \times \left(\frac{-3}{4}\right)^{-2}$

(d) $\left(\frac{-7}{8}\right)^{0} \times \left(\frac{3}{4}\right)^{-3} \times \left(\frac{2}{3}\right)^{-2}$ (e) $(2^{-1} \times 5^{-1})^{-1} \div 4^{-1}$ (f) $(4^{-1} + 8^{-1})^{-1} \div \left(\frac{2}{3}\right)^{-1}$

(g) $\left(\frac{-1}{4}\right)^{-3} \div \left(\frac{3}{8}\right)^{-2}$ (h) $(3^{-1} \div 4^{-1})^{2}$

3. Find reciprocal :

(a) $\left[\left(\frac{3}{7}\right)^{2}\right]^{5} \times \left(\frac{7}{3}\right)^{-12}$ (b) $\left(\frac{-5}{11}\right)^{-3} \div \left(\frac{-5}{11}\right)^{-4}$

4. Find the rational number which should be multiplied with $\left(\frac{-3}{2}\right)^{-3}$ so that the product is $\left(\frac{9}{8}\right)^{-2}$.

5. Find the rational number with which $(-6)^{-1}$ should be multiplied so that the product is 9^{-1}.

6. **Find the value of *x* If :**

(a) $7^{2x+1} \div 49 = 243$ (b) $3^{2x+1} \div 9 = 27$

7. If $x = \left(\frac{3}{4}\right)^{-2} \times \left(\frac{6}{9}\right)^{-2}$, find the value of x^{-3}.

8. **Show that:**

(a) $2^7 \times 2^{-5} = 2^{7+(-5)}$ (b) $(-27)^{-9} \times (-27)^5 = (-27)^{-9+5}$

(c) $\left(\frac{-2}{3}\right)^{-3} \times \left(\frac{-2}{3}\right)^{-4} = \left(\frac{-2}{3}\right)^{-3+(-4)}$ (d) $\left(\frac{8}{3} \times \frac{7}{11}\right)^2 = \left(\frac{7}{11}\right)^2 \times \left(\frac{8}{3}\right)^2$

9. **Find the value of *x* such that :**

(a) $\left(\frac{3}{4}\right)^{-5} \times \left(\frac{3}{4}\right)^{-3} = \left(\frac{3}{4}\right)^{x-2}$ (b) $\left(\frac{343}{8}\right)^{x} \times \left(\frac{343}{8}\right)^{5} = \left(\frac{7}{2}\right)^{18}$

Using Exponents for Scientific Notations

Sometimes we come across very large or very small numbers. Which when expressed in digits or numbers become very difficult to understand. Such numbers are written in the form of powers. In science we come across very large numbers such as —

(a) Distances of star, sun and moon etc.

(b) Ages of earth and universe.

(c) Speed of light and other rays.

As well as very small numbers such as —

(a) Size of atoms and molecules.

(b) Size of unicellular organisms.

(c) Size of blood cells and other cells.

Expressing these numbers in the form of powers is called Scientific Notation.

In scientific notation we simply shift the decimal after the extreme left non zero digit.

Example 1 : Change the number 9000000000000 to scientific notation.

Solution : 12 zeroes succeed the digit 9

it can be written as 9×10^{12}

Example 2 : Change the number 9365002.01 to scientific notation.

Solution : $\frac{9365002.01}{1 \times 10^6} \times 10^6$

$= 9.365002 \times 10^6$

$= 9.4 \times 10^6$ approximately.

Example 3 : Change 0.000000934 to scientific notation.

Solution : $\frac{0.000000934 \times 10^{7}}{10^{7}} = \frac{9.34}{10^{7}}$

$= 9.34 \times 10^{-7}$

Example 4 : Change $\frac{3}{1000000}$ into scientific notation.

Solution : $\frac{3}{1000000} = \frac{3}{10^{6}} = 3 \times 10^{-6}$

Short Cut Method

Example 2 : 9365002.01

Solution : 9.365002×10^{6} or 9.4×10^{6}

Because the decimal has shifted six places towards the left.

Example 3 : $0.000000934 = 9.34 \times 10^{-7}$

Because the decimals has shifted seven places towards the right.

The digits on the left of the decimal are denoted by the letter k. The power of 10 is denoted by n. Therefore the scientific notations are numbers of the form of $k \times 10^{n}$.

That is, when we say that $n = 5$. It means that the power of the 10 is 5 or 10^{5}.

Example 4 : Change the number 936520003.03 to —

(a) $n = 5$ (b) $n = 7$ (c) $n = 8$

Solution : (a) $n = 5$

$= K \times 10^{5}$

Shift the point five places towards the left

$9365.2000303 \times 10^{5}$ or 9365.2×10^{5}

(b) $n = 7$

$= K \times 10^{7}$

$= 93.652000303 \times 10^{7}$ or 93.65×10^{7}

(c) $n = 8$

$K \times 10^{8}$

$= 9.3652000303 \times 10^{8}$ or 9.4×10^{8}

Exercise 4.5

1. **Express in scientific notation or in the form of $k \times 10^{n}$ with value of n given:**

(a) 190000000, $n = 8$ (b) 12300000000, $n = 9$

(c) 0.0000000000000037, $n = -15$ (d) 0.0000000066, $n = -9$

2. **Write the following numbers in the usual form:**

(a) 9.5×10^{7} (b) 9.8×10^{9} (c) 6.5146939×10^{7}

(d) 3.8×10^{11} (e) 1.001×10^{10} (f) 6.5×10^{5}

3. **Express the number 71865000000 in the form of $k \times 10^n$, where:**

(a) $n = 10$ (b) $n = 9$ (c) $n = 7$ (d) $n = 6$ (e) $n = 8$

4. **Express the number 0.00003984 in the form of $K \times 10^n$ where :**

(a) $n = -7$ (b) $n = -6$ (c) $n = -5$

5. **Reexpress the following statements with their numbers in the form of scientific notation with k = 1**

(Hint k = No of digits counted from left to right)

(a) The speed of light is approximately 300000 km/second in the vacuum.

(b) The speed of light is exactly 299792.5 km/second in the vacuum.

(c) The universe is approximately 8,000,000,000 years old.

(d) The earth is approximately 6,000,000,000 years old.

(e) The mean distance of sun from the earth is 150,000,000 km.

(f) The mass of the earth is 5980,000,000,000,000,000,000,000,kg.

(g) The unit angstrom (Å) is used to measure radii of atoms and molecules and wavelength.

1Å = 0.0000000001 m.

(h) Each day in Delhi 1050000 kg of pollutants are released.

(i) The earth has 1,353,000,000 km^3 of sea water.

(j) The sea water contains 13,61,10,00000 kg of gold.

(k) The unit micron is used to measure microorganisms. 1 micron = $\frac{1}{1000,000}$ m

Negative rational numbers as exponents

Example 5 :

$$(27)^{-2/3} = \left(\frac{1}{(27)^{2/3}}\right) = \left(\frac{1}{\left[(27)^2\right]^{1/3}}\right) = \left(\frac{1}{\sqrt[3]{27 \times 27}}\right)$$

Solution :

$$= \frac{1}{\sqrt[3]{3 \times 3 \times 3 \times 3 \times 3 \times 3}} = \frac{1}{3 \times 3} = \frac{1}{9}$$

Laws of exponent for rational exponents. If *x* and *y* are any rational numbers different from zero and *a, b* are any two integers then we have the following laws of rational exponents.

(i) $x^a \times x^b = x^{a+b}$ (ii) $\frac{x^a}{x^b} = x^{a-b}$ (iii) $\left(\frac{x}{y}\right)^a = \frac{x^a}{y^a}$

Exercise 4.6

1. **Express each of the following in exponential form:**

(a) $\sqrt[5]{34}$ (b) $\sqrt[4]{27}$ (c) $\sqrt[11]{25}^{-2}$ (d) $\sqrt[7]{\frac{10}{7}}$

2. **Express each of the following as radicals:**

(a) $21^{1/8}$ (b) $27^{3/4}$ (c) $(335)^{7/5}$ (d) $\left(\frac{6}{19}\right)^{1/9}$

3. **Express each of the following with positive indices:**

(a) x^{-4} (b) $x^{-1/2}$ (c) $x^{-3/4}$ (d) $\frac{2}{5}x^{-7/8}$

4. **Simplify :**

(a) $x^{1/2} \times x^{3/2}$ (b) $\frac{x^{4/3}}{x^{1/3}}$ (c) $(x^{1/2})^4$ (d) $(x^5)^0$

5. **Evaluate the following :**

(a) $8^{2/3}$ (b) $27^{-2/3}$ (c) $\frac{1}{16^{-3/4}}$ (d) $\left(\frac{64}{729}\right)^{1/6}$

6. Determine x so that:- (a) $5^{1/3} \times 5^{1/6} = 5^{-x}$ (b) $800 = 8 \times 10^8 \times x^{-3/2}$.

7. By what number should we multiply $81^{3/16}$ so that the product becomes $3^{5/4}$?

8. If $4^x - 4^{x-1} = 24$ then find the value of $(2x)$.

9. Determine x and y so that $3^{x+y} = 81$ and $81^{x-y} = 3$.

10. **Evaluate the following**

(a) $(3^2+4^2)^{1/2}$ (b) $(5^2+12^2)^{1/2}$ (c) $\sqrt[3]{7} \times \sqrt[3]{49}$ (d) $(0.04)^{3/2}$

Radicals : If a is a rational number and n is a positive integer such that *the* n^{th} root of a i.e. $\sqrt{a}$ or $a^{1/n}$ is an irrational number then it is called $a^{1/n}$ radicals.

Example 6 : $\sqrt{5}$ or $5^{1/2}$ since 5 is a rational number and 2 is a positive integer such that $5^{1/2}$ or $\sqrt{5}$ is an irrational number. So $\sqrt{5}$ is a radical of index 2.

Pure radical : A radical that contains no radical factor other than 1 is called a pure radical.

Example 7 : $\sqrt{3}$ $\sqrt[5]{2}$ and $\sqrt[4]{3}$ are pure radicals.

Mixed Radicals : A radical which has a rational factor other than unity. The other factor being irrational is called a mixed radical.

Example 8 : $5\sqrt[2]{3}$ $\sqrt[3]{12}$ and $2\sqrt[4]{5}$ are mixed radicals.

Simplest form of a square root radical : A square root radical is said to have in simplest form if –

(i) There is no fraction in the radical.

(ii) No perfect square is a factor of radical.

Example 9 : Express $\sqrt{\frac{125}{63}}$ in its simplest form:

Solution : we have $\sqrt{\frac{125}{63}} = \frac{\sqrt{125}}{\sqrt{63}}$

$$= \frac{\sqrt{5 \times 5 \times 5}}{\sqrt{3 \times 3 \times 7}} = \frac{\sqrt{5^2 \times 5}}{\sqrt{3^2 \times 7}}$$

$$= \frac{\sqrt{5^2}}{\sqrt{3^2}} \times \frac{\sqrt{5}}{\sqrt{7}} = \frac{5}{3} \times \frac{\sqrt{5}}{\sqrt{7}}$$

$$= \frac{5}{3} \times \sqrt{\frac{5 \times 7}{7 \times 7}} = \frac{5}{3} \times \frac{\sqrt{5 \times 7}}{\sqrt{7^2}}$$

$$= \frac{5}{3} \times \frac{\sqrt{35}}{7} = \frac{5}{21} \times \sqrt{35}$$

Example 10 : Simplify :

(i) $\sqrt{18}+\sqrt{50}-\sqrt{32}$ (ii) $\sqrt{84}\div\sqrt{7}$ (iii) $\frac{1}{6-\sqrt{3}}$

Solution : (i) $\sqrt{18}+\sqrt{50}-\sqrt{32} = \sqrt{9\times 2} + \sqrt{25\times 2} - \sqrt{16\times 2}$

$= \sqrt{3^2\times 2} + \sqrt{5^2\times 2} - \sqrt{4^2\times 2}$

$= 3\sqrt{2} + 5\sqrt{2} - 4\sqrt{2}$

$= (3+5-4)\sqrt{2} = 4\sqrt{2}$

(ii) $\sqrt{84}\div\sqrt{7} = \frac{\sqrt{84}}{\sqrt{7}} = \sqrt{\frac{84}{7}} = \sqrt{12} = \sqrt{2^2\times 3} = 2\sqrt{3}$

(iii) $\frac{1}{6-\sqrt{3}} = \frac{1}{(6-\sqrt{3})}\times\frac{6+\sqrt{3}}{6+\sqrt{3}} = \frac{6+\sqrt{3}}{6^2-(\sqrt{3})^2} = \frac{6+\sqrt{3}}{36-3} = \frac{6+\sqrt{3}}{33}$

Multiplying the numerator and denominator by $6+\sqrt{3}$

Exercise 4.7

1. Express the following radicals in exponential form:

(a) $\sqrt{5}$ (b) $\sqrt[3]{7}$ (c) $\sqrt[4]{\frac{3}{4}}$ (d) $\sqrt[8]{\frac{71}{2159}}$

2. Express the following as radicals in each case. Find the radical and the index:

(a) $16^{1/2}$ (b) $125^{1/3}$ (c) $\left(\frac{6}{17}\right)^{1/19}$

3. Express each of the following as mixed radicals :

(a) $\sqrt{18}$ (b) $\sqrt{405}$ (c) $\sqrt{108}$ (d) $\sqrt{300}$

4. Express each of the following as pure radicals :

(a) $2\sqrt{6}$ (b) $7\sqrt{5}$ (c) $4\sqrt{5}$ (d) $\frac{3}{2}\sqrt{\frac{3}{2}}$ (e) $10\sqrt{13}$

5. Express each of the following as a mixed radicals in the simplest form :

(a) $\sqrt{125}$ (b) $\sqrt{112}$ (c) $\sqrt{192}$ (d) $\sqrt{75}$

6. Simplify :

(a) $\sqrt{6}\times\sqrt{3}$ (b) $\sqrt{96}\div\sqrt{12}$ (c) $\sqrt{300}-\sqrt{48}+\sqrt{75}-\sqrt{27}$ (d) $(7\sqrt{2}+5)(7\sqrt{2}-5)$

7. Simplify :

(a) $\frac{\sqrt{126}\times\sqrt{63}\times\sqrt{45}}{\sqrt{147}\times\sqrt{243}}$ (b) $(\sqrt{5}+\sqrt{2})^2+(\sqrt{5}-\sqrt{2})^2$

Points to Remember :

- Exponents are powers to the numbers called bases.
- Very large and very small numbers are expressed in the form of exponents for convenience.
- $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^{n}$
- $\left(\frac{a}{b}\right)^{0} = 1$
- $\left(\frac{a}{b}\right)^{m} \times \left(\frac{a}{b}\right)^{n} = \left(\frac{a}{b}\right)^{m+n}$
- $\left(\frac{a}{b}\right)^{m} \div \left(\frac{a}{b}\right)^{n} = \left(\frac{a}{b}\right)^{m-n}$ if $m>n$
- $\left(\frac{a}{b} \times \frac{c}{d}\right)^{n} = \left(\frac{a}{b}\right)^{n} \times \left(\frac{c}{d}\right)^{-n}$
- $\left(\frac{a}{b} \times \frac{c}{d}\right)^{-m} = \frac{1}{\left(\frac{a}{b} \times \frac{c}{d}\right)^{m}} = \left(\frac{b}{a} \times \frac{d}{c}\right)^{m}$
- $\left(\frac{a/b}{c/d}\right)^{n} = \frac{\left(\frac{a}{b}\right)^{n}}{\left(\frac{c}{d}\right)^{n}}$
- $\left(\frac{a}{b}\right)^{-1} = \frac{b}{a}$
- A radical that contains no rational factors other than I is called a pure radical.
- A radical which has a rational factor other than unity. The other factor being irrational is called a mixed radical.

EXERCISE

1. MULTIPLE CHOICE QUESTIONS (MCQs) :

Tick (✓) the correct options.

(a) Which of the following is correct for $(-1)^{20}$?

(i) -1 ☐ (ii) 0 ☐ (iii) 1 ☐ (iv) 20 ☐

(b) The reciprocal of $\frac{-2}{22}$ is equal to–

(i) $\frac{2}{22}$ ☐ (ii) $\frac{-2}{22}$ ☐ (iii) $\frac{21}{2}$ ☐ (iv) none of these ☐

(c) What is the value of 'm' for which $(-4)^{m+1} \times (-4)^{-2} = -64$?

(i) 4 ☐ (ii) -4 ☐ (iii) 3 ☐ (iv) 64 ☐

(d) What is the value of $11^{7/3} \div 11^{1/3}$?

(i) 111 ☐ (ii) 11 ☐ (iii) 144 ☐ (iv) 121 ☐

(e) $\frac{64}{343}$ Can be written as.

(i) $\frac{4^3}{7^2}$ ☐ (ii) $\left(\frac{4}{7}\right)^{-3}$ ☐ (iii) $\left(\frac{4}{7}\right)^{3}$ ☐ (iv) $\left(\frac{7}{3}\right)^{3}$ ☐

(f) The product of 4^5 and 4^3 is equal to –

(i) 16^5 ☐ (ii) 16^3 ☐ (iii) 4^2 ☐ (iv) 4^8 ☐

(g) What is value of 'x' in $3^{4x} = \frac{1}{81}$?

(i) 1 ☐ (ii) –1 ☐ (iii) 0 ☐ (iv) –2 ☐

(h) $2^9 \div 2^4$ is equal to –

(i) 2^5 ☐ (ii) 2^{13} ☐ (iii) 4^9 ☐ (iv) 4^4 ☐

2. Write the base and the exponent in each of the following :

(a) $\left(\frac{-1}{9}\right)^8$ (b) $(-18)^3$ (c) $(12)^{-15}$ (d) $\left(\frac{3}{19}\right)^{-3}$

(e) $(2^7)^{-6}$ (f) $\left\{\left(\frac{1}{3}\right)^{-2}\right\}^{-3}$ (g) $(2 \times 3)^2$ (h) $2^3 \div 2^2$

3. Simplify the following:

(a) $6^{15} \div 6^7$ (b) $\left(\frac{2}{5}\right)^3 \div \left(\frac{2}{5}\right)^5$ (c) $(ab)^5 \div (ab)$

(d) $\left(-\frac{3}{8}\right)^5 \div \left(-\frac{3}{8}\right)^7$ (e) $\left(\frac{1}{9}\right)^2 \times \left(\frac{1}{9}\right)^5$

4. Find the value of '*x*' in each of the following:

(a) $5^x = 125$ (b) $(2 \times 2)^x = 2^8$ (c) $\left(-\frac{2}{3}\right)^x = \frac{16}{81}$ (d) $(a^3 \times a^2) = a^x$

5. Simplify the following:

(a) $\frac{4^3 \times 5^a \times b^3}{4^2 \times 9^3 \times b^2}$ (b) $(6^\circ + 7^\circ)^2$ (c) $(2^\circ \times 3^\circ \times 4^\circ)^2$ (d) $\left(\frac{-1}{3}\right)^4 \times \left(\frac{-1}{3}\right)^5$

6. Find the value of *x* for each of the following:

(a) $x^5 \div x^3 = \frac{9}{16}$ (b) $\left(\frac{4}{15}\right)^3 \times \left(\frac{4}{15}\right)^{-6} = \left(\frac{4}{15}\right)^{2x+1}$

7. Find the value of p so that $\left(\frac{4}{5}\right)^3 \times \left(\frac{4}{5}\right)^{-3} = \left(\frac{4}{5}\right)^{3p}$

8. Simplify the following:

(a) $a^2 \times b^2$ (b) $\left(\frac{1}{3}\right)^3 \times \left(\frac{2}{3}\right)^3$ (c) $\left(\frac{1}{5}\right)^{-8} \times \left(\frac{5}{7}\right)^{-8}$ (d) $\left(\frac{1}{4}\right)^{-10} \times \left(\frac{2}{5}\right)^{-10}$

HOTS

1. The value of $(2^3)^2$ is equal to ....

2. The value of $\left(\frac{x}{y}\right)^0 \times \left(\frac{2}{3}\right)^3$ is ....

Lab Activity

Objective : To verify the law of exponents experimentally when the bases are different, i.e. $x^n \times y^n = (x \times y)^n$.

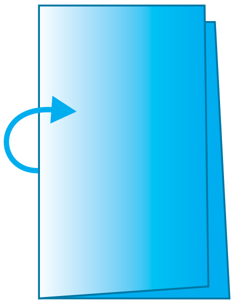
Materials Required : Glazed paper, white papers (to note the results), sketch pens, fevicol and chart paper.

Procedure :

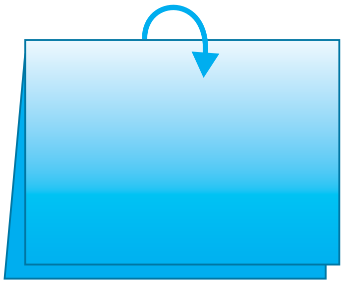
1. Take a glazed paper and fold it twice as shown.

Step 1:

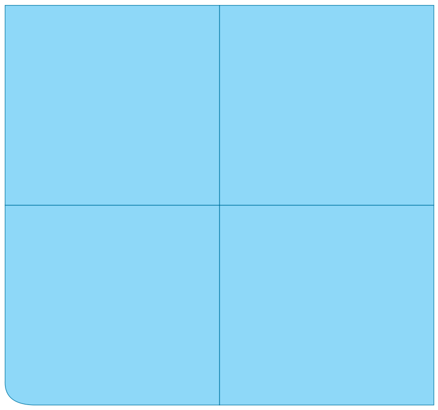
Step 2:



Step 3:



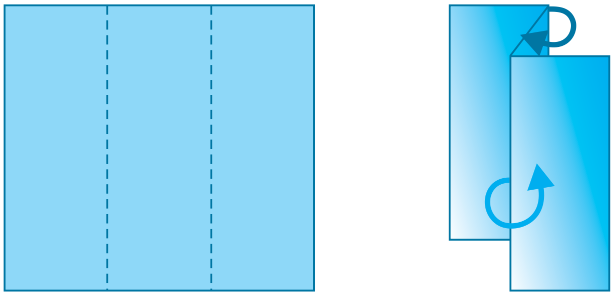
Step 4 : Unfold the paper



Now the creased paper represents $2^2 = 4$.

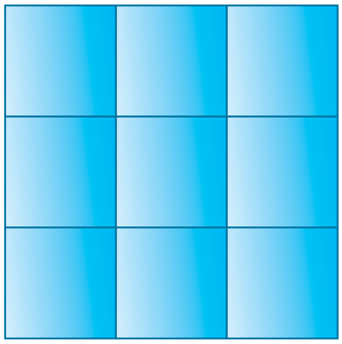
2. Take another paper and divide it into three parts, colour them and fold then as shown.

Step 1:



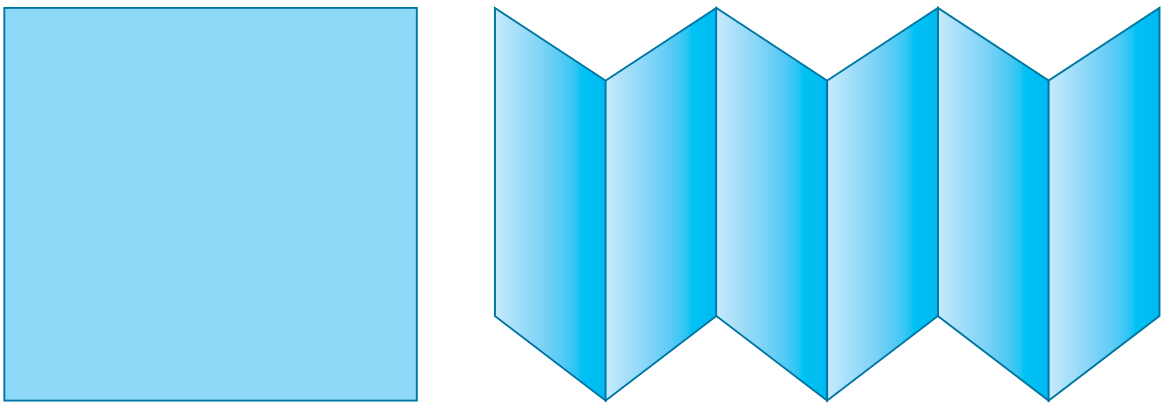
Step 2: Divide the folder paper again into three parts and fold again it as shown.

Step 3 : Unfold the paper.

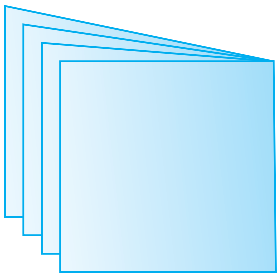


This creased paper represents $3^2 = 9$.

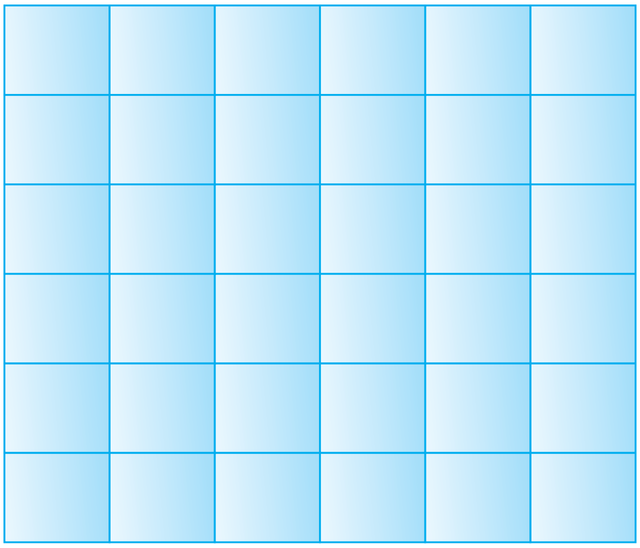
3. Take another paper and divide it into 6 parts.



Step 2 : Fold it as shown.



Step 3 : Unfold the paper



This creased paper represents $6^2 = 36$

So $2^2 \times 3^2 = 4 \times 9 = 36 = 6^2$

it is verified that $2^2 \times 3^2 = 36 = 6^2$

i.e. $x^n \times y^n = (x \times y)^n$

5 Squares and Square Roots

We have studied the numbers with powers. **The numbers with the power of two are called squares and numbers itself are called square roots.** The number which can be expressed in the form of square roots are called **perfect squares**.

Perfect squares can be expressed in the form of square roots by prime factorisation method or long division method. The numbers which are not perfect squares can also be changed to approximate square root number called **approximate square roots**.

If x and y are natural numbers such that $x = y^2$ then x is the square of the number y called **square number** of **perfect square**. All natural numbers are not perfect square. Fox example, 6,18, 60,90 are not perfect squares. Because they cannot be expressed in the form of square roots. The numbers from 1 to 100. There are only 100 perfect squares from 1 to 10,000. Some of the numbers which are perfect square are given in the table.

Number	Square	Number	Square	Number	square
1	1	8	64	15	225
2	4	9	81	16	256
3	9	10	100	17	289
4	16	11	121	18	324
5	25	12	144	19	361
6	36	13	169	20	400
7	49	14	196	25	625

Characters of Perfect Squares

1. The square numbers of perfect squares usually end with 0, 1, 4, 5, 6, or 9. But it is not necessary that all numbers ending 0, 1, 4, 5, 6 or 9 are perfect squares. Of course the perfect squares never end with digits 2, 3, 7 and 8.

2. (a) The squares of number ending in 1 and 9 with 1.

Example : $1^2 = 1$ $19^2 = 361$ $99^2 = 9801$ $9^2 = 81$

$21^2 = 441$ $101^2 = 10201$ $11^2 = 121$ $29^2 = 841$

(b) The numbers end with 2 or 8 their squares end with 4.

Example : $2^2 = 4$ $12^2 = 144$ $22^2 = 484$

$8^2 = 64$ $18^2 = 324$ $28^2 - 784$

(c) The squares of numbers ending with 3 and 7 end with 9.

Example : $3^2 = 9$ $13^2 = 169$ $23^2 = 529$

$7^2 = 49$ $17^2 = 289$ $27^2 = 729$

(d) The numbers which end in 4 or 6, their perfect squares end in 6.

Example : $4^2 = 16$ $\quad 14^2 = 196$ $\quad 24^2 = 576$

$6^2 = 36$ $\quad 16^2 = 256$ $\quad 26^2 = 676$

(e) The perfect squares of numbers that end in 0 or 5 end in 0 or 5.

Example : $5^2 = 25$ $\quad 15^2 = 225$ $\quad 25^2 = 625$

$10^2 = 100$ $\quad 20^2 = 400$ $\quad 30^2 = 900$

(f) The perfect squares leave a remainder of 0 or 1, when they are divided by 3.

Example :

12	16	27	85
3)3 6	3)4 9	3)8 1	3)2 5 6
−3	−3	−6	−2 4
×6	1 9	2 1	1 6
− 6	− 1 8	2 1	−1 5
××	1	××	1

(g) If x is a perfect square then $2x$ is never a perfect square.

Example : $5^2 = 25$, 25 is a perfect square.

$2 \times 25 = 50$, 50 is not a perfect square.

That is, if a perfect square is doubled. It would never be a perfect square.

(h) For every natural number n. We have

$(n+1)^2 - n^2 = (n+1+n)(n+1-n) = [(n+1)+n)$

Example : (i) $25^2 - 24^2 = 25 + 24 = 49$

(ii) $69^2 - 68^2 = 69 + 68 = 137$

The difference of squares of two numbers is equal to their sum, i.e, for every natural number n, we have.
$(n+1)^2 - n^2 = (n+1+n)(n+1-n)$
$= \{(n+1)+n\}$
for example, $28^2 - 27^2 = 28+27$
$=55$

(i) For every natural number n. We have

n^2 = Sum of first n odd numbers.

Example : $7^2 = 1+3+5+7+9+11+13 = 49$

(j) For natural a, b and c, $(a^2 + b^2) = c^2$

Such numbers are called pythagorean Triplets.

(k) If natural number $a > 1$, then we have

$(2a, a^2 - 1, a^2 + 1)$ as a Pythagorean Triplet.

Example : Let a natural number $a = 3$. Find the Pythagorean Triplets related to a.

Solution : $2a = 2 \times 3 = 6$, $\quad a^2 - 1 = 3^2 - 1 = 8$, $\quad a^2 + 1 = 3^2 + 1 = 10$.

There Pythagorean Triplets are 6, 8, and 10.

Example : Find the sum without adding, $1+3+5+7+9+11+13+15$

Solution : $8^2 = 64$

Example : Express 81 as the sum of odd numbers.

Solution : $81 = 9^2 = 1+3+5+7+9+11+13+15+17$

Square Roots

Let the two numbers be x and y, such that $x = y^2$ then y is the square root of x. If x is a perfect square its square root will be an integeral square root. If x is not a perfect square its square root will not be an integeral square root.

Properties of Square Roots

1. The square roots of even numbers are even and square roots of odd numbers are odd.
2. If a number ends in even number of zeroes. It is a perfect square. The numbers of zeroes of its square root will be half of zeroes of the perfect square.
3. All negative numbers are not perfect squares. Therefore negative numbers have no square roots.
4. If a number has a square root then its units digits must be 0, 1, 4, 5, 6 or 9.
5. Numbers ending in 2, 3, 7, or 8 are not perfect squares. Hence they have no square roots.

Finding Square Roots by Prime Factorisation Method

Observe prime factorisation of 8, 12 and 18

$8 = 2 \times 2 \times 2$, $12 = 2 \times 2 \times 3$ $18 = 2 \times 3 \times 3$

Can the numbers 8, 12 and 18 have square roots? No! They do not have square roots because their prime factorisation is not paired. Prime factorisations of perfect squares should have the following characters.

If x is a prime factor of number y then $x \times x$ is a prime factor of y^2. Conversely if $x \times x$ is a factor of y^2 then x is a factor *of* y and x is a prime number.

Exercise 5.1

1. Write True or False .

(a) Squares of numbers such as 11^2, 111^2, 1111^2 form beautiful patterns.

(b) Perfect squares leave a remainder of 0 or 1 when divided by 7.

(c) The perfect square of numbers ending in 0 always end in 0.

(d) Double of perfect squares never form squares.

(e) Numbers ending with 2, 3, 7, 8 never form perfect squares.

(f) Numbers which end in 0, 1, 4, 5 or 6 are always perfect squares.

(g) The squares of numbers ending in 1 or 9 may or may not end in 1 or 9.

2. Which of the following numbers are not perfect squares.

(a) 5017 (b) 32453 (c) 9728

3. Find unit digit of the following numbers if squared without actual calculation.

(a) 62958 (b) 88990 (c) 36827

(d) 1234 (e) 372 (f) 61

(g) 2863 (h) 350 (i) 99999

4. Verify that the following numbers are not perfect square.

(a) 3567 (b) 3058 (c) 63453

5. **If the following numbers are squared, which of the following would have an even number as its square root?**

6521, 3332, 53904, 69, 30, 37

6. **Which of the following numbers will have an odd number at its units place if squared?**

(a) 12345 (b) 999 (c) 2221

(d) 6356 (e) 2934 (f) 6358

7. **Find the sum without adding :**

(a) 1 + 3 + 5 + 7 + 9

(b) 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15

(c) 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19

(d) 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21 + 23

8. **Express the following as the sum of odd numbers.**

(a) 49 (b) 81 (c) 36 (d) 121

9. **The smallest numbers of Pythagorean Triplets are –**

(a) 3 (b) 5 (c) 7 (d) 8

Find the other two numbers.

10. **The biggest numbers of Pythagorean Triplets are –**

(a) 17 (b) 37 (c) 82

Find the other two Pythagorean Triplet numbers.

11. **Fill in the blanks :**

(i) $1^2 = 1$

(ii) $2^2 = 1 + 3$

(iii) $3^2 = 1 + 3 + 5$

(iv) $4^2 = 1 +$ .... + ... + 7

(v) $7^2 = 1 +$ .... + ... + ... + ... + ... + ...

How to Find Square Roots by Prime Factorisation

Find prime factorisation of the perfect square. Group the similar primes in pairs. Out of each pair of prime numbers of prime factorisation, choose one prime. Find the product of prime numbers choosen. This product is the square root of perfect square.

Example 1 : Find the square root of 6400 by prime factorisation method

Solution : $6400 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5$

$\sqrt{6400} = 2 \times 2 \times 2 \times 2 \times 5 = 80$

2	6400
2	3200
2	1600
2	800
2	400
2	200
2	100
2	50
5	25
5	5
	1

Example 2 : What is the least number with which 9408 should be multiplied to make it a perfect square?

Solution : $9408 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 7 \times 7 \times 3$

In a prime factorisation of 9408 all the numbers except 3 are in pairs. To make it pair we need another 3.

Therefore, the least number with which 9408 should be multiplied to make it perfect square is 3.

$9408 \times 3 = 28224$

2	9408
2	4704
2	2352
2	1176
2	588
2	294
3	147
7	49
7	7
	1

Example 3 : Find the least number with which 2352 should be divided to make it a perfect square.

2352 = 2 x 2 x 2 x 2 x 7 x 7 x 3

In the prime factorisation we see that all the factors are in pairs except 3. Had the number 3 not been these in the factorisation the number would have been a perfect number. To remove it we have to divide it

by 3. $\dfrac{2 \times 2 \times 2 \times 2 \times 3 \times 7 \times 7}{3} = 784$

2	2352
2	1176
2	588
2	294
3	147
7	49
7	7
	1

Example 4 : Find the least square number or perfect square divisible by each of the numbers 4, 8, 9 and 10.

Solution : The LCM of 4, 8, 9, & 10 = 360

$= 2 \times 2 \times 2 \times 5 \times 3 \times 3$

In the prime factorisation the factors 2×5 do not form a pair.

$360 \times 2 \times 5 = 3600$

3600 is the least square number divisible by 4, 8, 9, and 10.

2	4, 8, 9, 10
2	2, 4, 9, 5
2	1, 2, 9, 5
5	1, 9, 5
3	9,
3	3,
	1,

Example 5 : An army commander has 2025 soldiers with him. He wants to arrange the soldiers in such a way that number of rows of soldiers are equal to the number of soldiers. Find the number of soldiers in each row.

Solution : Let the number of soldiers be $= x$

Let the number rows be $= x$

Total number of soldiers $= x \times x = x^2$

$x^2 = 2025 = 5 \times 5 \times 3 \times 3 \times 3 \times 3$

$\sqrt{2025} = 5 \times 3 \times 3 = 45$

There are 45 rows of soldiers. Each row has 45 soldiers.

5	2025
5	405
3	81
3	27
3	9
3	3
	1

Example 6 : Show that 63504 is perfect square. Also, find the number whose square is 63504.

Resolving 63504 into prime factors we obtain

Solution : $63504 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 7 \times 7$

Grouping the factors in pairs of equal factors we obtain $65504 = (2 \times 2) \times (2 \times 2) \times (3 \times 3) \times (3 \times 3) \times (7 \times 7)$

Clearly no factor is left over in grouping the factors in pairs in equal factors. So 63504 is a perfect square.

2	63504
2	31752
2	15876
2	7938
3	3969
3	1323
3	441
3	147
7	49
7	7
	1

Again 63504 $= (2 \times 2 \times 3 \times 3 \times 7)$

$=$ [Grouping first factor in each group]

$= 252$

Thus 63504 is the square of 252.

Example 7 : Which of the following perfect squares are squares of even numbers : 121, 225, 256, 1296, 6561?

Solution : We know that squares of even numbers are always even.

256 and 1296 are squares of even numbers.

$256 = 16^2$, $1296 = 36^2$

Example 8 : Which of the following triplets are Pythagorean ?

(1,2,3), (3,4,5) , (6,8, 10), (1,1,1)

Solution : We know that the three natural numbers *m*, *n* and *p* are called Pythagorean triplets if $m^2 + n^2 = p^2$.

(i) $1^2 + 2^2 = 3^3 \Rightarrow 1 + 4 = 9 \Rightarrow 5 = 9$

But $5 \neq 9$

(1,2,3) are not Pythagorean triplets.

(ii) $3^2 + 4^2 = 5^2 \Rightarrow 9 + 16 = 25 \Rightarrow 25 = 25.$

(3, 4, 5) are Pythagorean triplets.

(iii) $6^2 + 8^2 = 10^2 \Rightarrow 36 + 64 = 100 \Rightarrow 100 = 100$

(6, 8, 10) are Pythagorean triplets.

(iv) $1^2 + 1^2 = 1^2 \Rightarrow 1 + 1 = 1 \Rightarrow 2 = 1$

But $2 \neq 1$

(1, 1, 1) are not Pythagorean triplets.

Exercise 5.2

1. Which of the following numbers are perfect squares?

(a) 484 (b) 941 (c) 576 (d) 2500

2. Find the smallest number by which the given numbers must be multiplied so that the perfect square.

(a) 23805 (b) 12150 (c) 7688

3. **Which of the following numbers are perfect squares?**

11, 12, 16, 32, 36, 50, 64, 79, 81, 111, 121

4. **Using prime factorization method, find which of the following numbers are perfect squares.**

189, 225, 2048, 343, 441, 2916, 11025, 3549

5. **Find the greatest number of two digits which is a perfect square.**

6. **Find the square root of each of the following by prime factorization.**

(a) 441 (b) 1764 (c) 4096 (d) 8281

7. **Which of the following numbers are not perfect squares?**

(a) 81 (b) 92 (c) 121 (d) 132

8. **Guess and verify the square roots of –**

(a) 27×27 (b) 196 (c) 38×38

9. Final the square roots of 121 and 169 by the method of repeated subtraction.

10. **Write the possible unit digits of the square roots of the following numbers which of these numbers are odd square roots.**

(a) 9801 (b) 99856 (c) 998001

11. **Write the prime factorization of the following numbers are hence find their square roots.**

(a) 7744 (b) 9604 (c) 5929 (d) 7056

Some short-cuts to find squares : In order to find the square of a number we multiply the given number by itself. The multiplication is convenient for small numbers but for large numbers multiplication may be laborious and time consuming. In this section, we shall discuss some short methods for finding the squares of natural numbers without using actual multiplication.

Column method : The method is based upon an old Indian method of multiplying two numbers.

This method used the identity $(a+b)^2 = a^2 + 2ab + b^2$ for finding the square of a two digit number ab (Where a is the ten digit and b is the unit digit). We follow the following steps to final the square of two digit numbers ab (where a is the tens digit and b is the units digits)

Step 1 : Make three columns and write the values of a^2, $2 \times a \times b$ and b^2 respectively. As an illustration let us take ab = 57

Column I	Column II	Column III
a^2	$2 \times a \times b$	b^2
25	70	49

Step 2 : Underline the unit digit of b^2 (In column III) and write it below the column. Now add the tens digit of b^2 column to the result of $2 \times a \times b$ in column II.

Column I	Column II	Column III
a^2	$2 \times a \times b$	b^2
25	70 +4	9
	= 74	9

Step 3 : Underline the unit digit in column II and write it below the column. Now add the tens digit of column II to the result column a^2 and write the result below the same column.

Column I	Column II	Column III
a^2	$2 \times a \times b$	b^2
25 +7 = 32	70 +4 = 4	49
	4	9

Step 4 : Under the number in column I.

Column I	Column II	Column III
a^2	$2 \times a \times b$	b^2
25 +7 = 32	70 +4 = 74	49
32	4	9

Step 5 : Write the underlined digits 4 at bottom of each column to obtain the square of the given number.

In this case, we have $57^2 = 3249$.

Diagonal Method. This method is applicable to final the square of any number irrespective of the number of digit in the number. We follow the following steps to final the square of a number by this method.

Step 1 : Obtain the numbers and count the number of digits in it.

Step 2 : Draw the diagonals of each sub square. As an illustration, let the number to be squared be 349.

Step 3 : Write the digits of the numbers to be squared along the left vertical side and top horizontal side of the squares as shown below.

Example :

1 0 + 1 = 1

2 Carry 1
1 + 9 + 1 + 1 = 12

1 Carry 1
2 +2 + 1 + 2 + 2 + 2 +11

8 Carry 2
7+3 + 6 +3 + 7 + 2 = 28

0 Carry 2
6 + 8 + 6 = 20

3	4	9
0 / 9	1 / 2	2 / 7
1 / 2	1 / 6	3 / 6
2 / 7	3 / 6	8 / 1

349 = 121801

Example : Find 68^2 :

4 3 + 1 = 4

6 Carry 1
4 + 6 + 4 + 2 = 16 **6**

2 Carry 2
8 + 6 + 8 = 22 **8**

9 4

6	8
3 / 8	4 / 8
4 / 8	6 / 4

$68^2 = 4624$

Remark : The diagonal method can be applied to final square of any number irrespective of the number of digits.

Example 1: (1) Find the squares of the following number using column method. Verify the result by finalising the square using the usual multiplication.

1. 25 2. 37 3. 54 4. 71 5. 96

(2) Find the squares of the following number, using diagonal method.

1. 98 2. 273 3. 348 4. 295 5. 171

(3) Find the squares of the following numbers.

1. 127 2. 575 3. 512 4. 95 5. 852

(4) Find the squares of the following number using the identity.

$(a+b)^2 = a^2 + 2ab + b^2$

1. 405 2. 510 3. 211 4. 625

(5) Find the squares of the following number using the identity.

$(a-b)^2 = 2a - 2ab + b^2$

1. 395 2. 99 3. 575 4. 498

Square Root : The square root of a number a is that number which when multiplied by itself gives a as the product.

Thus if b is the square root of a number a then

$b \times b = a$ or $b^2 = a$

Thus square root of a number a is denoted by $\sqrt{a}$ it follows this form such that

$b = \sqrt{a} \Leftrightarrow b^2 = a$.

i.e. b is the square root of a if and only if a is the square of b.

Example 2 : (1) 4 $=\sqrt{2}$ because $2^2 = 4$

(2) 49 $=\sqrt{7}$ because $7^2 = 49$

(3) 324 $=\sqrt{18}$ because $18^2 = 324$

Remark : Since $4 = 2^2 = (-2)^2$ therefore 2 and –2 can both be the square roots of 4. However we agree that square root of a number will be taken to positive square root only thus $\sqrt{4} = 2$.

Properties of square roots :

Property 1 : If the units digits of a number is 2, 3, 7 or 8 then it does not have a square in N (the set of natural numbers.)

Property 2 : The square root of an even square number is even and that root of and odd square number is odd.

Property 3 : If a number has a square root is N, then its units digit must be 0, 1, 4, 5, 6 or 9.

Unit digit of Square	0	1	4	5	6	9
Unit digit of root	0	1 or 9	2 or 8	5	4 or 6	3 or 7

Property 4 : Negative numbers have no square root in the system of rational numbers.

Square Root of a Perfect Square by Prime Factorization

In order to final the square root of a perfect square by prime factorization. We follow the following steps.

Procedure.

Step 1: Obtain the given number.

Step 2 : Resolve the given number into prime factors by successive division.

Step 3 : Take one factor from each pair.

Step 4 : Find the product of factor.

2	8100
2	4050
3	2025
3	675
3	225
3	75
5	25
	5

Example 3 : Find the square root of 8100.

Solution : $8100 = \underline{2\times2}\times\underline{3\times3}\times\underline{3\times3}\times\underline{5\times5}$

$\sqrt{8100} = 2\times3\times3\times5 = 90$

Example 4 : Find the smallest number by which 9408 must be divided so that it becomes a perfect square. Also, Find the square root of the perfect square so obtained.

2	9408
2	4704
2	2352
2	1176
2	588
2	294
3	147
7	49
	7

$9408 = \underline{2\times2}\times\underline{2\times2}\times\underline{2\times2}\times\underline{7\times7}\times3.$

Hence, as 3 doesn't make any pair and dividing by 3 will make the 9408 a perfect square.

Square root of the obtained perfect square will be —

$2\times2\times2\times7 = 56$

Example 5 : Find the square root of the following by means of factors :

(i) 529 (ii) 298116

23	529
23	23
	1

Solution : (i) 529 $= 23 \times 23$

$\therefore \sqrt{529} = \sqrt{23 \times 23}$

$= 23$

(ii) 298116 $= 2 \times 2 \times 3 \times 3 \times 7 \times 7 \times 13 \times 13$

$\therefore \sqrt{298166} = \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{7 \times 7} \times \underline{13 \times 13}$

$= 2 \times 3 \times 7 \times 13 = 546$

2	298116
2	149058
3	74529
3	24843
7	8281
7	1183
13	169
13	13
	1

Example 6. 1764 students are siting in an auditorium in such a manner that there are as many students in a row as there are rows in the auditorium. How many rows are there in the auditorium?

Solution : Because number of rows and number of students in a row are equal. Therefore we must find the square root of 1764.

$1764 = 2 \times 2 \times 3 \times 3 \times 7 \times 7$

$\therefore \sqrt{1764} = \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{7 \times 7}$

$= 2 \times 3 \times 7$

$= 42$

$\therefore$ No. of rows in the auditorium = 42

2	1764
2	882
3	441
3	147
7	49
7	7
	1

Square roots of perfect squares by method of long division

When the square numbers are very large. The method of finding their square roots by prime factorization becomes very lengthy and difficult also. In such cases, we use the method of long division to find the square root.

Let us illustrate the division method by the taking the number 54776.

Step 1 : Place a bar over every pair of digits starting form the unit digit.

Step 2 : Bring down the number under the next bar to the right of the remainder.

$\sqrt{54776} = 234$

	234
2	$\overline{5}\,\overline{47}\,\overline{76}$
	4
43	147
	129
469	1876
	1876
	0

Step 3 : Double the quotient and enter it with a blank on the right or the next digit of the next possible divisor.

Step 4 : Guess the largest possible digit to fill in the blank and also become the new digit in the quotient.

Step 5 : Bring down the number under the next bar to the right of the new remainder.

Step 6 : Repeat above steps till all bars have been considered.

Example 7 : (1) Find the square root of : (i) 4489 (ii) 46656 (iii) 54756 (iv) 390625

(i)

	67
6	$\overline{44}\,\overline{89}$
	36
127	889
	889
	×

$\therefore \sqrt{4489} = 67$

(ii)

	216
2	$\overline{4}\,\overline{66}\,\overline{56}$
	4
41	× 66
	41
426	2556
	2556
	×

$\therefore \sqrt{46656} = 216$

(iii)

	234
2	$\overline{5}\,\overline{47}\,\overline{56}$
	4
43	147
	129
464	1856
	1856
	×

$\therefore \sqrt{54756} = 234$

(iv)

	625
6	$\overline{39}\,\overline{06}\,\overline{25}$
	36
122	306
	244
1245	6225
	6225
	×

$\therefore \sqrt{390625} = 625$

(2) Find the least number which must be subtracted from 18265 to make it a perfect square. Also, find the square root of the resulting number.

	135
6	$\overline{1}\,\overline{82}\,\overline{65}$
	1
23	82
	69
265	1365
	1325
	40

Now we find the remainder in the last steps is 40. This means if 40 be subtracted from the given number the remainder will be zero and the new number will be a perfect square.

Hence the required least number = 40

$18265 - 40 = 18225$

Also $18225 = (135)^2$

Exercise 5.3

1. **Do the following numbers have perfect double powers in their prime factorisation ?**

   (a) 60 (b) 64 (c) 81 (d) 98

2. **Find square roots by prime factorisation.**

   (a) 7056 (b) 8100 (c) 9216

   (d) 11025 (e) 225

3. **Justify if the numbers are perfect squares and if so find their roots.**

   (a) 15876 (b) 17424

4. Find the least number with which the numbers 1575 should be multiplied and divided to make it a perfect square.

5. By what number should the number 726 be multiplied to make it a perfect square?

6. The students of a class arranged a picnic. Each of the students contributed as many rupees as the students in the class. The total contribution was ₹ 1225. Find the strength of the class.

7. Find the least square number which is exactly divisibly by each of the numbers 3, 5, 8, 12, 15 and 20.

8. Find the least square number which is exactly divisible by each of the numbers 3, 5, 6, 9, 15 and 20.

Square Root by Long Division Method

Solved Example : Find square root of $10\frac{2}{3}$, Correct up to two decimal places.

Solution : $10\frac{2}{3} = \frac{32}{3} = 10.66666......$

	3.265
3	10.666666
	-9
62	1 66
	-1 24
646	4266
	-3876
6525	39066
	32625
	6441

$\therefore \sqrt[10]{\frac{2}{3}}$ =3.265n
=3.27
(up to 2 decimal 6525 places)

Steps of Long Division Method.

1. Group the digits of perfect square in pairs by underlining it. Starting with the digits in units place. Each pair of the digits and the remaining unpaired digits if any are called the periods.
2. The paired or unpaired digits at the highest place value is the first period. Find the largest number whose square is just equal to or just less than the first period. Write this number as divisor as well as the quotient.
3. Subtract the product of square of the divisor from the first period and bring down the digits of the next period. These digits along with the remaining digits if any become the new dividend.
4. Now double the quotient or the divisor as both are equal and write the number by the side of the dividend.
5. Think of another digit which when multiplied by itself and the dividend brought down is just equal to or just less than the dividend.
6. Write the product below the dividend accordingly and subtract. If remainder is not zero repeat the procedure till we get 0 at the remainder place. The quotient obtained is the square root of the perfect square.

Advantages of Long Division Method of Finding Square Root, Over Prime Factorisation Method

1. The long division method is a quicker method to find square roots of perfect squares.
2. Long division methods are suitable for finding square root of long and big perfect square numbers.
3. Long division method can be used to obtain square roots in decimals of numbers, which are not perfect squares.

Example : Find square root of 17424 by long division method.

Solution : $\sqrt{17424}$ = ?

	132
1	17424
+1	1
23	74
+3	69
262	0524
+2	0524
264	000

$\therefore \sqrt{17424} = 132$

Example : Find the square root of 10609 by long division method

Solution : $\sqrt{10609}$ = ?

	103
1	10609
+1	1
20	006
+ 0	00
203	0609
+3	0609
206	000

$\therefore \sqrt{10609} = 103$

Example : Evaluate — $\sqrt{66049}$

Solution :

	257
2	66049
+2	4
45	260
5	225
507	03549
+7	03549
514	000

$\therefore \sqrt{66049} = 257$

Example : Find the greatest number of 4 digits which is a square number.

Solution : The greatest 4 digits number is 9999 trying to find square root of 9999.

	99
9	9999
+9	81
189	1899
+9	1701
198	0198

The remainder shows that 9999 is not a perfect square and 99^2 is 198 less than 9999.

Therefore, 9999 – 198 = 9801

Hence, 9801 is the greatest four digit number which is a perfect square.

Example : Find the smallest four digit number which is a perfect square.

Solution : The smallest four digit number is 1000. Let us try to find out the square root of 1000.

	32
3	1000
+3	9
62	100
+2	124

The division shows that 1000 is not a perfect square the required number is (124 – 100) + 1000 = 24 + 1000 = 1024 **Ans.**

Example : What should be subtracted from 66060 to make it a perfect square?

Solution : Let us try to find out the square roots by long division method.

	257
2	66060
+2	4
45	260
+5	225
507	3560
+7	3549
514	0011

The remainder shows that the no. 66060 is greater by 11 to be a perfect square.

Required no is : 66060 – 11 = 66049

Example : What should be added to 5600 to make it a perfect square ?

	74
7	$\overline{56}\,\overline{00}$
+7	49
144	700
+4	576
148	124

Solution : Let us try to find out the square root of 5600 by long division method.

We observe that $74^2 = 5476 < 5600 < 5625 = 75^2$.

Therefore, the required number to be added is

$75^2 - 5600 = 5625 - 5600 = 25$

25 should be added to 5600 to make it a perfect square.

Example : Find the cost of creating a fence around a field whose area is 4 hectare at the cost of ₹ 45 per meter.

Solution :

1 hectare = $10{,}000 m^2$

Area of the field = $10{,}000 m^2 \times 4 = 40000 m^2$

Cost of fencing = Perimeter × Cost

Perimeter = 4 × Side

Area of square = S^2

$S = \sqrt{\text{Area of square field}}$

$= \sqrt{40{,}000\text{ m}^2}$

$= 200m$

Perimeter = $4 \times 200m = 800m$

Cost = $800m \times 45$

= ₹ 36000.

Exercise 5.4

1. $\sqrt{193600}$
2. $\sqrt{119025}$
3. $\sqrt{390625}$
4. $\sqrt{99856}$
5. $\sqrt{49284}$
6. $\sqrt{92416}$
7. $\sqrt{19600}$
8. $\sqrt{17956}$
9. $\sqrt{10404}$
10. $\sqrt{11449}$
11. $\sqrt{14161}$
12. $\sqrt{6241}$
13. Find the least number which must be subtracted from 390700 to make it a perfect square.
14. Find the least number which must be subtracted from 18625 to make it a perfect square.
15. Find the least number which must be subtracted from 19625 to make it a perfect square.
16. Find the least number which must be added to 92400 to make it a perfect square.
17. What should be added to 17900 to make it a perfect square?
18. What number should be added to 8400 to make it a perfect square? Find the square root of the number obtained.
19. Find the greatest number of four digits, which is a perfect square.
20. Find the cost of creating a fence around a square field of area 9 hectares at a cost of ₹ 25 per metre.

Square Roots of Rational Numbers

Law : 1 1. $\sqrt{m \times n} = \sqrt{m} \times \sqrt{n}$

2. $\sqrt{\dfrac{m}{n}} = \dfrac{\sqrt{m}}{\sqrt{n}}$

Example : Find the square of $\dfrac{49}{64}$

Solution : $\sqrt{\dfrac{49}{64}} = \dfrac{\sqrt{49}}{\sqrt{64}} = \dfrac{\sqrt{7^2}}{\sqrt{8^2}} = \dfrac{7}{8}$

Example : Find square root of $\dfrac{225}{441}$

Solution : $\sqrt{\dfrac{225}{441}} = \dfrac{\sqrt{225}}{\sqrt{441}} = \dfrac{\sqrt{15^2}}{\sqrt{21^2}} = \dfrac{15}{21}$

Example : Evaluate $\dfrac{11025}{15876}$

Solution :

	105
1	$\overline{1}\,\overline{10}\,\overline{25}$
+1	1
20	× 10
+0	0
205	1025
5	1025
	0000

	126
1	$\overline{1}\,\overline{58}\,\overline{76}$
+1	1
22	058
+2	44
246	1476
6	1476
	×

$= \dfrac{\sqrt{11025}}{\sqrt{15876}} = \dfrac{\sqrt{105^2}}{\sqrt{126^2}} = \dfrac{\sqrt{105^2}}{\sqrt{126^2}} = \dfrac{105}{126}$

Example : Evaluate $\sqrt{0.2916} = 0.54$

Solution :

	0.54
5	0.2916
+5	25
104	416
4	416
	000

Example : Evaluate $\sqrt{4\dfrac{29}{49}}$

Solution : $\sqrt{4\dfrac{29}{49}} = \sqrt{\dfrac{225}{49}} = \sqrt{\dfrac{15^2}{7^2}} = \dfrac{15}{7}$

Solution by Rule — I

Example : Evaluate $\sqrt{0.0196}$

Solution : $\sqrt{0.0196} = 0.14$

	0.14
1	0.0196
+1	1
24	96
4	96
	00

Solution by Rule —II

$$\sqrt{0.0196} = \sqrt{\frac{196}{10000}} = \sqrt{\frac{14^2}{100^2}} = \frac{14}{100} = 0.14$$

Example : Evaluate $\sqrt{\frac{10000}{1000000}}$

Solution : $\sqrt{\frac{10000}{1000000}} = \frac{\sqrt{100^2}}{\sqrt{1000^2}} = \frac{100}{1000} = 0.1$

Example : Evaluate $\sqrt{156.25}$

Solution by Rule —I

Solution : $\sqrt{1546.25} = 12.5$

	12.5
1	156.25
+1	1
22	56
+2	44
245	1225
5	1225
	0000

Solution by Rule —II

$$\sqrt{156.25} = \sqrt{\frac{15625}{100}} = \sqrt{\frac{125^2}{10^2}} = \frac{125}{10} = 12.5$$

Example : Find square root of 3 to three places of decimal.

Solution : $\sqrt{3}$

	1.73
1	3.0000
+1	1
27	200
+7	189
343	1100
3	1029
	071

$\sqrt{3} = 1.73$

Exercise 5.5

1. Find square roots:

(a) $\frac{625}{729}$ (b) $\frac{121}{256}$ (c) $\frac{64}{225}$ (d) $\frac{16}{81}$

2. **Evaluate :**

(a) $\sqrt{\frac{2116}{15129}}$ (b) $\sqrt{\frac{361}{625}}$ (c) $\sqrt{\frac{110889}{308025}}$ (d) $\sqrt{\frac{16641}{4489}}$

3. **Find the square roots :**

(a) $\frac{80}{405}$ (b) $3\frac{33}{289}$ (c) $4\frac{73}{324}$ (d) $3\frac{13}{36}$ (e) $23\frac{394}{729}$ (f) $56\frac{569}{1225}$

4. **Evaluate :**

(a) $\sqrt{98} \times \sqrt{162}$ (b) $\sqrt{\frac{1183}{2023}}$

5. **Evaluate the square root :**

(a) 0.0169 (b) 0.1 (c) 23.1 (d) 1.7

(e) 84.8241 (f) 9.3025 (g) 16.81 (h) 7.29

6. **Find square roots of the following numbers.**

(a) 0.2916 (b) 1.0816 (c) 10.0489 (d) 9.8596 (e) 75.69

7. Evaluate $\sqrt{2}$ up to two places of decimals.

8. A square and a rectangle have equal areas. The length of the rectangle is 13.6 metres and breadth 3.4 metres. Find length of sides of the square.

9. Find square of $\sqrt{\frac{243}{363}}$

10. Evaluate $\sqrt{1\frac{56}{169}}$

11. Evaluate $\sqrt{0.8}$ up to two places of decimal.

12. Find the square root of the following numbers in decimal form :

(a) 84.8241 (b) 0.813604 (c) 0.008464 (d) 0.0576 (e) 0.000169

13. What is that fraction which when multiplied by itself gives 227.798649?

14. **Find the square root of :**

(a) $\sqrt{4\frac{29}{49}}$ (b) $\sqrt{407\frac{37}{121}}$

15. **Find the square root of the following fractions to two decimal places:**

(a) $\frac{3}{8}$ (b) $1\frac{2}{7}$ (c) $\frac{0.625}{12}$

16. What is the fraction which when multiplied by itself gives 0.00053361 ?

17. Find the square root of 924.831 up to three decimal places.

18. In a basket there are 1250 flowers. A man goes for worship and puts as many flowers in each temple as there are templesin the city. Thus, he needs 8 baskets of flowers. Find the number of temples in the city.

Points to Remember :

- A natural number n is perfect square if $n = m^2$ for some natural number m.
- The square of a natural number is the product of the number with number itself Thus $a^2 = (a \times a)$.
- A perfect square number is never negative.
- A number ending in 2, 3, 7 or 8 is never a perfect square.
- All perfect square numbers ending in zeroes have even numbers of zeroes. A perfect square number ending in odd number of zeroes are never perfect squares.
- The square of an even number is always even and the square of an odd number is always odd.
- There are no natural numbers m and n such that $m = 2n^2$.
- For any natural number n we have n^2= sum of the first n odd numbers.
- For any natural number n greater than 1 we have, $(2n, n^2 - 1, n^2 + 1)$ called Pythagorian Triplets.
- The square roots of a number 'a' is that number which when multiplied by itself gives 'a' as the product written in the form of $\sqrt{a} \times \sqrt{a} = a$.
- By prime factorisation of number we can find out the square root of the number.
- The square root of a number can also be found by long division.
- In the division method of finding square roots, the pairing of integral part of the number starts from right to left and for decimal part it starts from left to right.
- Approximate values of square roots could be found for those numbers which are not perfect squares.
- if '*a*' and '*b*' are not perfect squares then to find $\sqrt{\frac{a}{b}}$ we can first convert them to decimal numbers and then use division method to find their square roots.
- If *a* and *b* are natural numbers then
  (i) $\sqrt{a \times b} = \sqrt{a} \times \sqrt{b}$ (ii) $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$
- $\sqrt{n}$ is never a rational number if n is not a perfect square.
- For finding the square root of a decimal fraction, make the numbers of decimal places even by adding zeroes. Put the decimal point in the square root as soon as the integral part is exhausted.

EXERCISE

1. MULTIPLE CHOICE QUESTIONS (MCQs) :

Tick (✓) the correct options.

(a) Which of the following is a perfect square?

(i) 47 (ii) 22 (iii) 63 (iv) 25

(b) Which of the following is not a perfect square?

(i) 48 (ii) 49 (iii) 4 (iv) 36

(c) The square of a proper fraction is .... a fraction.

(i) greater than (ii) smaller than (iii) equal to (iv) None

(d) The sum of first 'n' odd natural numbers is equal to —

(i) n^2 (ii) $n^2 + 1$ (iii) n+1 (iv) 2n

(e) For any natural number, n > 1, which of the following is a Pythagorean triplet?

(i) 2n–1, 2n, 2n+1 (ii) $2n, n^2 - 1, n^2 + 1$

(iii) 2n, 2n–1, 2n+1 (iv) None of these

(f) For every n natural numbers $\{(n+1)^2 - n^2\} = ?$

(i) {(n+1)+n} (ii) {(n+1)–n}

(iii) {(n–1)+n} (iv) None of these

(g) Which of the following methods is used to find the square of a number?

(i) Column Method (ii) Lattice Method

(iii) Both (a) and (b) (iv) None of these

2. Find the possible number of digits in the square of the following numbers :

(a) 8 (b) 65 (c) 125 (d) 1060

3. Write the possible digit at the ones place in the square root of the following numbers :

(a) 9801 (b) 99856 (c) 998001 (d) 657666025

4. Find the square of the following numbers using column method :

(a) 19 (b) 53 (c) 72 (d) 89

5. Find the square of the following numbers using diagonal method :

(a) 17 (b) 145 (c) 289 (d) 678

6. Find the square root of the following numbers by successive subtraction :

(a) 36 (b) 49 (c) 100 (d) 225

7. Find square root of the following numbers by using their ones and tens digits :

(a) 324 (b) 625 (c) 5929 (d) 18496

8. Using prime factorisation, find the square root of the following numbers :

(a) 256 (b) 1444 (c) 5184 (d) 90000

HOTS

1. Find $\sqrt{12996}$ and hence evaluate $\sqrt{0.012996} + \sqrt{1.2996} + \sqrt{129.96}$.

2. Evaluate $\dfrac{\sqrt{0.7569}+\sqrt{0.4761}}{\sqrt{0.7569}-\sqrt{0.4761}}$

Lab Activity

Objective : To Identify and write appropriate numbers on the number line.

Materials Required : A pen.

Procedure : Fill in the missing numbers on each number line given below.

1.


2.


3.


4.


5.


6 Cubes and Cube Roots

The cube of a number 'n' = $n \times n \times n = n^3$

Let $n_1 = 2$

$n_2 = 3$

$n_3 = 5$

Then $(n_1)^3 = 2^3 = 2 \times 2 \times 2 = 8$

$(n_2)^3 = 3^3 = 3 \times 3 \times 3 = 27$

$(n_3)^3 = 5^3 = 5 \times 5 \times 5 = 125$

The natural numbers 8, 27 and 125 are perfect cubes because they are cubes of natural numbers n_1, n_2 and n_3 respectively. These numbers can also be prime factorised to form cubes of some natural numbers.

Perfect Cubes

Prefect cubes are those natural numbers which can be expressed as the product of triplets of equal factors.

The definition of perfect cubes is a test to find if a natural number is a perfect cube. Resolve it into prime factors. If it forms the triplets of the same factors it is a perfect cube.

Example : $2^3 = 2 \times 2 \times 2 = 8$ that is the cube 2 is 8.

$4^3 = 4 \times 4 \times 4 = 64$ that is the cube of 4 is 64.

Perfect Cube : A natural number is said to be a perfect cube if it is the cube of some natural number. In other words, a natural number n is a perfect cube if there exists a natural number m whose cube is n. i.e

$$n = m^3$$

Example 1: Find - cube root of rational number

i.e $\sqrt[3]{\dfrac{125}{512}}$

Solution :

$$\frac{\sqrt[3]{125}}{\sqrt[3]{512}} = \frac{\sqrt[3]{5 \times 5 \times 5}}{\sqrt[3]{8 \times 8 \times 8}} = \frac{5}{8}$$

Example 2 : Find the cube of:

(i) $\dfrac{-8}{11}$ (ii) $\dfrac{7}{9}$ (iii) 1.5 (iv) 0.08

(i) $\left(\dfrac{-8}{11}\right)^3 = \dfrac{-8 \times -8 \times -8}{11 \times 11 \times 11} = \dfrac{-512}{1331}$

(ii) $\left(\frac{7}{9}\right)^3 = \frac{7\times7\times7}{9\times9\times9} = \frac{343}{729}$

(iii) $(1.5)^3 = \left(\frac{15}{10}\right)^3 = \frac{15\times15\times15}{10\times10\times10} = \frac{3375}{1000} = 3.375$

(iv) $(0.08)^3 = \left(\frac{8}{100}\right)^3 = \frac{8\times8\times8}{100\times100\times100} = \frac{512}{1000000} = 0.000512$

Example 3 : Is 256 a perfect cube?

Solution : Grouping the factors in triplets of equal factors we get

$256 = (2\times2\times2)(2\times2\times2)\times2\times2$

Clearly after grouping there are two triplets of 2 and one doublet, that indicates that 256 is not a perfect cube.

2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

Example 4 : Is 216 a perfect cube? What is that number whose cube is 216?

Solution : 216 into Prime factors, we get

$216 = (2\times2\times2)\times(3\times3\times3)$

We find that the prime factors of 216 can be grouped into triplets of equal factors and no factor is left over

Therefore, 216 is a perfect cube

Taking one factor from each triplet. We obtain $2\times3 = 6$.

Thus 216 is the cube of 6.

2	216
2	108
2	54
3	27
3	9
3	3
	1

Some Properties Of Cubes Of Natural Numbers

1 Cubes of all even natural numbers are even.

2. Cubes of all odd natural numbers are odd.

3. Cubes of negative integers are negative.

Example 5 : What is the smallest number by which 392 must be multiplied so that the product is a perfect cube.

Solution : $392 = (2\times2\times2)\times7\times7$

Grouping the factors in triplets of equal factors we get

$392 = (2\times2\times2)\times7\times7$

We find that 2 occurs as a prime factor of 392 thrice but 7 occurs as a prime factor only twice. Thus if we multiply 392 by 7, 7 will also occur as a prime factor thrice and the product will be $2\times2\times2\times7\times7\times7$ which is a perfect cube. Hence we must multiply 392 by 7 so that the product becomes a perfect cube.

2	392
2	196
2	98
7	49
7	7
	1

Example 6 : What is the smallest number by which 8640 must be divided so that the quotient is a perfect cube?

Solution : $8640 = \underline{2\times2\times2}\times\underline{2\times2\times2}\times\underline{3\times3\times3}\times5$

We note that 2, 2 and 3 occurs as a prime factor of 8640 thrice but 5 occurs as a prime factor only once.

If we divide 8640 by 5, the quotient would be $\underline{2\times2\times2}\times\underline{2\times2\times2}\times\underline{3\times3\times3}$ which is a perfect cube. Therefore, we must divide 8640 by 5 so that the quotient is a perfect cube.

2	8640
2	4320
2	2160
2	1080
2	540
2	270
3	135
3	45
3	15
5	5
	1

Example 7 : If one side of the cube is 16 meters, find its volume.

Solution : One side of the cube = 16 m

Volume of the cube = $(\text{Side})^3$

$= (16)^3 = 16 \times 16 \times 16$

$= 4096\ m^3$

Exercise 6.1

1. **Find the cubes of following numbers.**

   (a) 7 (b) 21 (c) 40 (d) 100

2. **Write the cubes of all natural numbers between 1 and 10 and verify the following statements.**

   (a) Cubes of all odd natural numbers are odd.

   (b) Cubes of all even natural numbers are even.

3. **Which of the following are perfect cubes?**

   (a) 64 (b) 108 (c) 1000 (d) 1728 (e) 243

4. Find the volume of cube whose surface area is 384 m^2.

5. **Which of the following numbers are not perfect cubes?**

   (a) 64 (b) 216 (c) 243 (d) 1728

6. **Find the cubes of:**

   (a) $\frac{4}{9}$ (b) $\frac{-13}{8}$ (c) 0.001 (d) –2.9

7. Prove that if a number is triple then its cube is 27 times the cube of the given number.

8. Find the smallest number by which 8788 must be divided so that the quotient is a perfect cube.

9. **Which of the following are cubes of even natural numbers?**

   512, 729, 1000, 3375, 13824

10. **Which of the following are cubes of odd natural numbers?**

    125, 343, 1728, 4096, 32788, 6859

11. If one side of the cube is 14 metres, find its volume.

12. **Write (T) or (F) for the following statements.**

    (a) 3375 is a perfect cube.

    (b) 243 is not a perfect cube.

    (c) No cube can end with exactly two zeroes.

    (d) If a^2 ends in an even number of zeros, then a^3 ends in an odd number of zeros.

(e) There is no perfect cube which ends in 4.

(f) Cubes of all even natural numbers are odd.

(g) The cube of a number is that number raised to the power3.

(h) For an integer a, a^2 is always greater than a^3.

13. Find the cube root of each of the following natural numbers.

(a) 4913 (b) 1728 (c) 17576 (d) 35937 (e) 1157625

14. Find the smallest number which when multiplied with 3600 will make the product a perfect cube. Further find the cube root of the product.

15. The volume of a cube is $9261000 m^3$. Find the side of the cube.

16. Find the cube roots of each of the following integers.

(a) –125 (b) –8000 (c) –3375 (d) –753571

17. Find the cube roots of each of the following rational numbers.

(a) $\frac{-125}{729}$ (b) $\frac{64}{1331}$ (c) $\frac{10648}{12167}$ (d) $\frac{9261}{42875}$

18. Find the cube roots of each of the following numbers.

(a) 8×125 (b) -1728×216 (c) -729×-15625

19. Evaluate—

(a) $\sqrt[3]{36} \times \sqrt[3]{384}$ (b) $\sqrt[3]{96} \times \sqrt[3]{144}$ (c) $\sqrt[3]{100} \times \sqrt[3]{270}$

20. The volume of a cubical box is 13.824 cubic metres. Find the length of each side of the box.

21. Show that—

(a) $\frac{\sqrt[3]{729}}{\sqrt[3]{1000}} = \sqrt[3]{\frac{729}{1000}}$ (b) $\frac{\sqrt[3]{-512}}{\sqrt[3]{343}} = \sqrt[3]{\frac{-512}{343}}$

22. Find the cube roots of the following number by finding their units and tens digits.

(a) 389017 (b) 110592 (c) 46656

23. Find the side of a cube whose volume is $\frac{24389}{216} m^3$.

24. Find the units digit of the cube root of the following numbers.

(a) 226981 (b) 13824 (c) 571787

25. Divide 88209 by the smallest number so that the quotient is a perfect cube. What is that number? Also, find the cube root of the quotient.

Cube Roots : The cube root of a number x is that number whose cube is given x.

The cube root of x is denoted by the symbol $\sqrt[3]{x}$

Thus $\sqrt[3]{8} = 2$ $\sqrt[3]{27} = 9$ $\sqrt[3]{64} = 4$ $\sqrt[3]{125} = 5$

Cube Root Of A Perfect Cube By Factors : In order to find the cube root of a perfect cube by factors, we follow the following procedure.

(1) Obtain the given number.

(2) Resolve it into prime factors.

(3) Group the factors in triplets such that all the three factors in each triplet are equal.

(4) Take one factor from each triplet formed in step III.

(5) Find the product of factors obtained in step IV this product is the requested cube root.

The above procedure is illustrated in the following examples.

2	13824
2	6912
2	3456
2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
3	3
	1

Examples 8 : Find the cube root of 13824

$13824 = (2\times2\times2) \times (2\times2\times2) \times (2\times2\times2) \times (3\times3\times3)$

Taking one factor from each triple we get

$\sqrt[3]{13824} = 2\times2\times2\times3 = 24$

Cube Root of A Negative Integral Perfect Cube

If a is a positive integer then - a is negative integer.

We know that $(-a)^3 = -a^3$

So $\sqrt[3]{-a^3} = -a$ $\sqrt[3]{-x^3} = -x$

In general we have $\sqrt[3]{-x} = (-x)^{\frac{1}{3}} = -3\sqrt{x}$

Example 9 : Find the cube root of –1728.

Solution : We have $\sqrt[3]{-1728} = -\sqrt[3]{1728}$

Now resolving 1728 in prime factors, we get

$1728 = (2\times 2\times2) \times (2\times 2\times2) \times (3\times3\times3)$

$\therefore \sqrt[3]{1728} = 2 \times 2 \times 3 = 12$

Hence $\sqrt[3]{-1728} = -\sqrt[3]{1728} = -12$

2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
3	3
	1

Cube root of a rational number : If x and a are two rational numbers such that $x^3 = a$. Then we say that x is the cube root of a and we write: $x = \sqrt[3]{a}$

Example10 : Find the cube root of each of the following numbers: $\frac{1331}{4096}$

11	1331
11	121
11	11
	1

Solution : We have : $\sqrt[3]{\frac{1331}{4096}} = \frac{\sqrt[3]{1331}}{\sqrt[3]{4096}}$

$1331 = 11 \times 11 \times 11$

$4096 = (2\times2\times2)\times(2\times2\times2)\times(2\times2\times2)\times(2\times2\times2)$

$= \therefore \sqrt[3]{1331} = 11$ and $\therefore \sqrt[3]{4096} \quad 2\times2\times2\times2 = 16$

Hence $\sqrt[3]{\dfrac{1331}{4096}} = \dfrac{\sqrt[3]{1331}}{\sqrt[3]{4096}} = \dfrac{11}{16}$

2	4096
2	2048
2	1024
2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

Example 11 : Multiply 2592 by the smallest number so that the product is a perfect cube. Also, find the cube root of the product.

Solution : $2592 = \underline{2\times2\times2}\times\underline{3\times3\times3}\times2\times2\times3$

We know that if a number is to be perfect cube then each of its prime factors must occur thrice or in multiples of three. Hence, the smallest number by which the given number must be multiplied in order that the product is cube is $2\times3\times3 = 18$

Product $= 2592 \times 18$

$= 46656$

$= \underline{2\times2\times2}\times\underline{3\times3\times3}\times\underline{2\times2\times2}\times\underline{3\times3\times3}$

$\therefore \sqrt[3]{46656} = 2\times3\times2\times3 = 36$

2	2592
2	1296
2	648
2	324
2	162
3	81
3	27
3	9
3	3
	1

Solved Examples

Example : Show that 160 is not a perfect cube. Find prime factorisation of 160.

2	160
2	80
2	40
2	20
2	10
5	5
	1

Solution : $160 = 2\times2\times2\times2\times2\times5$

In the prime factorisation of 160 we find that, we have only one triplet of same factor. The three other number of the prime factorisation do not form the triplets of the same factor.

Therefore the natural number 160 is not a perfect cube.

Example : Show that 343 is perfect cube. Also find the number whose cube is 343.

7	343
7	49
7	7
	1

Solution : $343 = 7\times 7\times 7 = 7^3$

343 is the perfect cube of 7.

Example : Show that 216 is a perfect cube. Also the number whose cube is 216.

2	216
2	108
2	54
3	27
3	9
3	3
	1

Solution : $216 = 2\times2\times2\times3\times3\times3$

The number 216 is a perfect cube.

To find the number whose cube is 216. Group the triplets and take one factor from each of the triplets and find their product.

$2 \times 3 = 6$ or $6^3 = 216$

Example : Write 216 in the form of triplets of only one type of factors.

Solution : $216 = 6 \times 6 \times 6$

Example : Find the prime factorisation of 2560.

Solution : $2560 = 2\times2\times2\times2\times2\times2\times2\times2\times2\times5$

2	2560
2	1280
2	640
2	320
2	160
2	80
2	40
2	20
2	10
5	5
	1

In the prime factorisation we see that we have three triplets of similar factors. We are left with a factor which does not form a triplet.

To make it a triplet we should multiply it with $5\times5 = 25$

The least number with which 2560 should be multiplied to make it perfect square is 25.

Example : Find the number with which 3087 may be multiplied to make it a perfect cube.

Solution : $3087 = 3\times3\times7\times7\times7$

3	3087
3	1029
7	343
7	49
7	7
	1

It may be multiplied with 3 to make it a perfect cube.

Perfect cube $= 3087 \times 3 = 9261$

$3\times7 = 21$

The number is a cube of 21

or $21^3 = 9261$

Example : Find the least number with which 3087 may be divided to make it a perfect cube.

Solution : $3087 = 3\times3\times7\times7\times7$

3087 ¸ (3×3)

= Perfect cube or 3087 ¸ 9

= Perfect cube.

Cube of a Rational Number

$$\left(\frac{a}{b}\right)^3 = \frac{a^3}{b^3}$$

Solved Examples

Example : Evaluate

Solution : $$\left(\frac{2}{3}\right)^3 = \frac{2^3}{3^3} = \frac{2\times2\times2}{3\times3\times3} = \frac{8}{27}$$

Properties of Cubes

1. Cubes of even number are always even.
2. Cubes of odd numbers are always odd.
3. Cubes of negative integers are always negative.
4. Cubes of positive integers are always positive.

Example : Evaluate—

Solution : $\left(\frac{-3}{5}\right)^3 = \frac{-3^3}{5^3} = \frac{-3 \times -3 \times -3}{5 \times 5 \times 5} = \frac{-27}{125}$

Example : Find cubes of—

(i) 0.06

SOLUTION BY RULE – 1

(i) $(0.06)^3 = 0.06 \times 0.06 \times 0.06$

$= 0.000216$

SOLUTION BY RULE – II

$(0.06)^3 = \left(\frac{6}{100}\right)^3 = \frac{6}{100} \times \frac{6}{100} \times \frac{6}{100}$

$= \frac{216}{1000000} = 0.000216$

Example : Find cube root of $\sqrt[3]{\frac{-729}{1331}}$

Solution : $\sqrt[3]{\frac{-729}{1331}}$

$= \frac{\sqrt[3]{-729}}{\sqrt[3]{1331}} = \frac{-(\sqrt[3]{3 \times 3 \times 3 \times 3 \times 3 \times 3})}{\sqrt[3]{11 \times 11 \times 11}}$

$\Rightarrow \frac{-9}{11}$

Cubes of Numbers 1 to 25

$1^3 = 1$	$6^3 = 216$	$11^3 = 1331$	$16^3 = 4096$	$21^3 = 9261$
$2^3 = 8$	$7^3 = 343$	$12^3 = 1728$	$17^3 = 4913$	$22^3 = 10648$
$3^3 = 27$	$8^3 = 512$	$13^3 = 2197$	$18^3 = 5832$	$23^3 = 12167$
$4^3 = 64$	$9^3 = 729$	$14^3 = 2744$	$19^3 = 6859$	$24^3 = 13824$
$5^3 = 125$	$10^3 = 1000$	$15^3 = 3375$	$20^3 = 8000$	$25^3 = 15625$

Exercise 6.2

1. **Find cubes –**

(a) 11 (b) 21 (c) 15 (d) 100

(e) 2.5 (f) 3.5 (g) 0.8 (h) 1.2

2. **Evaluate–**

(a) $\left(1\frac{2}{3}\right)^3$ (b) $\left(1\frac{3}{10}\right)^3$ (c) $\left(\frac{1}{15}\right)^3$

3. **Which of the following numbers are perfect cubes?**

(a) 9261 (b) 5324 (c) 3375 (d) 27000

(e) 243 (f) 343 (g) 800000 (h) 125

4. **Which of the following are cubes of even numbers?**

(a) 8000 (b) 1331 (c) 216

(d) 729 (e) 3375 (f) 4096

5. Find the least number with which 196 should be multiplied to make it a perfect cube.

6. Which is the least number with which 1372 may be multiplied to make it a perfect cube?

7. Which is the least number with which 1600 may be divided to make it a perfect cube?

8. Find the least number with which 28561 should be divided to make it a perfect cube?

9. What is the least number with which 1323 should be multiplied to make it a perfect cube?

10. Convert into cubes of natural numbers.

(a) $\frac{216}{2197}$ (b) $\frac{125}{512}$ (c) 8000

11. **Justify–**

(a) If x leaves a remainder of 1 when divided by 5, then x^3 also leaves a remainder of 1 when divided by 5.

(b) If x is an even natural number then x^3 is also an even number.

(c) If x is an odd number then x^3 is also an odd number.

(d) If x is a negative integer then x^3 is also a negative integer.

(e) If a^2 ends in 4 then a^3 ends in 8.

(f) If a^2 ends in 9 then a^3 ends in 7.

(g) If a^2 ends in 6 then a^3 ends in 6.

Cube Roots

Cube Root : The cube root of a natural number 'n' is that number whose prime factors are triplets of m, then m is the cube root of n. They are written as $\sqrt[3]{\ }$ symbolically.

Example : Evaluate $\sqrt[3]{27}$ and $\sqrt[3]{125}$

Solution : $\sqrt[3]{27} = \sqrt[3]{3\times3\times3} = 3$

$\sqrt[3]{125} = \sqrt[3]{5\times5\times5} = 5$

Example : Find cube roots of the following perfect cubes.

(i) 2197 (b) 4913 (c) 8000

13	2197
13	169
13	13
	1

Solution : $\sqrt[3]{2197} = \sqrt[3]{13\times13\times13} = 13$

Properties of Cube Roots

1. $\sqrt[3]{-x} = -\sqrt[3]{x}$ 2. $\sqrt[3]{xy} = \sqrt[3]{x} \times \sqrt[3]{y}$ 3. $\sqrt[3]{\frac{x}{y}} = \frac{\sqrt[3]{x}}{\sqrt[3]{y}}$

4. $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

5. Cubes of number ending in 0, 1, 4, 5, 6 and 9 ends in 0, 1, 4, 5, 6 and 9 respectively.

6. The cube of number ending in 2 ends in 8.

7. The cube of a number ending in 8 ends in 2.

8. The cube of a number ending in 3 ends in 7.

9. The cube of a number ending is 7 ends in 3.

10. (i) $(1^3 - 0^3) = 1$ (ii) $2^3 - 1^3 = 7$ (iii) $3^3 - 2^3 = 19$

(iv) $4^3 - 3^3 = 37$ (v) $5^3 - 4^3 = 61$ (vi) $6^3 - 5^3 = 91$

(vii) $7^3 - 6^3 = 127$ (viii) $8^3 - 7^3 = 169$ (ix) $9^3 - 8^3 = 217$

(x) $10^3 - 9^3 = 271$ (xi) $11^3 - 10^3 = 331$ (xii) $12^3 - 11^3 = 397$

These number can be used to find cube roots by repeated subtraction method.

11. (i) $1^3 = (1^3 - 0^3)$

(ii) $2^3 = (1^3 - 0^3) + (2^3 - 1^3)$

(iii) $3^3 = (1^3 - 0^3) + (2^3 - 1^3) + (3^3 - 2^2)$

12. (i) $1^3 = (1 + 0\times 6)$

(ii) $2^3 = (1 + 0 \times 6) + (1 + 1\times6)$

(iii) $3^3 = (1 + 0 \times 6) + (1 + 1\times6) + (1 + 1\times6 + 2\times6)$

13. (i) $1^3 = (1 + 1\times0\times3)$

(ii) $2^3 = (1 + 1\times0\times3) + (1 + 2\times1\times3)$

(iii) $3^3 = (1 + 1\times0\times3) + (1 + 2\times1\times3) + (1 + 3\times2\times3)$

14. (i) $1^3 = 1$

(ii) $2^3 = 1 + 7$

(iii) $3^3 = 1 + 7 + 19$

(iv) $4^3 = 1 + 7 + 19 + 37$

(v) 5^3 = 1 + 7 + 19 +37 + 61

(vi) 6^3 = 1 + 7 + 19 +37 + 61+91

(vii) 7^3 = 1 + 7 + 19 +37 + 61+91+ 127

(viii) 8^3 = 1 + 7 + 19 +37 + 61+91+ 127 + 169

(ix) 9^3 = 1 + 7 + 19 +37 + 61+91+ 127 + 169 + 217

(x) 10^3 = 1 + 7 + 19 +37 + 61+91+ 127 + 169 + 217 + 271

Method of Finding Cube Roots by Prime Factorisation

1. Find prime factorisation and express the number as products of primes.
2. Make group of triplets of similar factors.
3. Chose one prime out of one triplet and find the product.
4. This product is the cube root.

17	4913
17	289
17	17
	1

2	8000
2	4000
2	2000
2	1000
2	500
2	250
5	125
5	25
5	5
	1

Example : Find cube roots of the following perfect cubes.

(a) 4913 (b) 8000

Solution : (a) $\sqrt[3]{4913} = \sqrt[3]{17 \times 17 \times 17} = 17$

(b) $\sqrt[3]{8000} = \sqrt[3]{\overline{2 \times 2 \times 2} \times 2 \times 2 \times 2 \times 5 \times 5 \times 5}$

Example : Evaluate $\sqrt[3]{-2744}$

Solution : $\sqrt[3]{-2744} = \sqrt[3]{-14 \times -14 \times -14} = -14$

2	2744
2	1372
2	686
7	343
7	49
7	7
	1

Example : Evaluate $\sqrt[3]{64 \times 343}$

Solution : $\sqrt[3]{64 \times 343} = \sqrt[3]{2 \times 2 \times 2 \times 2 \times 2 \times 2} \times \sqrt[3]{7 \times 7 \times 7}$

$= \sqrt[3]{4 \times 4 \times 4} \times \sqrt[3]{7 \times 7 \times 7}$

$= 4 \times 7 = 28$

Example : Resolving 6859

Solution : Resolving 6859 into prime factors, we have

We seen;

19	6859
19	1125
19	375
	1

(After grouping no factor is left)

∴ 6859 = 19x19x19

Grouping into triplets of identical prime factors, we have

6859 = (19x19x19)

Hence Perfect cube

Example : Evaluate $\sqrt[3]{125 \times -64}$

Solution :

$$\sqrt[3]{125 \times -64} = \sqrt[3]{125} \times \sqrt[3]{(-64)}$$

$$= \sqrt[3]{5 \times 5 \times 5} \times - \sqrt[3]{(4 \times 4 \times 4)}$$

$$= 5 \times -4 \quad = -20$$

Example : Evaluate $\sqrt[3]{\frac{-125}{512}}$

Solution :

$$\sqrt[3]{\frac{-125}{512}} = \frac{\sqrt[3]{125}}{\sqrt[3]{512}} = \frac{-\sqrt[3]{5 \times 5 \times 5}}{\sqrt[3]{8 \times 8 \times 8}} = \frac{-5}{8}$$

Exercise 6.3

1. Evaluate –

(a) $\sqrt[3]{3375}$ (b) $\sqrt[3]{4096}$ (c) $\sqrt[3]{9261}$ (d) $\sqrt[3]{46656}$

(e) $\sqrt[3]{91125}$ (f) $\sqrt[3]{389017}$ (g) $\sqrt[3]{551368}$ (h) $\sqrt[3]{531441}$

2. Find cube roots –

(a) $\frac{125}{216}$ (b) $\frac{-27}{125}$ (c) $\frac{-64}{343}$ (d) $\frac{729}{2197}$ (e) $\frac{343}{166375}$ (f) $\frac{3375}{4913}$

3. Evaluate –

(a) $\sqrt[3]{\frac{729}{1000}}$ (b) $\sqrt[3]{\frac{3375}{-9000}}$

4. What is the least number with which 6075 should be multiplied to make it a perfect cube? Also find the cube root of that number.

5. What is the least number by which 120393 should be divided so that the quotient has cube root?

6. Find the number with 33275 may be divided, so that the quotient has a cube root.

Alterative Methods of Finding Cube Roots and Cubes

Find cube roots by repeated subtraction :

Solved Example : Evaluate $\sqrt[3]{64}$ by repeated subtraction of numbers. (1, 7. 19, 37, 61, 91, 127, 169, 217, 331, 397...........)

Solution: Refer to values of (13 – 03), (23 – 13), (33 – 23) discussed in the section of characters of cubes, which are 1, 7, 19, ...

Subtract each of the numbers given in the brackets one by from the perfect cube.

$$\begin{array}{r} 64 \\ -1 \\ \hline 63 \\ -7 \\ \hline 56 \\ -19 \\ \hline 37 \\ -37 \\ \hline 0 \end{array}$$

We get the remainder 0 after subtracting 4 times.

$$\therefore \sqrt[3]{64} = 4$$

Solved Example : Evaluate $\sqrt[3]{216}$ by repeated subtraction method.

Solution : No. of subtraction = 6

$$\sqrt[3]{216} = 6$$

```
 216
 - 1
 215
  -7
 208
 -19
 198
 -37
 152
 - 61
  91
 - 91
   0
```

Solved Example : What may be subtracted from 350 to make it a perfect cube? Find by repeated subtraction method.

Solution : After repeated subtraction we get a remainder of 7. 7 should be subtracted from 350 to make it a perfect cube.

Number of subtraction = 7

Hence $350 - 7 = 343$

$$\sqrt{343} = 7$$

```
 350
 - 1
 349
  -7
 342
 -19
 323
 -37
 286
 - 61
 225
 - 91
 134
-127
   7
```

Exercise 6.4

1. Verify by repeated subtraction method if the following numbers are perfect cubes.

(a) 572 (b) 750 (c) 246

2. What should be subtracted from 139 to make it a perfect cube?

3. What should be added to 335 to make it a perfect cube. Find by repeated subtraction method.

4. Find the unit digit of the cube root of the following perfect cubes. Without actual calculation.

(a) 1331 (b) 1728 (c) 2197

(d) 2744 (e) 3375 (f) 4096

5. State true or false.

(a) Cube of 24 will end in 6. ....

(b) Cube of 69 will end in 1. ....

(c) Cube of 37 will end in 3. ....

(d) Cube of 42 will end in 4. ....

(e) Cube of 88 will end in 2. ....

Points to Remember :

- The cube of a number is the number raised to the power of 3.
- The cube of an even number is even and the cube of an odd number is odd.
- The square of a number cannot be negative, but the cube of a number can be negative.
- For any positive numbers a and b,

$$\sqrt[3]{a \times b} = \sqrt[3]{a} \times \sqrt[3]{b}$$

$$\sqrt[3]{\frac{a}{b}} = \frac{\sqrt[3]{a}}{\sqrt[3]{b}}$$

$$\sqrt[3]{-a^3} = -\sqrt[3]{a^3} = -a$$

EXERCISE

1. MULTIPLE CHOICE QUESTIONS (MCQs) :

Tick (✓) the correct options.

(a) Which of the following numbers is a perfect cube?

(i) 121 ☐ (ii) 145 ☐ (iii) 343 ☐ (iv) 215 ☐

(b) Which of the following numbers is not a perfect cube?

(i) 1331 ☐ (ii) 1441 ☐ (iii) 2197 ☐ (iv) 3375 ☐

(c) $\sqrt[3]{64 \times 27} = ?$

(i) 12 ☐ (ii) 4 ☐ (iii) 3 ☐ (iv) 8 ☐

(d) $\sqrt[3]{125} = ?$

(i) 5 ☐ (ii) 25 ☐ (iii) 125 ☐ (iv) 50 ☐

(e) $\sqrt[3]{\frac{-343}{512}} = ?$

(i) $\frac{4}{5}$ ☐ (ii) $\frac{6}{7}$ ☐ (iii) $\frac{-7}{8}$ ☐ (iv) $\frac{8}{9}$ ☐

2. Which of the following numbers are not perfect cubes?

(a) 25 (b) 512 (c) 13824 (d) 42875

3. Find the cube root of the following numbers by successive subtraction.

(a) 216 (b) 343 (c) 1331 (d) 2744

4. Find the cube root of the following numbers by using their ones and tens digits.

(a) 4096 (b) 6859 (c) 42875 (d) 103823

5. Find the cube root of the following numbers.

(a) 2744 (b) 5832 (c) 42875 (d) 74088

6. Find the smallest number by which 29160 should be divided so that the quotient becomes a perfect cube.

HOTS

1. If a divides b, then a^3 divides b^3. Is it true? Give reason to support your answer.
2. Find the value of the infinite product given below :

$$P = \frac{7}{9} \times \frac{26}{28} \times \frac{63}{65} \times \ldots \times \frac{k^3 - 1}{k^3 + 1} \times \ldots$$

Lab Activity

Objective : To find out the cube roots through a pattern by activity method.

Materials Required : Chart paper, geometry box, sketch pens.

Procedure : Write cubes of the first twenty natural numbers as follows :

Natural numbers	Cubes
1	1
2	8
3	27
4	64
5	125
6	216
7	343
8	512
9	729
10	1000
11	1331
12	1728
13	2197
14	2744
15	3375
16	4096
17	4913
18	5832
19	6859
20	8000

Using the numbers from the difference of cubes, make a pattern by completing the following table in your notebook.

1	1^3
1+7	2^3
1+7+19	3^3
1+7+19+37	4^3
1+7+19+37+61	5^3
1+7+19+37+61+91	6^3
1+7+19+37+61+91+127	7^3
	8^3
	9^3
	10^3
	11^3
	12^3
	13^3
	14^3
	15^3
	16^3
	17^3
	18^3
	19^3
	20^3

7 Algebraic Expressions and Identities

The word algebra derives its name from a book of mathematics titled Al –jabr wail Maquabaiahin 9th century by an Arab mathematician ibn Musa at Khuwarizmi. Moreover the first mathematicians to use algebraic notations were two ancient mathematicians, Aryabhata (inventor of zero) and Brahmagupta. They used ya (;) for the variable. This abbreviation ya corresponds to x of modern mathematics.

Algebraic Expressions

The symbols in mathematics which have a fixed value are called constants such as 1, 2, 3, 4, 5........ The symbols whose values are variable are called variables such as x, y, z, a, b, c, k, l and m etc. Expressions having variables and constants connected by fundamental variables operations of mathematics are called '**Algebraic Expressions**'. The operations of mathematics are +, –, × and ÷.

Terms : The parts of expression which are separated by the signs of positive (+) and negative (–) are called its terms.

Types of Algebraic Expressions

[Two add two or more algebraic expression, we collect the terms from each and them.]

Algebraic expressions are classified into different categories according to the number of terms they contain.

1. **Simple Expressions or Monomial** are the algebraic expressions which consist of **only one term**.

Examples : $3x$, $2x^2$, $3x^2y$, $8\,xyz$, etc.

2. **Binomial** are the algebraic expressions which consist of **two terms**.

Examples : $2x^2 + y$, $-4x^2 - xy$, $3x^2y + 4y^2x$, $-2x^2yz + 3xy$, etc.

3. **Trinomial** are the algebraic expressions which consist of **three terms**.

Examples : $2x^2y + 3xy + 2x^2$, $3x^2yz - 2x^2y + 5z$, etc.

4. **Polynomial or Compound Expressions** are the algebraic expressions which consist of **two or more terms**.

Examples : $4x - 5$,
$3x^2 - 4xy + 2z$,
$3a^2b - 5a + 6a^2 + 7b$,
$2a^2 - 4ab + 16a^2 + 7$, etc.

An algebraic expression of the form,

$$a + bx + cx^2 + dz^3 + ex^4 + ...$$

where, $a, b, c, d, e,$ .... are the constants

and x is a variable and is known as polynomial in x.

Degree of an Algebraic Expression

If an expression does not have any index or power, its index and power is regarded to be 1. Hence that is the degree of the polynomial.

For example : $5x + 2y + z$, $2x + 7y$ and 5a have no power on them. Therefore all the expressions have a degree of 1.

In $5a^2 + 3a$, $3k^2 + 7k$ and $5a^2$ the highest power of each expressions is 2. Therefore degree of each polynomial is 2.

In the expressions $x^3 + x$, $5a^3 + 2$, the highest power of each term is 3. So they are polynomials of third degree.

In multiplied polynomials the powers of literal factors of the expressions are added.

For example : In the polynomials (i) $3x^2y^5$, (ii) $\frac{1}{2}a^3a^2$

The degree of expression (i) is $5 + 2 = 7$

The degree of expressions (ii) is $3 + 2 = 5$.

Or we can say that (i) is a polynomial of seventh degree and (ii) is a polynomial of fifth degree.

Solved example : Change the following expression to algebraic expressions.

(i) The difference of 7 and a number.

(ii) The sum of a number and 5.

(iii) Product of a number and 11.

(iv) Multiply a number by 3 and add 5.

Solution : Let the number be 'a' then :

(i) $7 - a$ (ii) $a + 5$ (iii) $11a$ (iv) $3a + 5$

Like and Unlike Expressions

The expressions having the same literal factors are called like terms and those having different literal factors are called unlike terms.

Solved Example : Separate pairs of expressions into—

(i) Like and (ii) Unlike terms

(a) $-7a^2y, ya^2$ (c) $\frac{1}{3}$ Klm^2, $\frac{1}{3}$ Klm

(b) $-3a^2y, -3ay^2$ (d) $5xy^2z, -\frac{3}{4}xzy^2$

Solution : (a) and (d) are pairs of like terms

(ii) *b* and *c* are pairs of unlike terms.

Addition of Algebraic Expressions

While adding an algebraic expression, we collect the like terms and add them. The sum of sevral like terms is the sum of the coefficients of the like terms.

An algebraic expression is a combination of numerals, literal numbers (letters) and the symbols of the fundamental operations.

Solved Example : Add the following algebraic expressions—

(i) $5x^2y, -4x^2y, 3x^2y$

(ii) $m^2 - 3mn, m^2 + mn, mn + n^2$

(iii) $6a + 8b - 5c, 2b + c - 4a, a - 3b - 2c$

Solution : (i) $5x^2y, -4x^2y, 3x^2y$

$5x^2y + (-4x^2y) + 3x^2y$ (monomials)

$= 5x^2y - 4x^2y + 3x^2y$

$= (5 - 4 + 3)\,x^2y$ (by collecting the coefficients of the like terms)

$= 4x^2y$

(ii) $m^2 - 3mn, m^2 + mn$ and $mn + n^2$

$= (m^2 + m^2) + (-3mn + mn + mn) + n^2$ (by collecting like terms)

$= 2m^2 - mn + n^2$

(iii) $6a + 8b - 5c, 2b + c - 4a$ and $a - 3b - 2c$

To solve this expression, we shall use the vertical method and write the like terms in columns

$$\begin{array}{rrrr} & 6a & +\ 8b & -\ 5c \\ - & 4a & +\ 2b & +\ c \\ + & a & -\ 3b & -\ 2c \\ \hline & 3a & +\ 7b & -\ 6c \\ \hline = & 3a & +\ 7b & -\ 6c \end{array}$$

(trinomials)

Subtraction of Algebraic Expressions

Two add two or more algebraic expressions, we collect the like terms from each and add them. While subtracting, we write the given algebraic expressions in two rows in such a way that the like terms occur one below the other. The expression from which we have to subtract is in the first row and the expression which is to be subtracted is in the second row.

Solved Example : (i) $7x^2y^2$ from $-11x^2y^2$

(ii) $11\,ab^2 + ab$ from $16\,ab^2 - 3ab$

(iii) $4a + 5b - 3c$ from $6a - 3b + c$

Solution : (i) $7x^2y^2$ from $-11x^2y^2$

$(-11x^2y^2) - (7x^2y^2)$ (monomials)

$= -11x^2y^2 - 7x^2y^2$

$= -17x^2y^2$

(ii) $11\,ab^2 + ab$ from $16\,ab^2 - 3ab$

$= (16\,ab^2 - 3ab) - (11\,ab^2 + ab)$ (binomials)

$= 16\,ab^2 - 11\,ab^2 - ab$

$= (16\,ab^2 - 11\,ab^2) + (-3ab - ab)$

$= 5ab^2 - 4ab$

(iii) $4a + 5b - 3c$ from $6a - 3b + c$

We shall use the vertical method to solve this algebraic expression. We will write like terms below each other and change the signs of the terms to be subtracted.

$$\begin{array}{rrrrrrl} & 6a & - & 3b & + & c & \text{(trinomials)} \\ - & (4a & + & 5b & - & 3c) & \text{(by changing the signs)} \\ \hline \\ \hline = & 2a & - & 8b & + & 4c & \end{array}$$

$$\begin{array}{rrrrrr} & 6a & - & 3b & + & c \\ - & (4a & + & 5b & - & 3c) \\ \hline & 2a & - & 8b & + & 4c \\ \hline \end{array}$$

Solved Example : From the sum of $5a + 6b - 7c$ and $9a - 16b - 2c$ subtract $7a - 10b + 9c$.

Solution: Adding the two expressions we get,

$$\begin{array}{rrrrrr} & 5a & + & 6b & - & 7c \\ + & 9a & - & 16b & - & 2c \\ \hline & 14a & - & 10b & - & 9c \\ \hline \end{array}$$

Now by subtracting $7a - 10b + 9c$ from the above sum we get,

$$\begin{array}{rrrrrr} & 14a & - & 10b & - & 9c \\ - & (7a & - & 10b & + & 9c) \\ \hline \\ \hline = & 7a & - & 18c & & \end{array}$$

or

$$\begin{array}{rrrrrr} & 14a & - & 10b & - & 9c \\ - & 7a & + & 10b & - & 9c) \\ \hline & 7a & + & 0 & - & 18c \\ \hline \end{array}$$

Exercise 7.1

1. **Identify like and unlike terms.**
   - (a) $x, \frac{1}{3}x, 9x$
   - (b) $3xy, 4xy, 9xy$
   - (c) $x^3y^3z^3, -xyz, x^2y^2z^2$
   - (d) $5a^2b, -4a^2b, 5ba^2$
   - (e) $7x^2y, xy^2, -5xy^2$
   - (f) $3abc, 5ab, 7ac^2$
   - (g) $3x^2yz, -3x^2yz, -7x^2zy$
2. **Pick out like terms from each group of expressions.**
   - (a) $xy, yz, 5yx$
   - (b) $a, 7b$, and $5b$
   - (c) $5b, 3a, 2a$
   - (d) $6p, 8x, zy, 3x$
   - (e) $ab, a^2, 11a, 3a^2$
   - (f) $a^2z, 2az, 3az^2, 4a^2z$
   - (g) $9x, 7y, x, 8b, 6x$
3. **Change the statements to algebraic expressions.**
   - (a) Subtract 7 from a number
   - (b) 3 added to a number multiplied with 5.
   - (c) The sum of number and 11 multiplied by 7.
   - (d) 2 subtracted from the product of 6 and a number
   - (e) 7 multiplied to the difference of 8 and a number.
4. **Find the degree of the following polynomials.**
   - (a) $\frac{3}{5}$
   - (b) $4a+3b+c$
   - (c) $3x^2y^3 - 5x^2yz^2 - 3x^3$
   - (d) $5x^6 - y^3 + 7xy^8$
   - (e) $5x^4 - 3x^2 + 4x + 5$

5. **Classify the following polynomials as monomials, binomials and trinomials :**

(a) $2x + 3y$ (b) $m^2 + 4n$ (c) $\frac{3}{7}x^2$

(d) $8x^2 - 9y^2 + 3xy$ (e) $\frac{2}{9}x^3 - x - 2$ (f) $-3x$

6. **Add the following :**

(a) $2x - 5y - 4z$ and $3x + 5y - 4z$

(b) $-3a^2 - 2b^2 + 4, -a^2 - 5b^2 - 8$ and $2a^2 + 4b^2 + 6$

(c) $2x^2 - 3x + 6,\ x - 3x^2, -4 + x^2, -2x + 7x^2 - 3$

(d) $2x^2 + 4y^2 + 2, -2x^2 - 5y^2 + 7, -x^2 + y^2$

(e) $3mn + 4pq + 2, -3pq - 4mn - 1, -pq - mn$

7. **Subtract the following :**

(a) $(2a - b)$ from $(5a + 2b - 3)$

(b) $(-m^2 - 2n)$ from $(6m^2 - 3n + 8)$

(c) $(-5p + q + 7s)$ from $(7p - 8q + r)$

(d) $(-4x^2 + 5y^2 - xy + 9)$ from $(-x^2 - 4y^2 + xy - 7)$

(e) $(4p^2q - 3pq + 5pq^2 - 8p + 7q - 10)$ from $(18 - 3p - 11q + 5pq - 2pq^2 + 5p^2q)$

8. Find the perimeter of triangle whose sides are $m+n$, $-2m+n$ and $4m + 3n$, respectively, in terms of m and n.

9. Subtract $-2x^2 + y^2 - xy + x$ from the sum of $x^2 + 4y^2 - 6xy$, $x^2 - y^2 + 2xy$, $y^2 + 6$ and $x^2 - 4xy$.

Multiplication of Algebraic Expressions

Laws Of Signs For Multiplication

(i) The product of two numbers having like signs is positive.

(ii) The product of two numbers having unlike signs is negative.

If x and y are two variables, then

$(+x)\ (+y) = (+xy)$

$(-x)\ (-y) = (+xy)$

$(+x)\ (-y) = (-xy)$

$(-x)\ (+y) = (-xy)$

Rule Of Multiplication : While multiplying two or more variables, the powers of like variables are added.

For example : $x^2 \times x^3 = x^{2+3} = x^5$

Multiplication of A Monomial by A Monomial

Rule : Product of two monomials
= (Product of their numerical coefficients) × (Product of their variables)

Solved Example : If $a - \frac{1}{a} = 3$, find the value of $a^2 + \frac{1}{a^2}$

Solution: we have, $a - \frac{1}{a} = 3$

Squaring both sides, we get $\left(a - \frac{1}{a}\right)^2 = 3^2$

$$\Rightarrow a^2 + \frac{1}{a^2} - 2 \times a \times \frac{1}{a} = 9$$

$$\Rightarrow a^2 + \frac{1}{a^2} - 2 = 9$$

$$\Rightarrow a^2 + \frac{1}{a^2} = 9 + 2 = 11$$

Solved Example : If $a + \frac{1}{a} = 8$. Find the value of $a^2 + \frac{1}{a^2}$

Solution: we have, $a + \frac{1}{a} = 8.$

Squaring both sides. we get $\left(a + \frac{1}{a}\right)^2 = 8^2$

$$\Rightarrow a^2 + \frac{1}{a^2} + 2 \times a \times \frac{1}{a} = 64$$

$$\Rightarrow a^2 + \frac{1}{a^2} = 64 - 2 = 62$$

Multiplication of A Polynomial by A Monomial

Rule : To multiply a polynomial by a monomial, we commonly use distributive law.

$a \times (b+c) = a \times b + a \times c$

Solved Example : Find the product of :

$6a^2b^2 \times (2a^2 - 4ab + 5b^2)$

Solution: $6a^2b^2 \times (2a^2 - 4ab + 5b^2)$

$= (6a^2b^2 \times 2a^2) + [6a^2b^2 \times (-4ab)] + (6a^2b^2 \times 5b^2)$

$= 12a^4b^2 - 24a^3b^3 + 30a^2b^4$

Solved Example : Simplify : $a(a^2+a+1)+5$ and find its value for $a = -1$

Solution: $a(a^2+a+1)+5 = a^3 + a^2 + a + 5$

Substituting, $a = -1$, we have

$(-1)^3 + (-1)^2 + (-1) + 5$

$= -1 + 1 - 1 + 5$

$= 4$

Thus, the value of $a(a^2+a+1)+5$ for $a = -1$ is 4.

Solved Example : Subtract : $3l(l-4m+5n)$ from $4l(10n-3m+2l)$

Solution: We have, $3l(l-4m+5n) = 3l^2 - 12lm + 15ln$

and, $4l(10n-3m+2l) = 40ln - 12lm + 8l^2$

On subtraction, we have,

	$40ln$	$-12lm$	$+8l^2$
+	$15ln$	$-12lm$	$+3l^2$
	(–)	(+)	(–)
	$25ln$	+ 0	$+5l^2$

Thus, $\{4l(10n - 3m + 2l)\} - \{3l(l - 4m + 5n)\} = 25ln + 5l^2$

$= 5l(5n + l)$

Multiplication of A Binomial by A Binomial

Rule : Let us consider two binomials ($a+b$) and ($c+d$), respectively. Using distributive law of multiplication over addition, we have

$$(a+b) \times (c+d) = a \times (c+d) + b \times (c+d)$$
$$= (a\times c + a\times d) + (b\times c + b\times d)$$
$$ac + ad + bc + bd$$

Solved Example : Multiply: $(2pq + 3q^2)$ and $(3pq+2q^2)$

Solution: $(2pq + 3q^2) \times (3pq + 2q^2)$

$$= 2pq\,(3pq + 2q^2) + 3q^2\,(3pq + 2q^2)$$
$$= \{(2pq) \times (3pq) + (2pq) \times (2q^2)\} + \{(3q^2) \times (3pq) + (3q^2) \times (2q^2)\}$$
$$= 6p^2q^2 + 4pq^3 + 9pq^3 + 6q^4$$
$$= 6p^2q^2 + 13pq^3 + 6q^4$$

Multiplication of A Binomial by A Trinomial or Multiplication of A Polynomial by A Polynomial

Golden Rule; Product of two monomials = (product of their numerical co-efficients) x (product of their variables) known method of (Horizontal method)

By using the distributive low of multiplication over addition twice, we may find the product of two binomials

$$(a+b) \times (c+d) = a \times (c+d) + b \times (c+d)$$
$$= (a \times c) + (a \times d) + (b \times c) + (b \times d)$$
$$= ac + ad + bc + bd$$

Exercise 7.2

1. Find the product of following monomials :

(a) $5a^2b$ and $-2ab^2$ (b) $-6x^2y$ and $-4x^2y^2$

(c) $-4xy^2$ and $-2x^3y$ (d) $-3xy$ and $7x^3y^3$

2. Obtain the volume of rectangular boxes with the following length, breadth and height, respectively :

(a) $2xy, 3x^2y, 4xy^2$ (b) $2p, 6q, 7r$

(c) $3x, 4xy, 2xyz$ (d) a, a^2, a^3

3. Simplify the expressions and evaluate them as directed :

(a) $x(x-3)+2$, for $x = 1$

(b) $3y\,(2y-7)-3(y-4)-63$, for $y = -2$

4. Find the area of a rectangle whose sides are $5x$ and $2y$.

5. Find the area of a square whose sides are $4a^2$.

6. **Solve :**

(a) $\{5x(3-x)\}+\{6x^2-13x\}$ (b) $\{2ab(a+b)\}-\{3ab(a-b)\}$

7. **Find the product of :**

(a) $(a^2)\times(2a^{22})\times(4a^{26})$ (b) $\left(\frac{2}{3}xy\right)\times\left(\frac{-9}{10}x^2y^2\right)$

(c) $\left(\frac{-10}{3}pq^3\right)\times\left(\frac{6}{5}p^3q\right)$ (d) $x\times x^2\times x^3\times x^4$

8. **Simplify :**

(a) $(a+b)\quad(a^2-ab+b^2)$

(b) $(2x^2-5x+4)\times(x^2+7x-8)$

(c) $(3a^2+b^2)\times(2a^2+3b^2)$

(d) $(2m^3-5m^2-m+7)\times(3-2m+4m^2)$

Division of Algebraic Expressions

An algebraic expression of the form.

$a+bx+cx^2dz^3+ex^4+$ ....

When, a, b, c, d, e...... are the constants and x is a variable and is known as polynomial in x.

(i) The product of two numbers having like signs is positive.

(ii) The product of two numbers having unlike signs is negative.

Rule of Division : If x is a variable and a and b are positive integers, such that $a > b$, then

$$x^a \div x^b = x^{a-b}$$

Division of A Monomial by A Monomial

Rule : Quotient of two monomials

= (Quotient of their numerical coefficients) × (Quotient of their variables)

Solved Example :

Divide :

(i) $15x^2y^5$ by $-3xy$

(ii) $-12x^3y^2z$ by $-2xyz$

Solution: (i) $\frac{15x^2y^5}{-3xy} = \left(\frac{15}{-3}\right)x^{(2-1)}y^{(5-1)} = -5xy^4$

(ii) $\frac{-12x^3y^2z}{-2xyz} = \left(\frac{-12}{-2}\right)x^{(3-1)}y^{(2-1)}z^{(1-1)} = 6x^2y^1z^0 = 6x^2y$

Division of A Polynomial by A Monomial

Golden Rule and method:- Quotient of two monomials = (Quotient of their numerical co-efficients) X(Quotient of their variables). i.e; $\frac{15x^2y^5}{-3xy} = \left(\frac{15}{-3}\right)x^{(2-1)}y^{(5-1)} = -5xy^4$

Rule : To divide a polynomial by a monomial, divide each term of the polynomial by the monomial.

Solved Example : Divide :

$9x^6+24x^5-18x^3$ by $3x^2$

Solution: $(9x^6+24x^5-18x^3)\div 3x^2$

$= \frac{9x^6}{3x^2} + \frac{24x^5}{3x^2} - \frac{18x^3}{3x^2}$

$= 3x^4+8x^3-6x$

Division of A Polynomial by A Polynomial

To divide a polynomial by an another polynomial, follow the steps given below:

Step -1 : Arrange the terms of the divisor and dividend in descending order of their degrees.

Step -2 : Divide the first term of the dividend by the first term of the divisor to obtain the first term of the quotient.

Step -3 : Multiply all the terms of the divisor by the first term of the quotient and then subtract the result from the dividend.

Step -4 : The remainder obtained (if any), becomes the new dividend.

Step -5 : Repeat the above process, until you get the remainder either as 0 or a polynomial of degree less than that of a divisor.

Solved Example : Divide :

(x^2-4x+4) by $(x-2)$

Solution:

$x-2\,\big)\,x^2-4x+4\,\big(\,x-2$

x^2-2x

$-\quad+$

$-2x+4$

$-2x+4$

$+\quad-$

0

$\therefore$ Quotient = $x-2$, Remainder = 0

Solved Example : Divide :

$(2x^3+x^2-5x-2)$ by $(2x+3)$

Solution:

$$
\begin{array}{r|l}
2x+3 & 2x^3+x^2-5x-2 \;(\,x^2-x-1 \\
 & 2x^3+3x^2 \\
 & -\quad\; - \\
\hline
 & -2x^2-5x \\
 & -2x^2-3x \\
 & +\quad\; + \\
\hline
 & -2x-2 \\
 & -2x-3 \\
 & +\quad + \\
\hline
 & 1 \\
\hline
\end{array}
$$

$\therefore$ Quotient $= x^2-x-1$, Remainder $= 1$

Exercise 7.3

1. Divide :

(a) $45x^2y^3$ by $5xy$ (b) $32abc^3$ by $-4ac$

(c) $-64a^2b^2c$ by $-8abc$ (d) $-24xyz^2$ by $12xyz$

2. Divide :

(a) $4x^4-12x^2+36x$ by $4x$ (b) $6x^2y-8xy+10xy^2$ by $-2xy$

(c) $9a^2b^2-12ab^2+15a^2b^3$ by $3ab$ (d) $21a^4+14a^3-7a^2$ by $-7a^2$

3. Divide the following and write the quotient and remainder :

(a) $(2x^2+3x+1)$ by $(x+1)$ (b) $5a^3-15a^2+12a+3$ by $(3-3a+a^2)$

(c) $(x^4+4x^3-2x^2+10x-25)$ by $(x+5)$ (d) $(6m^2-31m+47)$ by $(2m-5)$

Standard Identities

An **identity** is an equation which is true for all values of the variables.

Identity - 1 : $(a+b)^2 = (a^2+2ab+b^2)$

Proof : We have,

$(a+b)^2 = (a+b)(a+b)$

$= a(a+b)+b(a+b)$ { $\because$ Using distributive law}

$= a^2+ab+ba+b^2$

$= a^2+2ab+b^2$ [$\because ab=ba$]

Thus, $(a+b)^2 = a^2+2ab+b^2$

Identity - 2 : $(a-b)^2 = (a^2-2ab+b^2)$

Proof : We have,

$(a-b)^2 = (a-b)(a-b)$

$= a(a-b)-b(a-b)$ { ∵ Using distributive law}

$= a^2-ab-ba+b^2$

$= a^2-2ab+b^2$ [∵ $ab = ba$]

Thus, $(a-b)^2 = a^2-2ab+b^2$

Identity - 3 : $(a+b)(a-b) = (a^2-b^2)$

Proof : We have,

$(a+b)(a-b) = a(a-b)+b(a-b)$ { ∵ Using distributive law}

$= a^2-ab+ba-b^2$

$= a^2-b^2$ [∵ $ab = ba$]

Thus, $(a+b)(a-b) = a^2-b^2$

Identity - 4 : $(x+a)(x+b) = x^2+(a+b)x+ab$

Proof : We have,

$(x+a)(x+b) = x(x+b)+a(x+b)$ { ∵ Using distributive law}

$= x^2+xb+ax+ab$

$= x^2+(a+b)x+ab$

Thus, $(x+a)(x+b) = x^2+(a+b)x+ab$

Application of Identities

Solved Example : Solve the following :

(i) $(x+5)(x+5)$ (ii) $(2x+3y)(2x+3y)$

(iii) $(x-2)(x-2)$ (iv) $(4x-3y)(4x-3y)$

Solution:

(i) $(x+5)(x+5)$

$= (x+5)^2$ { ∵ Using laws of exponents}

$= x^2+2\times x\times 5+5^2$ { ∵ Using $(a+b)^2 = (a^2+2ab+b^2)$}

$= x^2+10x+25$

(ii) $(2x+3y)(2x+3y)$

$= (2x+3y)^2$ { ∵ Using laws of exponents}

$= (2x)^2+2(2x)(3y)+(3y)^2$ { ∵ Using $(a+b)^2 = (a^2+2ab+b^2)$}

$= 4x^2+12xy+9y^2$

(iii) $(x-2)(x-2)$

$= (x-2)^2$ { ∵ Using laws of exponents}

$= x^2-2\times x\times 2+2^2$ { ∵ Using $(a-b)^2 = (a^2-2ab+b^2)$}

$= x^2-4x+4$

(iv) $(4x-3y)(4x-3y)$

$= (4x-3y)^2$ { ∵ Using laws of exponents}

$= (4x)^2-2(4x)(3y)+(3y)^2$ { ∵ Using $(a-b)^2 = (a^2-2ab+b^2)$}

$= 16x^2-24xy+9y^2$

Solved Example : Find the following squares by using the identities :

(i) $\left(\frac{2}{3}m+\frac{3}{2}n\right)^2$ (ii) $(0{\cdot}4p-0{\cdot}5q)^2$

Solution: (i) $\left(\frac{2}{3}m+\frac{3}{2}n\right)^2$

Using identity $(a+b)^2 = a^2+2ab+b^2$, we have

$= \left(\frac{2}{3}m\right)^2 + 2\left(\frac{2}{3}m\right)\left(\frac{3}{2}n\right)+\left(\frac{3}{2}\right)^2$

$= \frac{4}{9}m^2+2mn+\frac{9}{4}n^2$

(ii) $(0{\cdot}4p–0{\cdot}5q)^2$

Using identity $(a–b)^2 = a^2-2ab+b^2$, we have

$= (0{\cdot}4p)^2–2\times(0{\cdot}4p)\times(0{\cdot}5q)+(0{\cdot}5q)^2$

$= 0{\cdot}16p^2-0{\cdot}4pq+0{\cdot}25q^2$

Solved Example : Evaluate using identities :

(i) $(2x+3y)(2x–3y)$ (ii) $(7a^2+8b^2)(7a^2–8b^2)$

Solution: Using the identity $(a+b)(a–b) = (a^2–b^2)$, we have

(i) $(2x+3y)(2x–3y)$

$= (2x)^2-(3y)^2$

$= 4x^2-9y^2$

(ii) $(7a^2+8b^2)(7a^2–8b^2)$

$= (7a)^2-(8b)^2$

$= 49a^2-64b^2$

Solved Example : Using $(x+a)(x+b) = x^2+(a+b)x+ab$, find :

(i) 103×104 (ii) $9{\cdot}7\times9{\cdot}8$

Solution: (i) $103\times104 = (100+3)(100+4)$

$= (100)^2+(3+4)\times100+3\times4$

$= 10000+7\times100+12$

$= 10000+700+12$

$= 10712$

(ii) $9{\cdot}7\times9{\cdot}8 = (9+0{\cdot}7)(9+0{\cdot}8)$

$= (9)^2+(0{\cdot}7+0{\cdot}8)\times9+0{\cdot}7\times0{\cdot}8$

$= 81+1{\cdot}5\times9+0{\cdot}56$

$= 81+13{\cdot}5+0{\cdot}56$

$= 95{\cdot}06$

Exercise 7.4

1. Solve the following :

(a) $(x+8)(x+8)$ (b) $(x+11)(x+11)$

(c) $(4x+7y)(4x+7y)$ (d) $(x–12)(x–12)$

(e) $(3x–5y)(3x–5y)$ (f) $(3a–\frac{1}{2})(3a–\frac{1}{2})$

2. Find the following squares by using the identities :

(a) $(2x+3y)^2$ (b) $(4p-3q)^2$ (c) $(6x^2-5y)^2$

(d) $(xy+3z)^2$ (e) $(0{\cdot}4p-0{\cdot}5q)^2$ (f) $(2xy+5y)^2$

3. **Evaluate the following using identities :**

(a) $\left(\frac{5}{3}x+\frac{3}{5}y\right)\left(\frac{5}{3}x-\frac{3}{5}y\right)$

(b) $(3x+7y)(3x-7y)$

(c) $(0\cdot8x+0\cdot3y)(0\cdot8x-0\cdot3y)$

(d) 983^2-17^2

(e) $12\cdot1^2-7\cdot9^2$

4. **Using $(x+a)(x+b)=x^2+(a+b)x+ab$, find :**

(a) $6\cdot3\times6\cdot4$ (b) 208×205 (c) 58×52 (d) $3\cdot7\times3\cdot2$

5. **Show that :**

(a) $(9p-5q)^2+180pq=(9p+5q)^2$

(b) $\left(\frac{4}{3}m-\frac{3}{4}n\right)^2+2mn=\frac{16}{9}m^2+\frac{9}{16}n^2$

(c) $a^2+3a+2=(a+1)(a+2)$

6. **Using $a^2-b^2=(a+b)(a-b)$, find :**

(a) 102^2-98^2 (b) $(0\cdot51)^2-(0\cdot49)^2$

Points to Remember :

- Expressions that contain one, two or three terms are called monomials, binomials and trinomials, respectively. In general, any expression with three or more terms can be called a polynomial.
- Terms with same literal factors are called like terms. If the literal factors are not the same, they are called unlike terms.
- Only like terms can be added or subtracted.
- When a monomial is multiplied by another monomial, the product is a monomial.
- While multiplying a polynomial by a monomial, every term in the polynomial is to be multiplied by the monomial.
- While multiplying two polynomials, every term in one polynomial is to be multiplied by every term in the other polynomial.
- While dividing a polynomial by a monomial, every term of the polynomial is to be divided by the monomial.
- To divide a polynomial by a polynomial, the long division method is used.
- On performing long division, we arrange the dividend and the divisor in the standard form, that is, arrange them in descending order of the divisor.
- On performing long division, if the remainder is 0, the divisor is a factor of the dividend.
- The degree of the remainder has to be less than the degree of the divisor.
- An identity is an equation which is true for all values of the variables that it contains.
- The important and useful identities are:
  Identity 1: $(a+b)^2=a^2+2ab+b^2$
  Identity 1: $(a-b)^2=a^2-2ab+b^2$
  Identity 3: $(a+b)(a-b)+a^2-b^2$ (It is called the difference of two squares.)
  Identity 4: $(x+a)(x+b)=x^2+(a+b)x+ab$

EXERCISE

1. MULTIPLE CHOICE QUESTIONS (MCQs) :

Tick (✓) the correct options.

(a) $4x^2y + 3x^3$ is a—

(i) Monomial ☐ (ii) Binomial ☐ (iii) Trinomial ☐ (iv) None of these ☐

(b) Is x and y are two variables, then $(-x) \times (-y)$ = ?

(i) xy ☐ (ii) $(-x)(y)$ ☐ (iii) $x \times (-y)$ ☐ (iv) None of these ☐

(c) The area of a square of side $6x$ is—

(i) $6x$ ☐ (ii) $12x^2$ ☐ (iii) $36x^2$ ☐ (iv) $36x$ ☐

(d) $x^a \div x^b$ = ?

(i) x^{a-b} ☐ (ii) x^{b-a} ☐ (iii) x^{a+b} ☐ (iv) None of these ☐

(e) $(a-b)^2$ = ?

(i) $(a^2-2ab+b^2)$ ☐ (ii) $(a^2+2ab+b^2)$ ☐ (iii) (a^2+b^2) ☐ (iv) (a^2-b^2) ☐

(f) The quotient when $-56xyz^3$ divided by $8xyz$ is—

(i) $-7z^2$ ☐ (ii) $7z^2$ ☐ (iii) $8xyz$ ☐ (iv) $-56z^3$ ☐

2. The age of father is $13xy - 6x^2 + 4a^2 - 1$. The age of the son is $25x^2 + 16xy - 3b^2 - 2$. Find the difference of their ages.
3. The perimeter of a triangle is $6a^2 - 4a + 9$ and two of its sides are $a^2 - 2a + 1$ and $3a^2 - 5p + 3$. Find the length of the third side of the triangle.
4. Add $3x^2 + 4x - 2$ to the product of $(3x - 4)$ and $(x + 5)$.
5. From the product of $(xy + y + 1)$ and $(y - 6)$ subtract $4xy^2 + 9y^2$.
6. Add the product of $(4ab + b)$ and $(b - 7)$ to the product of $(-3ab + 1)$ and $(b + 2)$.
7. What should be subtracted from $14a^2 + 13a - 15$ to make it divisible by $7a - 4$?
8. What should be subtracted from $4x^3 + 8x^2 + 8x + 7$ to make it divisible by $2x^2 - x + 1$?
9. The perimeter of a triangle is $7x^2 - 17xy + 5y^2 + 8$ and two of its sides are $4x^2 - 7xy + 4y^2 - 3$ and $5 + 6y^2 - 8xy + x^2$. Find the third side of the triangle.
10. The expression $2x^4 - x^3 - 3x^2 + 5x - 2$ should be divided by which expression to get $x^2 + x - 1$ as the quotient?

HOTS

1. If $a^2 + \frac{1}{a^2}$ 2 = 27, find the value of $a^4 + \frac{1}{a^4}$
2. If $x = 11$, find the value of

   (a) $x^2 + 2$ (b) $x^4 + 4$

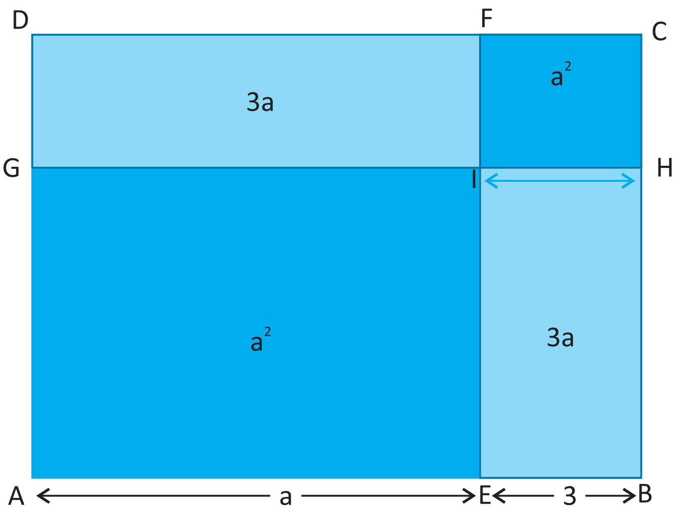
Lab Activity

Objective : To visualise and reinforce the concept of identities.

Materials Required : Chart paper / graph paper and colored sketch pens.

Procedure : To prove $(a + 3)^2 = a^2 + 6a + 9$ using a diagram proceed with the following steps:

Step 1 : Draw a square ABCD and divide it into two squares and two rectangles as shown using a black tram.



Step 2 : Let AE measure a units and EB measure 3 units since AB = AE + EB, we have AB = $(a + 3)$ units.

Step 3 : Shade the two squares is red colous and two rectangles in green colour to distinguish between square and rectangles.

Area of square ABCD = Area of square AEIG + Area of rectangle EBHI + Area of square IHCF + Area of rectangle DGIF

$(a + 3)^2 = a^2 + 3a + 3^2 + 3a$

$(a + 3)^2 = a^2 + 6a + 9$ sq. units.

We can put any numerical value of a to verify the equation.

8 Factorisation

We have learnt multiplication of algebraic identities. The number that we get after the multiplication is called the product. The expressions of which it is the factor can divide the product without any remainder. These numbers are called factors of the product.

Let us select a number say 12. The numbers that can divide it are called its factors. The numbers 1, 2, 3, 4, 6 and 12 can divide it. Therefore all these numbers are its factors. The number 12 can be written in the following forms.

$12 = 1 \times 12$	$12 = 4 \times 3$	$12 = 2 \times 6$
$12 = 6 \times 2$	$12 = 3 \times 4$	$12 = 12 \times 1$

The numbers which are written in the form of its factors is called the factor form. If the numbers are written in the form of products of prime factors. It is called the prime factor form. For example, 12m can be written in the form $2 \times 2 \times 3 = 12$. This is the prime factor form. The process of writing a number or expression in the form of products of its factors is called factorisation.

One As A Factor

Since 1 is the factor of every number. All numbers can be written in the form of the product of factor 1. Therefore when write a number in the factor form. We need not write the number in the form of factor of one.

Similarly all the numbers are factors of itself. Therefore we will also not write a number in factor form of the number itself. For example– $12 = 12 \times 1$, $5 = 5 \times 1$ $7 = 7 \times 1$

Factors of Algebraic Expressions

Consider a number 3ab. In this expression 3, a and b are factors of 3ab. In the factor form it can be written as– $3ab = 3 \times a \times b$.

Similarly the expression $5x^2y^2z$ and be written in the factor form as.

$5 \times x \times x \times y \times y \times z = 5x^2y^2z$

Common Factors

In algebraic expressions if the terms are two or more than two in number the factors which are common to both are called common factors.

For example is the binomial expressions.

$3a + 6$

$3 \times a = 3$ and a

$3 \times 2 = 6$

The terms 3a and 6 have a common factor 3. Hence both the terms can be written as

$3 \times a + 3 \times 2 = 3 \times (a + 2) = 3\,(a + 2) = 3a + 6.$

Example 1 : Factorise $12x^2y + 15xy^2$

Solution : $12x^2y = 12 \times x \times x \times y$

$15xy^2 = 15 \times x \times y \times y$

Example 2 : Factorise $15ab^2 + 18ab^2$

Solution : $15ab^2 = 3 \times 5 \times a \times b \times b$

$18ab^2 = 2 \times 3 \times 3 \times a \times b \times b$

The common factors are 3, a and b.

$15ab^2 + 18a^2b = 3ab(5b + 6a)$

Example 3 : Factorise $qp^2 + 42pq + 49q^2$

Solution : $qp^2 + 42\ pq + 49q^2$ is of the from,

$a^2 + 2ab + b^2$ where $a^2 = qp^2 = (3p)^2$, $b^2 = 49q^2$

$=(7q)^2$ and $2ab = 42pq = (2 \text{ x } 3p \text{ x } 7q)$

$\therefore\ qp^2 + 42pq + 49q^2 = (3p)^2 + 2 \text{ x } 3p \text{ x } 7q + (7q)^2$

$= (3p + 7q)^2$

Example 4 : Factorise $3ab + 3b + 5a + 5$

Solution : In the expression we have no common factor. However the first two terms and the last two terms do have common factors.

In such cases we find the factors of the first two terms and the last two terms separately.

$$3ab = 3 \times a \times b$$
$$3b = 3 \times b$$
$$5a = 5 \times a$$
$$5 = 5 \times 1$$

Let us find the common factor of 3ab and 3b

$$3ab + 3b = 3b\,(a + 1)$$
$$5a + 5 = 5\,(a + 1)$$

or

$$3ab + 3b + 5a + 5 = 3b\,(a + 1) + 5\,(a + 1)$$

We can see that in the former and the later pairs of terms $(a + 1)$ is common we can write it as – = $(a + 1)\,(3b + 5)$

Such a method of factorisation is called factorisation by regrouping. As we grouped the first and last two terms individualy. Then we regrouped them by finding another common factor.

Example 5 : Factorise $12ab - 8b + 12 - 18a$

Solution : Steps of factorisation–

Is there any common factor of all the terms? We see that there is no common factor for all the four terms other than 4.

Since we do not have any common factor. Try to regroup them. We can regroup the first and last two terms.

$12ab - 8b + 12 - 18a$

$4b(3a - 2) + 6(2 - 3a)$

Do we get a common factor after regrouping. No! we don't. Think! What can be done to get a common factor?

Let us see by changing the order of the last terms and see.

$$12ab - 8b - 18a + 12 \qquad \frac{12ab}{4b} = 3a, \qquad \frac{-8b}{4b} = -2$$

$$= 4b(3a - 2) - 6(3a - 2) \qquad \frac{-18a}{-6} = 3a, \qquad \frac{12}{-6} = -2$$

$$= (3a - 2)(4b - 6)$$

Exercise 8.1

Factorise–

1. $14a^3 + 21a^4b - 28a^2b^2$
2. $-5 - 10p + 20p^2$
3. $9a^3 - 6a^2 + 12a$
4. $8x^2 - 72xy + 12x$
5. $18a^3b^3 - 27a^2b^3 + 36a^3b^2$
6. $24a^3 - 36a^2b$
7. $10a^3 - 15a^2$
8. $36a^3b - 60a^2b^3c$
9. $16m^2 - 24mn$
10. $15xy^2 - 20x^2y$
11. $12a^2b^3 - 21a^3b^2$
12. $12a + 15$
13. $14a - 21$
14. $9a - 12a^2$

Factorisation by Identities

The three identities that we will use for factorisation are–

1. $(a + b)^2 = a^2 + 2ab + b^2$
2. $(a - b)^2 = a^2 - 2ab + b^2$
3. $(a + b)(a - b) = a^2 - b^2$

Example 1 : Factorise $2a^2 + 9a + 10$

Solution : Try to match the expression with identities above. The expression does not match the identity 3 as it has only two terms, while our expression has three terms. Morever the terms of the identity are squared. In our expression only one terms is squared.

It does not match identity number 2 also as one of the term in it is negative.

Our expression does not match identity 1 also, as it has two squared terms.

Such expressions should be expanded into four terms by splitting the middle term into two. Then they can be factorised by regrouping method.

Split the middle term in such a way that its sum is 9 and its product is equal to $(2 \times 10) = 20$

Two such numbers are 4 and 5

$$2a^2 + 9a + 10 = 2a^2 + 4a + 5a + 10$$
$$= 2a(a + 2) + 5(a + 2)$$
$$= (a + 2)(2a + 5)$$

Example 2 : Factorise $25a^2 - 16b^2$

Solution : $25a^2 - 16a^2$ using identify $a^2 - b^2 = (a + b)(a - b)$

Find square root of 25 and 16

$$\sqrt{25} = 5$$
$$\sqrt{16} = 4$$
$$= (5a)^2 - (4b)^2$$
$$= (5a + 4b)(5a - 4b)$$

Example 3 : Factorise $4y^2 - 20yz + 25z^2$

Solution : $4y^2 - 20yz + 25z^2$ is of the from.

$a^2 - 2ab + b^2$ where

$a^2 = 4y^2 = (2y)^2, b^2 = 25z^2$

$=(5z)^2$ and $2ab = 20yz = (2 x 2y x 5z)$

$\therefore 4y^2 - 20yz + 25z^2 = (2y)^2 - 2(2y)(5z) + (5z)^2$

$= (2y - 5z)^2$

Example 4 : Factorise $(302)^2 - (298)^2$

Solution : $(302)^2 - (298)^2$

$$= (302 + 298)(302 - 298)$$
$$= 600 + 4$$
$$= 2400$$

Example 5 : Factorise $a^2 - 7a + 12$

Solution : We have to find two numbers whose sum is 7 and the product is $(1 \times 12) = 12$. Arrange these numbers in such a way that we get a common expression. Two such numbers are 3 and 4.

$$
\begin{aligned}
a^2-7a+12 &= a^2-4a-3a+12\\
&= a(a-4)-3(a-4)\\
&= (a-4)(a-3)
\end{aligned}
$$

Exercise 8.2

Factorise the following expressions–

1. $a^2-4ab+4b^2$
2. $x^2y^2-6xyz+9z^2$
3. $1-6a+9a^2$
4. $1-2x+x^2$
5. $16a^2-24a+9$
6. $9a^2-12a+4$
7. $a^2-10a+25$
8. a^2-6a+9
9. $a^2+a+\frac{1}{4}$
10. $9a^2+24a+16$
11. $36a^2+36a+9$
12. $144m^2+120m+25$
13. $a^2+6ab+9b^2$
14. $9+6x+x^2$
15. $1+2x+x^2$
16. $\frac{4\times4-1.5\times1.5}{4+1.5}$
17. $\frac{1.1\times1.1-0.5\times0.5}{1.1+0.5}$
18. $\frac{2.5\times2.5-0.5\times0.5}{2.5+0.5}$
19. $\frac{1.37\times1.37-0.63\times0.63}{1.37+0.63}$

Points to Remember :

- The algebraic expressions which can be written in the form of products of expression, then each of these products are called its factors.
- The process of writing an expression in the form of its products is called factorisation.
- When a common monomial factor occurs in each term. Find HCF of all the terms of the given expression and divide each term by its HCF. Write the given expression as the product of this HCF and the quotient obtained.
- When a binomial is common in the given expressions. Find the common binomial and write the given expressions as the product of this binomial and quotient obtained on dividing the given expression by this binomial.
- In the regrouping method, arrange the terms of the given expressions in groups in such a way that all the groups have a common factor. Factorise each group and take out the factor which is common to each group.
- Identities used for factorisation are–
  - (a) $x^2+(a+b)x+ab=(x+a)(x+b)$
  - (b) $(a+b)^2=a^2+2ab+b^2$
  - (c) $(a-b)^2=a^2-2ab+b^2$
  - (d) $(a+b)(a-b)=a^2-b^2$

EXERCISE

1. MULTIPLE CHOICE QUESTIONS (MCQs) :

Tick (✓) the correct options.

(a) $(a+b)(a-b)$ is equal to —

(i) (a^2+b^2) ☐ (ii) (a^2-b^2) ☐ (iii) $(a+b)^2$ ☐ (iv) $(a-b)^2$ ☐

(b) $(a+b)^2$ is equal to —

(i) $(a+b)(a+b)$ ☐ (ii) $(a-b)(a-b)$ ☐ (iii) $(a+b)(a-b)$ ☐ (iv) None of these ☐

(c) $(a-b)^2$ is equal to —

(i) $(a+b)(a+b)$ ☐ (ii) $(a-b)(a-b)$ ☐ (iii) $(a+b)(a-b)$ ☐ (iv) None of these ☐

(d) $(5a^2-45b^2) = ?$

(i) $(5a-9b)(5a+9b)$ ☐ (ii) $(5a-9b)(9a+5b)$ ☐ (iii) $5(a-3b)(a+3b)$ ☐ (iv) $9(a-3b)(a+3b)$ ☐

2. Factorise–

(a) $14a^3+21a^4b-28a^2b^2$

(b) $-5-10p+20p^2$

(c) $9a^3-6a^2+12a$

(d) $8x^2-72xy+12x$

(e) $18a^3b^3-27a^2b^3+36a^3b^2$

(f) $24a^3-36a^2b$

(g) $10a^3-15a^2$

(h) $16a^2-24a+9$

(i) $a^2+6ab+9b^2$

(j) $9a^2-12a+4$

(k) $9+6x+x^2$

(l) $36a^3b-60a^2b^3c$

(m) $16m^2-24mn$

(n) $15xy^2-20x^2y$

(o) $12a^2b^3-21a^3b^2$

(p) $12a+15$

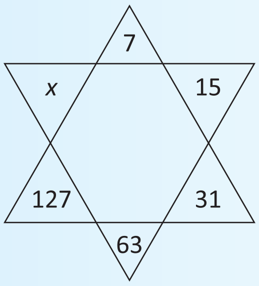
(q) $14a-21$

(r) $9a-12a^2$

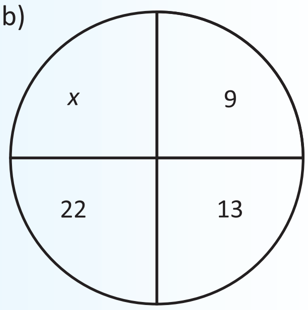
HOTS

1. Find the value of x in the following :

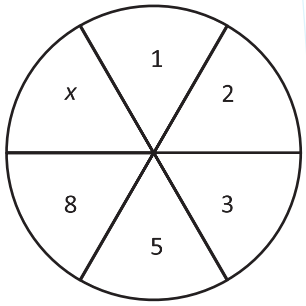
(a)



(b)



(c)



Lab Activity

Objective : To write a word problem in the form of an equation and then factorise it.

Materials Required : A pen.

Procedure : Write the equations and their fractions.

Problem	Equation	Factors
1. A number square plus eight times the same number minus three is equal to 38.		
2. Forty divided by a number plus seven times the number is equal to 10.		
3. A number plus its cube plus seven times then number added to 21 is 1.		
4. Twenty divided by a number minus the cube of another number plus 87 is seven.		
5. Thirty-nine times the square of a number is equal to three times the number plus 12.		

9 Linear Equations in One Variable

Two equal expressions are collectively called an equation. *e.g.* if expression $7x - 5$ and $4x + 13$ are equal, then $7x - 5 = 4x + 13$ is an equation.

1. To solve an equation means : to find the value of letter x this letter x is called the variable or unknown quantity.
2. The equation in which the variable i.e. x, y, or z etc. is in first order (*i.e.* is lightest power is one) is called equation.

Example 1 : Solve (i) $4x = 18 - 2x$

$$4x = 18 - 2x$$
$$4x + 2x = 18$$
$$6x = 18$$
$$x = \frac{18}{6}$$
$$= 3 \text{ Ans.}$$

(ii) $8 = 5x - 7$

$$8 = 5x - 7$$
$$8 + 7 = 5x$$
$$15 = 5x$$
$$\frac{15}{5} = 3x$$
$$x = 3$$

Example 2 : Solve $21 - 3(a - 7) = a + 20$

$$21 - 3a + 21 = a + 20$$
$$42 - 20 = a + 3a$$
$$22 = 4a$$
$$\therefore \quad a = \frac{22}{4} = 5\frac{1}{2}$$

Example 3 : Solve $\frac{y+2}{4} - \frac{y-3}{3} = \frac{1}{2}$

Solution : L.C.M of denominators 4, 3 and 2 = 12

$$12 \times \frac{y+2}{4} - 12 \times \frac{y-3}{3} = 12 \times \frac{1}{2}$$
$$3(y+2) - 4(y-3) = 6$$
$$3y + 6 - 4y + 12 = 6$$
$$-y = -12$$
$$y = 12$$

Example 4 : Solve (i) $\frac{5}{x} = \frac{7}{x-4}$

On cross – multiplying : we get

$$7x = 5(x-4)$$
$$7x = 5x-20$$
$$7x-5x = -20$$
$$2x = -20$$
$$x = -10$$

(ii) $\frac{a-2}{a+4} = \frac{a-3}{a+1}$

$$(a-2)(a+1) = (a-3)(a+4)$$
$$a^2+2a+a-2 = a^2-3a+4a-12$$
$$a^2+a-2 = a^2+a-12$$
$$a^2-a-a^2-a = -12+2$$
$$-2a = -10$$
$$a = 5$$

Exercise 9.1

Solve the following equations.

1. $20 = 6+2x$
2. $15+x = 5x+3$
3. $4x-13 = 7-x$
4. $1+5x = 10-x$
5. $\frac{3x+2}{x-6} = -7$
6. $3a-4 = 2(4-a)$
7. $2(7a-3) = 3(4a-2)$
8. $3(b-4) = 2(4-b)$
9. $7-x = x-1$
10. $\frac{x+2}{9} = \frac{x-4}{11}$
11. $-\frac{x-8}{5} = \frac{x-12}{9}$
12. $5(8x+3) = 9(4x+7)$
13. $3(x+1) = 12+4(x-1)$
14. $\frac{3x}{4} - \frac{1}{4}(x-20) = \frac{x}{4} + 32$
15. $3a - \frac{1}{5} = \frac{a}{5} + 5\frac{2}{5}$
16. $\frac{x}{3} - 2\frac{1}{2} = \frac{4x}{9} - \frac{2x}{3}$
17. $\frac{7x-1}{4} - \frac{1}{7}\left[2x - \frac{1-x}{2}\right] = 4$
18. $\frac{2x-(2x-3)}{3x-(4x+3)} = -1$
19. $\frac{2x-13}{5} - \frac{x-3}{11} = \frac{x-9}{5} + 1$
20. $\frac{3}{x+8} = \frac{4}{6-x}$

TO SOLVE PROBLEMS BASED ON EQUATIONS.

1. Read the problem carefully to find out what is given and what is to be known.
2. Represent the unknown quantity by x or by some other litter as a, b, y, z etc.
3. According to the conditions given in the problem, write the relation between knowns and unknowns.
4. Solve the equation to obtain the value of the unknown.

Example 5 : Find a number such that one–fifth of it is less than its one–fourth by 3.

Solution : Let the required number be x since, one–fifth of $x = \frac{x}{5}$ and one–fourth of it $\frac{x}{4}$ then according to the problem.

$$\frac{x}{4} - \frac{x}{5} = 3$$

$$\frac{5x - 4x}{20} = 3$$

$$x = 3 \times 20 = 60$$

Example 6 : The difference of the squares of two consecutive even natural numbers is 92. Taking x as the smaller of the two numbers form an equation in x and hence find the larger of the two.

Solution : Since the consecutive even natural numbers differ by 2 and it is given that the smaller of the two numbers is x. Therefore the next (larger) even number is $x + 2$.

According to the problems.

$$(x+2)^2 - x^2 = 92$$

$$x^2 + 4x + 4 - x^2 = 92$$

$$4x = 92 - 4 = 88$$

$$x = \frac{88}{4} = 22$$

$\therefore$ Larger even number $= x + 2 = 22 + 2 = 24$

Example 7: A rectangle is 8 *cm* long and 5 *cm* wide its perimeter is doubled when each of its sides is increased by x *cm* form an equation in x and find its new length.

Solution : Since length of the rectangle = 8*cm* and its width = 5 *cm*.

Its perimeter = 2 (length + width)

= 2 (8 + 5) = 26 *cm*

on increasing each of its side by x *cm*

Its new length = $(8 + x)$ *cm*.

and new width = $(5 + x)$ *cm*.

$\therefore$ Its new perimeter = $2(8 + x + 5 + x)$ *cm*

Given new perimeter = 2 times the original perimeter.

$$26 + 4x = 2 \times 26$$

$$4x = 52 - 26 = 26$$

$$x = \frac{26}{4} = 6.5 \text{ cm}.$$

Example 8 : A man is 24 years older than his son. In 2 years his age will be twice the age of his son. Find their present ages.

Solution : Let the present age of son be x years

$\therefore$ Present age of father = $x + 24$ years.

In 2 years

The man's age will be $x + 24 + 2$ = $x + 26$ years.

According to the problem $x + 26 = 2(x + 2)$

On solving we get $x = 22$

$\therefore$ Present age of man = $x + 24 = 22 + 24 = 46$ years

and present age of son x = 22 years.

Example 9: One day a boy walked from his house to his school at the speed of 4 km/hr and found that he was ten minutes late to the school. Next day he ran at the speed of 8 km/hr and found that he was 5 minute early to the school. Find the distance between his house and the school.

Solution: Since time = $\frac{\text{distance}}{\text{speed}}$

First day he takes $\frac{x}{4}$ hrs to reach the school and next day he takes $\frac{x}{8}$ hrs to reach the school.

Since, the difference of two timings = 10 minutes + 5 minutes = 15 minutes = $\frac{1}{4}$ hrs.

$\therefore \frac{x}{4} - \frac{x}{8} = \frac{1}{4}$

On solving, we get x = 2 km.

Exercise 9.2

1. Fifteen less then 4 times a number is 9. Find the number.
2. Three numbers are in the ratio of 4:5:6. If the sum of the largest and the smallest equals the sum of the third and 55, find the numbers.
3. 28 is 12 less than 4 times a number. Find the number.
4. Five less than 3 times a number is 20. Find the number.
5. Fifteen more than 3 times Neetu's age is the same as 4 times her age. How old is she?
6. A number decreased by 30 is the same as 3 times the number decreased by 14. Find the number.
7. A's Salary is same as 4 times B's salary. If together they earn ₹ 3750 a month. Find the salary of each.
8. Find the number whose fifth part increased by 5 is equal to its fourth part diminished by 5.
9. Six more than one–fourth of a number is two–fifths of the number. Find the number.
10. The length of a rectangle is 8cm more than its width. If the perimetre of the rectangle is 64 cm. Find its length and width.
11. The sum of three consecutive odd numbers is 57. Find the numbers.
12. Two years ago, Sahil was three times as old as his son and two years hence, twice his age will be equal to five times that of his son. Find their present ages. Check your solution.
13. Divide 105 into two parts so that one–fourth of one is equal to one–third of the other.
14. The length of a rectangle is 3m more than 5 times the width. The perimeter is 126m. Find the length and the width.
15. Find three consecutive even numbers whose sum is 234.
16. The first side of a triangle is 2cm longer then the second side. The third side is 5cm shorter than twice the second side. If the perimetre of the triangle is 49 cm find the lengths of its sides.

Points to Remember :

- An equation remains unaltered on:
  - (i) Adding the same number to each side of it.
  - (ii) Multiplying each side of it by the same number.
  - (iii) Dividing each side of it by the same number.
  - (iv) Consecutive integers are taken as x, x + 1 x +2 ....
- Consecutive even numbers are taken x, x + 2, x + 4.
- Consecutive multiples of 3 are taken x, x + 3 x + 6 .... and so on.

EXERCISE

1. MULTIPLE CHOICE QUESTIONS (MCQs) :

Tick (✓) the correct options.

(a) Which of these is not a linear equation?

(i) $3x + 5 = 12$ ☐ (ii) $4x^2 = 16$ ☐ (iii) $y + z - 2 = 0$ ☐ (iv) $\frac{3}{x} = 7$ ☐

(b) The solution of equation $3x - 1 = 5$, is ....

(i) 2 ☐ (ii) –2 ☐ (iii) $\frac{4}{3}$ ☐ (iv) none ☐

(c) A number when multiplied by 5 exceeds itself by 32. The number is ....

(i) 3 ☐ (ii) 4 ☐ (iii) 6 ☐ (iv) 8 ☐

(d) A number when added to its one fourth gives 40. The number is ....

(i) 16 ☐ (ii) 32 ☐ (iii) 36 ☐ (iv) 40 ☐

(e) Two consecutive natural numbers whose sum is 85 are ....

(i) 32, 53 ☐ (ii) 42, 43 ☐ (iii) 41, 44 ☐ (iv) 40, 45 ☐

2. Solve the following equations:

(a) $20 = 6 + 2x$ (b) $3(b - 4) = 2(4 - b)$ (c) $3a - \frac{1}{5} = \frac{a}{5} + 5\frac{2}{5}$

(d) $15 + x = 5x + 3$ (e) $7 - x = x - 1$ (f) $\frac{x}{3} - 2\frac{1}{2} = \frac{4x}{9} - \frac{2x}{3}$

3. Find the number whose fifth part increased by 5 is equal to its fourth part diminished by 5.

4. Six more than one–fourth of a number is two–fifths of the number. Find the number.

5. The length of a rectangle is 8cm more than its width. If the perimetre of the rectangle is 64 cm. Find its length and width.

6. The sum of three consecutive odd numbers is 57. Find the numbers.

7. Two years ago, Sahil was three times as old as his son and two years hence, twice his age will be equal to five times that of his son. Find their present ages. Check your solution.

HOTS

1. One dozen pencils is to be distributed between two children, so that the number of pencils the second child get double the number of pencils the first child got. How many pencils do the two children get?
2. How many kg of Basmati rice worth ₹96 per kg should be mixed with 15 kg of Basmati rice worth ₹80 per kg to obtain a mixture costing ₹90 per kg?

Lab Activity

Objective : To understand the balancing of an equation.

Materials Required : Weights like cards containing natural numbers, and variable like

X 0 1 2 3 4 5 6 7 8 9

Procedure : For each balance, calculate the unknown weights. Also, write sown and solve an equation for each situation.

Balance	Equation	Value of x
3 + x 5 + 2	3 + x = 5 + 2	x = 4
7 + x 4 + 6		
9 + 2 6 + x		
5 + 1 x + 4		
x + 8 5 + 4		
8 + 5 3 + x		
1 + 8 x + 2		

10 Profit, Loss, Discount and Compound Interest

Profit or Loss:

1. Suppose a man buys an article for ₹ 40 and sells it for ₹ 55 obviously he makes a profit of ₹ (55 – 40) = ₹ 15.
   Here ₹ 40 is the cost price C.P. ₹ 55 is the selling price (S.P.) and ₹ 15 is the profit.
   If the selling price of an article is more than the cost price there is a profit.
   Profit (or gain) = S.P – C.P
2. Now, the man buys the article for ₹ 55 and sells it for 40 he loses ₹ (55 – 40) = ₹ 15
   If the selling price is less than its cost price then there is a loss.
   Loss = C.P. – S.P.

Profit or loss is always calculated on cost price. Given above ₹ 15 is profit on ₹ 40 and in ₹ 15 is loss on ₹ 55 if profit or loss is calculated on ₹ 100. It is called profit percent or loss percent Profit % $= \frac{\text{Profit}}{\text{C.P.}} \times 100$ Loss% $= \frac{\text{Loss}}{\text{C.P.}} \times 100$

Example 1 : Mehta buys a table-fan for ₹ 600 and sells it for ₹ 750 find his gain and gain percent.

Solution : Gain = S.P. – C.P. = ₹ 750 – ₹ 600 = ₹ 150

$$\text{Gain } \% = \frac{\text{Gain}}{\text{C.P.}} \times 100 = \frac{150}{600} \times 100 = 25\%$$

Example 2 : A men buys an article for ₹ 5000 and sells it for 4500. What is his loss percent?

Solution : Loss = C.P. – S.P. = ₹ 5000 – ₹ 4500 = ₹ 500

$$\text{Loss } \% = \frac{\text{Loss}}{\text{C.P.}} \times 100 = \frac{500}{5000} \times 100 = 10\%$$

Example 3 : Jacob buys an old scooter for ₹ 4700 and spends ₹ 800 on its repairs if he sells the scooter for 5800. Find his gain or loss percent.

Solution :

Total C.P. of the scooter = ₹ 4700 + ₹ 800 = ₹ 5500

Since = S.P. = ₹ 5800

Gain = ₹ 5800 – ₹ 5500 = ₹ 300

$$\text{Gain } \% = \frac{300}{5500} \times 100 = 5\frac{5}{11}\%$$

Profit or loss percent is always on the total cost - price.

Example 4 : A fruit seller buys oranges at 5 for ₹ 4 and sells them at 4 for ₹ 5. Find his profit on percent. Find total no. of oranges he sold if he earns a total profit of ₹ 36.

Solution : Since C.P. of 5 oranges = ₹ 4

$$\text{C.P. of 1 orange} = ₹\frac{4}{5} = ₹\ 0.80$$

$$\text{Since S.P. of 4 oranges} = ₹\ 5$$

$$\text{S.P. of 1 orange} = ₹\frac{5}{4} = ₹\ 1.25$$

$$\text{Profit} = \text{S.P.} - \text{C.P.} = 1.25 - 0.80 = 0.45$$

$$\text{and profit\%} = \frac{\text{Profit}}{\text{C.P.}} \times 100 = \frac{0.45}{0.80} \times 100 = 56.25\%$$

$$\text{Also no of oranges sold} = \frac{\text{Total profit}}{\text{Profit of one orange}}$$

$$= ₹\frac{36}{0.45} = 80$$

In order to find profit or loss as percent always calculate the C.P. and S.P. of equal number of articles.

In Example 4 given above instead of finding C.P. and S.P. of one orange. If we find the C.P. and S.P. of 5 oranges or 20 oranges or 100 oranges etc. even then the profit percent will remain the same.

Example 5 : A radio is purchased for ₹ 1200 and sold for ₹ 1080. Find loss percent.

Solution : C.P. = ₹ 1200 S.P. = ₹ 1080

Loss = 1200 – 1080 = ₹ 120

$$\text{Loss\%} = \frac{120}{1200} \times 100 = 10\%$$

Example 6 : By selling 144 hens. Murphy lost the S.P. of 6 hens. Find her loss percent.

Solution :

$$\text{Let S.P. of 1 hen} = ₹\ 1$$

$$\text{S.P. of 144 hens} = ₹\ 144 \times 1 = ₹\ 144$$

$$\text{and loss} = \text{S.P. of 6 hens}$$

$$= 6 \times 1 = ₹\ 6$$

$$\text{C.P. of 144 hens} = \text{S.P.} + \text{Loss}$$

$$= ₹\ 144 + ₹\ 6 = ₹\ 150$$

$$\text{Therefore, loss \%} = \frac{\text{Loss}}{\text{C.P.}} \times 100 = \frac{6}{150} \times 100 = 4$$

Exercise 10.1

1. Find the profit or loss percent :

(a) C.P. = ₹ 275 Profit = ₹ 25

(b) S.P. = ₹ 320 Gain = ₹ 72

(c) S.P. = ₹ 250 Loss = ₹ 50

(d) C.P. = ₹ 400 S.P. = ₹ 450

(e) S.P. = ₹ 360 C.P. = ₹ 400

2. Hrithik goes from Agra to Delhi to buy an article. Which costs ₹ 6500 in Delhi. He sells this article for ₹ 8000 in Agra. Find his gain or loss percent consider that he spends ₹ 700 an transportation, food etc.

3. Moyna bought 10 note books for ₹ 40 and sold them at ₹ 4.75 per notebook find her gain percent.

4. A boy buys an old bicycle for ₹ 162 and spends ₹ 18 on its remains. He sells the bicycle for ₹ 171 find his gain or loss%.

5. A shopkeeper bought 300 eggs at 80 paisa each. 30 eggs were broken in transaction and then he sold the remaining eggs at one rupee each. Find his gain or loss percent.

6. A man sold his bicycle for 405 losing are length of its cost price finding : (a) his cost price (b) his loss percent

[Hint let C.P. = ₹ × then loss = ₹$\frac{x}{10}$ and $x - \frac{x}{10} = 405$

7. A man sold a radio set for ₹ 250 and gained one - nineth of its cost price find.

(a) His cost price

(b) His profit percent

8. A shopkeeper sells his goods at 80% of their cost price. What percent does he gain or loss?

9. The cost price of an article is 90% of its selling price. What is the profit or loss percent?

10. A shopkeeper mixes two variants of rice in ratio 3:1. The first variety costs ₹ 32/kg, while other costs ₹36/kg. If the mixed rice is sold at price of ₹ 28.05/kg. Find the profit/loss incurred by shopkeeper.

11. Shalley sold two sarees for ₹2185 each. On one saree she lost 5%, while on the other she gained 15%. Find her gain or loss per cent on the whole transaction.

12. Madan Lal purchased an old scooter for ₹ 12000 and spent ₹ 2850 on its overhauling. Then, he sold it to his friend Karambir for ₹ 13860. How much percent did he gain or loss?

TO FINALISE S.P. WHEN C.P. AND GAIN PERCENT OR LOSS PERCENT ARE GIVEN.

Example 7 : Girdhari bought a fountain pen for ₹ 12. For how much should he sell it to gain 15%?

Solution : Since, C.P. of the pen = ₹ 12

$\therefore$ Gain = 15% of the C.P.

$= ₹\ \frac{15 \times 12}{100} = ₹\ 1.80$

Since S.P. = C.P. + Gain

Alternative Method S.P. $= \frac{(100 \times \text{Gain\%})}{100} \times \text{C.P.}$

$= \frac{100 + 15}{100} \times ₹\ 12 = \frac{₹115 \times 12}{100} = ₹\ 13.80$

Example 8 : An article bought for ₹450 is sold at loss of 20% find its selling price.

Solution :

Since, C.P. = ₹450

Loss = 20% of ₹450 = ₹ $\frac{20 \times 450}{100}$ = ₹90

And S.P. = C.P. – Loss

= ₹450 – ₹90 = ₹360

$$\text{S.P.} = \frac{(100 - \text{loss\%})}{100} \times \text{C.P.}$$

$$\left(\frac{100-20}{100}\right) \times ₹450$$

$$= ₹\frac{80}{100} \times 450 = ₹360$$

To find C.P. when S.P. and gain percent or loss percent are given:

Example 9 : Raman sells an article for ₹360 at a gain of 20% find his cost price.

Solution :

Let C.P. of the article = ₹100

Gain = 20% of ₹100 = ₹20

and S.P. = ₹100 + ₹20 = ₹120

When S.P. is ₹120; C.P. = ₹100

When S.P. is ₹1 C.P = ₹ $\frac{100}{120}$

When S.P. is ₹360; C.P. = ₹ $\frac{100}{120} \times 360$ = ₹300

Alternative method : $\text{C.P.} \frac{100}{100 + \text{gain\%}} \times \text{S.P.} = \frac{100}{100+20} \times ₹360$

$$= ₹\frac{100 \times 360}{120} = ₹300$$

Example 10 : By selling an article for ₹382.50 a man losses 15% find its cost price.

Solution :

Let C.P. = ₹100

Loss = 15% of ₹100 = ₹15

and S.P. = ₹100 – ₹15 = ₹85

When S.P. is ₹85; C.P. = ₹100

When S.P. ₹1; C.P. = ₹ $\frac{100}{85}$

When S.P. is ₹382.50: C.P. = ₹ $\frac{100 \times 382.50}{85}$

= ₹450

$$\boxed{\text{C.P.} = \frac{100}{100 - \text{loss\%}} \times \text{S.P}}$$

$$\left(\frac{100}{100-15}\right) \times ₹382.50$$

$$= ₹\frac{100 \times 382.50}{85} = ₹450$$

Example 11 : Janet sells two watches for ₹ 198 each; gaining 20% on one and losing 20% on the other. Find his gain % or loss % on the whole transaction.

Solution : Since for one watch : S.P. = ₹ 198 and gain = 20%

$$\text{C.P.} = \frac{100}{(100+20)} \times 198$$

$$\boxed{\because \text{C.P.} = \frac{100}{(100 + \text{gain\%})}\text{S.P.}}$$

$$= ₹ 165$$

Since; for other watch : S.P. = ₹ 198 and loss 20%

$$\therefore \text{C.P.} = \frac{100}{(100-20)} \times ₹198$$

$$\therefore \text{C.P.} = ₹ 247.50$$

$$\boxed{\because \text{C.P.} = \frac{100}{100 - \text{loss\%}}\text{S.P.}}$$

Total C.P. of both the watches = ₹ 165 + ₹ 247.50 = ₹412.50

Total S.P. of both the watches = ₹ 198 + ₹ 198 = ₹ 396

Loss on the while = ₹ 412.50 – ₹ 396 = ₹ 16.50

and loss % = $\frac{16.50}{412.50} \times 100\% = 4\%$

Exercise 10.2

1. Find the selling price if :

(a) C.P. = ₹ 425 profit = 12%

(b) C.P. = ₹ 382 loss = 15%

(c) C.P. = ₹ 352 over heads = ₹ 28 and profit = 20%

(d) C.P. = ₹ 576 over heads = ₹ 44 and loss = 16%

2. Find the cost price if :

(a) S.P. = ₹ 121 gain = 10%

(b) S.P. = ₹ 475 loss = 5%

(c) S.P. = ₹ 430 gain = 7½%

(d) S.P. = ₹ 732 loss = 8.5%

3. By selling an article for ₹ 900; a man gain 20%. Find his cost price and the gain.

4. By selling an article ₹ 704; a person loses 12%. Find his cost price and the loss.

5. A man sells a radio set for ₹ 605 and gain 10%. At what price should he sell it in order to gain 16%.

6. A fruit-seller sells 8 bananas for a rupee gaining 25%. How many banana did he buy for a rupee.

7. A sells an article to B at a gain of 10% and B sells the same article to C at a gain of 12%. If C pays ₹ 616 for the article. Find how much did A pay for it?

8. Toshiba bought 100 hens for ₹ 8000 and sold 20 of them at a gain fo 5%. At what gain percent she must sell the remaining hens. So as to gain 20% on the whole.

9. A dealer gets ₹ 56 less by selling a chair on 8% gain instead of 15% gain. Find the C.P. of the chair?

10. Mr. Pratham sold his scooter for ₹ 6720 at a gain of 12%. He paid 2% of the selling price to the broker. Find his net gain as percent.

11. A man bought a piece of land for 15000. He sold 1/3 of this land at a loss of 5 percent. At what gain percent should he sell the remaining land in order to gain 8% on whole the transaction.

12. A shopkeeper sells an article at 15% gain. Had he sold it for ₹ 18 more he would have gained 18%. Find the cost price of the article.

13. By selling a silver necklace fo ₹ 657, a jeweller loses 8.75%. For how much did he purchase it?

Discount

In order to dispose off the old or damaged goods; some shopkeepers offer a reduction on the marked price of their articles. This reduction is called discount.

> (i) Discount is always given on marked price (M.P.) of the article. (ii) Selling price or price paid by the customer = M.P. – discount.

Example 12 : A shopkeeper offers a discount of 10% on a tea-set. Find the discount and net selling price of a tea-set which is marked at ₹ 450.

Solution :

M.P. of tea set = ₹ 450

Discount = 10% of ₹ 450 = ₹ 45

Net selling price = 450 – 45 = ₹ 405

Example 13 : A beadsman marks his goods at 35 percent above the cost price and then allows a discount of 15 percent. What profit percent does he save.

Solution :

Let the C.P. = ₹ 100

hence, marked price = ₹ (100+35) = ₹ 135

Discount = 15% of 135 = ₹ 20.75

∴ Selling price = ₹ (135 – 20.75) = ₹ 114.25

and hence profit percent = $\frac{₹(114.25 - 100)}{₹\ 100} \times 100 = 14.25\,\%$

Example 14 : A dealer allows his customers a discount of 25 percent and still gain 25 percent. Find the marked price of a article which costs the dealer ₹720.

Solution : Since, Cost price = ₹720 and gain = ₹25% of ₹720 = ₹180

S.P. = ₹(720 + 180) = ₹900

Now let he marks it at ₹100

Since, discount given = 25% of ₹100 = ₹25 S.P. = ₹(100 – 25) = ₹75

When S.P. = ₹75 M.P. = ₹100

When S.P. = ₹75 M.P. = ₹ $\frac{100}{75}$

When S.P. = ₹900 M.P. = ₹ $\frac{100 \times 900}{75}$

= ₹1,200

Example 15 : Find the single discount equivalent to two successive discounts of 20% and 10%.

Solution : Let the marked price of an article be ₹100.

Then, the discount given on it = ₹20

The price after the first discount = ₹(100 – 20) = ₹80

The next discount = 10% of ₹80

$= ₹\left(80 \times \frac{10}{100}\right) = ₹8$

The price after the second discount = ₹(80 – 8) = ₹72

The net S.P. = ₹72

The single discount equivalent to the given discounts = (100 – 72)% =28%

Exercise 10.3

1. If marked price of an article is ₹350 and it is sold at a cash discount of 15%. Find its selling price.
2. **Final the S.P. if.**
   (a) M.P. = ₹1300 and discount = 10%
   (b) M.P. = ₹500 and discount = 15%
3. After allowing a discount of $7\frac{1}{2}$ % on the marked price an article is sold for ₹555. Find its marked price.
4. An article marked for ₹650 is sold for ₹572. What percentage discount was it on?
5. A ready made garments shop in Delhi allows 20 percent discount on its garments and still makes a profit of 20 percent. Find the marked price of a dress which is bought by the shopkeeper for ₹400.
6. **Find discount in percent when.**
   (a) M.P. = ₹900 and S.P. = ₹873
   (b) M.P. = ₹500 and S.P. = ₹425

7. A cycle merchant allows 25% commission on his advertised price and still makes a profit of 20%. If he gain ₹ 60 over the sale of one cycle. Find his advertised price.

8. The marked price of a water cooler is ₹ 4650. The shopkeeper offers an off season discount of 18% on it. Find its selling price.

9. A lady shopkeeper allows her customer a 10% discount on the marked price of the goods and still gets a profit of 25%. What is the cost price of a fan for her marked at ₹ 1250?

10. The list price of a table fan is ₹ 480 and it is available to retailer at 25% discount for how much should a retailer sell it to gain 15%?

11. A publisher gives 32% discount on the printed price of a book to book sellers. What does a bookseller pay for a book whose printed price is ₹ 275?

12. An article marked at ₹ 800 is sold at a discount of 10%. Find its cost price if the dealer makes a profit of 20%. Also, find the profit percent if no discount had been allowed.

13. A dealer of scientific instruments allows 20% discount on the marked price of the instrument and still makes a profit of 25%. If his gain over the sale of a instrument is ₹ 150, find the marked price of the instrument.

14. Find the single discount which is equivalent to two successive discounts of 20% and 5%.

Compound Interest

So for, we had learnt about simple interest as the extra money paid by the borrower to the lender for the privilege of using the money. We have learnt earlier that if principal =₹ P, rate = R% per annum and time = T years then the simple interest given by the formula.

$$SI = \frac{P \times R \times T}{100}$$ [Here, P = Principal, R = Rate, T = Time.

In compound interest term. $C.I = P\left[1+\frac{R}{100}\right]^n - P$ OR $C.I = P\left[\left(1+\frac{R}{100}\right)^n - 1\right]$ [Here, n = Periods of time]

For example : If principle = 5000 and rate of interest = 10%

$$\text{S.I. for 1 year} = ₹\left[\frac{5000 \times 10 \times 1}{100}\right] = ₹500$$

$$\text{S.I. for 2 year} = ₹\left[\frac{5000 \times 10 \times 2}{100}\right] = ₹1000$$

Clearly in computing S.I. the principal remains constant throughout. But, the above method of computing interest is generally not used in banks.

COMPOUND INTEREST. If the borrower and the lender agree to fix up a certain interval of time (say, a year or a half-yearly or a quarter of a year etc.). So that the amount = (principal + interest) at the end of a interval becomes the principal for the next interval then the total interest over all the intervals, calculated in this way is called the compound interest and is abbreviated as C.I.

> If no conversion period is specified the conversion period is taken to be one year.

Example 1 : Find the compound interest on ₹ 1000 for two years at 4% per annum.

Solution :

Principal for the first year = ₹ 1000

Interest for the first year = ₹ $\left(\dfrac{100 \times 4 \times 1}{100}\right)$

Using interest = $\dfrac{P \times R \times T}{100}$

= ₹ 40

Amount at the end of first year = ₹ 1000 + ₹ 40 = ₹ 1040

Principal for the second year = ₹ 1040

Interest for the second year = $\left(\dfrac{1040 \times 4 \times 1}{100}\right)$

= 41.60

Amount at the end of second year = ₹ 1040 + ₹ 41.60 = ₹ 1081.60

∴ Compound interest = ₹ (1081.60 – 1000) = ₹ 81.60

Example 2 : Find the amount and the compound interest on ₹ 20,000 for 3 years at 10% per annum.

Solution : We first out the compound interest on ₹ 100 for 3 years at 10% per annum.

Interest on ₹ 100 at 10% for 1 year = ₹ 10

Thus amount at the end of the first year = ₹ (100 + 10) = ₹ 110

This forms the principal for the second year.

Interest for the second year = ₹ $\left(\dfrac{110 \times 10 \times 1}{100}\right)$ = ₹11

∴ Amount at the end of the second year = ₹ 110 + ₹ 11 = ₹ 121

Again this form the principal for the third year

∴ Interest for the third year = ₹ $\left(\dfrac{121 \times 10 \times 1}{100}\right)$ = ₹12.10

Amount at the end f the thirds year = ₹ 121 + ₹ 12.10 = 133.10 and C.I. = ₹ (133.10–100) = ₹ 33.10

Example 3 : Find the compound interest of ₹ 8000 for 1½ years at 10% per annum. Interest being payable half yearly.

Solution : We have.

Rate of interest = 10% per annum = 5% per half year

Time = 1 ½ years = 3 half - years.

Original principal = ₹ 8000

Interest for the first half year = ₹ $\left(\dfrac{8000 \times 5 \times 1}{100}\right)$ = 400

Amount at the end of the first half year = ₹ 8000 + ₹ 400 = ₹ 8400

Principal for the second half year = ₹ 8400

Interest for the second half year = ₹ $\left(\dfrac{8400 \times 5 \times 1}{100}\right)$ = 420

Amount at the end of the second half year = ₹ 8400 + ₹ 420 = ₹ 8820

Principal for the third half-year = ₹ 8820

Interest for the third half year = ₹ $\left(\frac{8820 \times 5 \times 1}{100}\right)$ = ₹441

Amount at the end of the third half year = ₹ 8820 + ₹ 441 = ₹ 9261

Compound interest = ₹ 9261 – ₹ 8000 = ₹ 1261

Example 4 : Compute the compound interest on ₹ 5000 for 1½ years at 16% per annum compounded half yearly.

Solution : Rate of interest = 16% per annum = 8% per half-year.

Time = 1½ years = 3 half – years.

Original principal = ₹ 5000

Interest for the first half year = ₹ $\left(\frac{5000 \times 1 \times 8}{100}\right)$ = ₹400

Amount at the end of the first half-year = ₹ (5000 + 400) = ₹ 5400

Principal for the second half-year = ₹ 5400

Interest for the second half-year = ₹ $\left(\frac{5400 \times 1 \times 8}{100}\right)$ = ₹432

Amount at the end of the second half-year = ₹ (5400 + 432) = ₹ 5832

Principal for the third half-year = ₹ 5832

Interest for the third half-year = ₹ $\left(\frac{5832 \times 1 \times 8}{100}\right)$ = ₹466.56

Amount at the end of the third half-year = ₹ (5832 + 466.56) = ₹ 6298.56

Compound interest = ₹ (6298.56 – 5000) = ₹ 1298.56

Exercise 10.4

1. Find the amount and the compound interest of ₹ 2500 for 2 years on 12% per annum.
2. What will be the compound interest on ₹ 4000 in two years when rate of interest is 5% per annum?
3. Find the difference between the simple interest and the compound interest on ₹ 5000 for 2 years at 8% per annum.
4. Subhra deposited ₹ 6250 to a company at 9.5% per annum compound interest for 2 years. Calculate the amount she will get after 2 years.
5. Lovely borrowed a sum of ₹ 12000 from a finance company at 5% per annum compound annually. Calculate the compound interest that Lovely has to pay to the company after three years.
6. Bindu borrowed ₹ 20,000 from her friend at 18% per annum simple interest. She lent it to Poly at the same rate but compounded annually. Find her gain after 2 years.
7. Find the compound interest at the rate of 10% per annum for two years on that principal which in two years at the rate of 10% per annum gives ₹ 200 as simple interest.
8. Dinesh deposited ₹ 7500 in a bank which pays him 12% interest per annum compounded quarterly, what is the amount which he receives after 9 months.

9. Find the compound interest on ₹ 1000 at the rate of 8% per annum for $1\frac{1}{2}$ years when interest is compounded half-yearly.

10. Palash received a sum of ₹ 40,000 on a loan from a finance company. If the rate of interest is 7% per annum compounded annually. Calculate the compound interest that Palash pays after 2 years.

Computation of Compound Interest

By Using Formula : In the premium section. We have discased some problems on the computation of compound interest. As you have seen that the method of computing compound interest was very lengthy. Specially when the period of time is very large in this section.

Formula : Let *p* be the principal and the rate of interest be R% per annum. If the interest is compounded annually. Then the amount A and the compound interest C.I. at the end of *R* years is given by.

$$A = P\left(\frac{1+R}{100}\right)^n \text{ ; n = no of years and C.I. = A − P}$$

Example 5 : Find the amount on ₹ 25000 at 12% per annum compound interest for 3 years. Also calculate the compound interest.

Solution : Here p = ₹ 25000 R = 12% per annum and *n* = 3 years.

$$\therefore \text{ Amount after 3 years} = P\left(1+\frac{R}{100}\right)^n$$

$$= ₹\left[25000\times\left(1+\frac{12}{100}\right)^3\right]$$

$$= ₹\left[25000\times\frac{28}{25}\times\frac{28}{25}\times\frac{28}{25}\right]$$

$$= ₹\left(\frac{175616}{5}\right) = ₹35123.20$$

Amount after 3 years = ₹ 35123.20

And compound interest = ₹ (35123.20 – 25000) = ₹ 10123.20

Example 6 : Find the compound interest on ₹ 15625 for 9 months at 16% per annum, compounded quarterly.

Solution : Here, Principal (P) = ₹ 15625,

Rate(r) = 16% p.a

= 4% per quarter

Time(T) = 9 months

= 3 quarters

Now, Amount, (A) $= P\left(1+\frac{R}{100}\right)^n = ₹15625\left(1+\frac{4}{100}\right)^n$

$= ₹\ 15625\left(\frac{26}{25}\right)^3 = ₹\ 15625\times\frac{26}{25}\times\frac{26}{25}\times\frac{26}{25}$

= ₹ 17567

Since, **Compound Interest = Amount - Principal**

∴ C.I = ₹ 17576 - ₹ 15625 = ₹ 1951

Example 7 : Shrey deposited in ₹ 7500 in *a* bank for 6 months at the rate of 8% interest compounded quarterly. Find the amount he received after 6 months.

Solution : Here, $p =$ ₹ 7500, R = 8% per annum and n = 6 months

$$\text{Amount after 6 months} = P\left(1+\frac{R}{100}\right)^n \qquad \frac{6}{12}\text{ year} = \frac{1}{2}\text{ year.}$$

$$= ₹\ 7500 \times \left(1+\frac{1}{2\times 100}\right)^2$$

$$= ₹\ 7500 \times \frac{51}{50} \times \frac{51}{50} = ₹7803.$$

Compound Interest when Time is a Fraction

When interest is compounded annually but time is a fraction.

Formula : If p = principal r = rate per annum and time $3\frac{3}{4}$ years.

$$A = P\left(1+\frac{R}{100}\right)^3 \times \left(1+\frac{\frac{3}{4}\times R}{100}\right)$$

Example 8 : Find the compound interest an ₹ 24000 at 15% per annum for $2\frac{1}{3}$ years.

Solution : Here, p = ₹ 24000, R = 15% per annum and time = $2\frac{1}{3}$ years.

$$\text{Amount after } 2\frac{1}{3}\text{ years} = P\left(1+\frac{R}{100}\right)^2 \times \left(\frac{1+\frac{1}{3}\times R}{100}\right)$$

$$= ₹\left[24000 \times \left(1+\frac{15}{100}\right)^2 \times \left(1+\frac{\frac{1}{3}\times 15}{100}\right)\right]$$

$$= ₹\left[24000 \times \left(\frac{115}{100}\right)^2 \times \left(\frac{105}{100}\right)\right]$$

$$= ₹\left[24000 \times \left(\frac{23}{22}\right)^2 \times \left(\frac{21}{20}\right)\right]$$

$$= ₹\,33327$$

Compound interest = ₹ (33327 – 24000) = ₹ 9327

Finding Principal When A., C.I., and N are Given

Example 9 : Find the principal. If the compound interest compound interest compound annually at the rate of 10% per annum for three years is ₹ 331.

Solution : Let the principal be ₹ 100 then

$$\text{Amount after three years} = ₹\left[100 \times \left(1+\frac{10}{100}\right)^3\right]$$

$$= ₹\left[100 \times \left(\frac{110}{100}\right)^3\right]$$

$$= ₹\,133.10$$

$$\text{Compound interest} = ₹\,(133.10 - 100) = 33.10$$

Now, If compound interest is ₹ 33.10 Principal = ₹ 100

$$\text{If compound interest is ₹ 1, Principal} = ₹\ \frac{100}{33.10}$$

$$\text{In compound interest is} = ₹\,331,$$

$$\text{Principal} = ₹\ \left(\frac{100 \times 331}{33.10}\right) = 1000$$

Hence principal ₹ 1000.

Finding the Interest Rate Percent Per Annum

Example 10 : At what rate percent per annum, compound interest will ₹ 10,000 amount to ₹ 13310 in three years?

Solution : Let the rate be R% per annum. We have, P = principal = ₹ 10000 A amount = ₹ 13310 and n = 3 years.

$$A = P\left(1 + \frac{R}{100}\right)^n$$

$$= 13310 = 10000\left(1 + \frac{R}{100}\right)^3$$

$$= \frac{13310}{10000} = \left(1 + \frac{R}{100}\right)^3$$

$$= \frac{1331}{1000} = \left(1 + \frac{R}{100}\right)^3$$

$$= \frac{11^3}{10^3} = \left(1 + \frac{R}{100}\right)^3 \qquad = \left(1 + \frac{R}{100}\right)^3 = \left(\frac{11}{10}\right)^3$$

$$\frac{R}{100} = \frac{11}{10} - 1 \quad \frac{R}{100} = \frac{1}{10} \quad R = \frac{100}{10} = 10$$

Hence rate = 10% per annum.

Example 11 : In what time will ₹ 1000 amount to ₹ 1331 at 10% perannum compound interest?

Solution : Let the time be *n* years.

$$\text{Then, the amount} = ₹\left[1000 \times \left(\frac{11}{10}\right)^n\right]$$

$$\therefore\ 1000 \times \left(\frac{11}{10}\right)^n = 1331$$

$$\text{or } \left(\frac{11}{10}\right)^n = \frac{1331}{1000} = \frac{11 \times 11 \times 11}{10 \times 10 \times 10} = \left(\frac{11}{10}\right)^n$$

$$\left(\frac{11}{10}\right)^n = \left(\frac{11}{10}\right)^3$$

So, $n = 3$

Hence, the required time is 3 years.

Exercise 10.5

1. **Compute the compound interest in each of the following by using the formula when–**
   (a) Principle = ₹ 3000, Rate = 5% time = 2 years
   (b) Principle = ₹ 5000, Rate = 10 percent per annum time = 2 years
   (c) Principle = ₹ 12800, Rate = 7.5% time = 3 year
2. Find the amount of ₹ 2400 after 3 years. When the interest is compounded annually at the rate of 20% per annum.
3. Find the amount of ₹ 4096 for 18 months at 12.5% per annum. The interest being compounded semi-annually.
4. Prasad lent out ₹ 10,000 for 2 years at 20% per annum, compounded annually. How much more he could earn if the interest be compounded half-yearly?
5. Deepali borrowed ₹ 15625 from the State Bank of India to buy a scooter. If the rate of interest be 16% per annum compounded annually, what payment will she have to make after 2 year, 3 months?
6. Compute the compound interest on ₹ 15625 for 9 month at 16% per annum compounded quarterly.
7. On what sum will the compound interest at 5% per annum for 2 years compounded annually be ₹ 164.
8. A sum amounts to ₹ 756.25 at 10% per annum in 2 years. Compounded annually find the sum.
9. In what time will ₹ 1000 amount to ₹ 1331 at 10% per annum compound interest?
10. The present population of a town is 2800. If it increases at the rate of 5% per annum. What will be its population after 2 years?
11. The cost of a machine is ₹ 175000. If its value depreciates at the rate of 20% per annum what will be its value after 3 years. Also find the total deprecation.
12. In a factory the production of scooters was 40,000 which rose to 48400 in 2 years. Find the rate of growth per annum.

Points to Remember :

- If S.P. > C.P. i.e. in case of profit.
  (i) Profit = S.P. – C.P.
  (ii) $\text{Profit\%} = \frac{\text{Profit}}{\text{C.P.}} \times 100$
  (iii) $\text{S.P.} = \text{C.P.}\left(\frac{100 + \text{Profit\%}}{100}\right)$
  (iv) $\text{C.P.} = \left(\frac{100 \times \text{S.P.}}{100 + \text{Profit\%}}\right)$
- Amount after n years is given by $A = P\left(1 + \frac{R}{100}\right)^n$
- If the principal remains the same through out the loan period, then the interest calculated on this principal is called the simple interest.
- If the rates be p% for the first year q% for the second year and r% is the third year then amount after 3 years $= P\left(1 + \frac{p}{100}\right) \times \left(1 + \frac{q}{100}\right) + \left(1 + \frac{r}{100}\right)$

EXERCISE

1. **MULTIPLE CHOICE QUESTIONS (MCQs) :**

   Tick (✓) the correct options.

   (a) The price at which goods are purchased is called —
   (i) cost price ☐ (ii) selling price ☐ (iii) profit ☐ (iv) loss ☐

   (b) If selling price is more than the cost price, what will happen?
   (i) profit ☐ (ii) loss ☐ (iii) no profit ☐ (iv) no loss ☐

   (c) If C.P. is ₹918, then the gain percent is —
   (i) 6% ☐ (ii) 8% ☐ (iii) 10% ☐ (iv) 12% ☐

   (d) There will be loss if —
   (i) S.P. > C.P. ☐ (ii) C.P. > S.P. ☐ (iii) C.P. = S.P. ☐ (iv) none of these ☐

2. **Find the profit or loss percent :**

   (a) C.P. = ₹479 Profit = ₹205

   (b) S.P. = ₹250 Loss = ₹50

   (c) C.P. = ₹400 S.P. = ₹450

   (d) S.P. = ₹360 C.P. = ₹400

3. **Find discount in percent when.**

   (a) M.P. = ₹900 and S.P. = ₹873

   (b) M.P. = ₹500 and S.P. = ₹425

4. What will be the compound interest on ₹4000 in two years when rate of interest is 5% per annum?
5. Trilok deposited ₹7500 in a bank which pays him 12% interest per annum compounded quartely. What is the amount which he receives after 9 months?
6. A shopkeeper sells his goods at 80% of their cost price. What percent does he gain or loss?
7. The cost price of an article is 90% of its selling price. What is the profit or loss percent?
8. By selling an article for ₹900; a man gain 20%. Find his cost price and the gain.
9. Subhra deposited ₹6250 to a company at 9.5% per annum compound interest for 2 years. Calculate the amount she will get after 2 years.

HOTS

1. What is the least number of complete years, in which a sum of money at 20% per annum compounded annually will become more than double?
2. At what rate per cent compound interest, does a sum of money become nine-fold in 2 years?

Lab Activity

Objective : To find a formula for future value by using compound interest.

Materials Required : Chart paper, geometry box and sketch pens.

Procedure : Suppose you open an account that pays a guaranteed interest rate, compounded annually. The balance in your account which it will grow to at some point in the future is known as the future value of your starting principal.

For calculating the future value, write P for your principal and r for the return expressed as percent.

Your balance will grow according to the following schedule :

Year	Balance
Now	P
1	$P+\frac{r}{100}P$
2	$\left(P+\frac{r}{100}P\right)\left(1+\frac{r}{100}\right)$

You can simplify it by noticing that you can keep out factors of $1+\frac{r}{100}$ for each line. If you do so, the balance comes to a simple pattern:

Year	Balance
Now	P
1	$P\left(1+\frac{r}{100}\right)$
2	$P\left(1+\frac{r}{100}\right)^2$
3	$P\left(1+\frac{r}{100}\right)^3$
4	$P\left(1+\frac{r}{100}\right)^n$

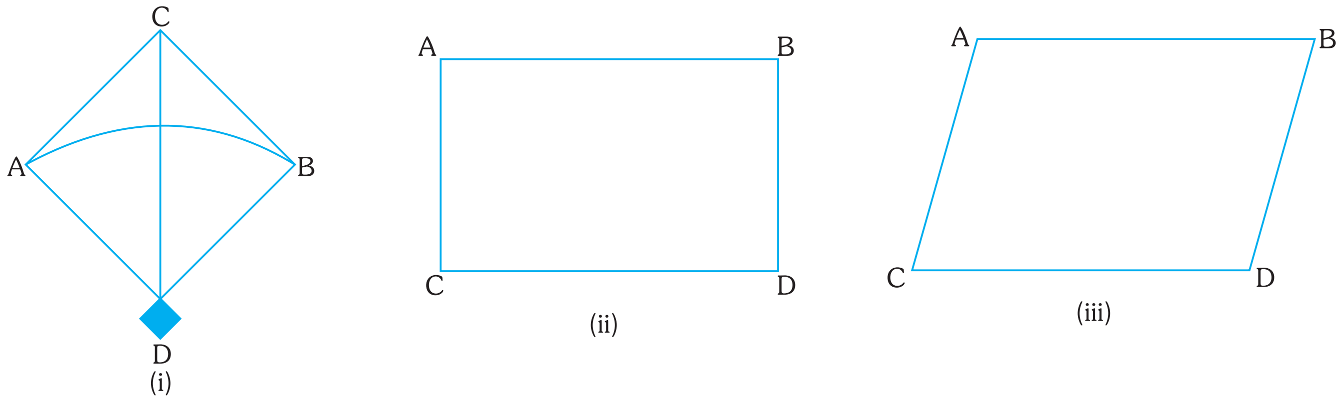
Let us take P - Rs 1000, r = 5%, n = 5 years

Year	Balance
Now	Rs 1000
1	$1000\left(1+\frac{5}{100}\right)$ = Rs 1050
2	$1000\left(1+\frac{5}{100}\right)^2$ = Rs 1102.50
3	$1000\left(1+\frac{5}{100}\right)^3$ = Rs 1157.62
4	$1000\left(1+\frac{5}{100}\right)^4$ = Rs 1215.50
5	$1000\left(1+\frac{5}{100}\right)^5$ = Rs 1276.28

11 Understanding Quadrilaterals

Quadrilateral Definition :

The figure made up of the four line segments is called the quadrilateral with vertices A, B, C and D.



Figures (ii), (iii) are quadrilaterals but fig (i) in not a quadrilateral, because the line segments AB, BC, CD and DA intersect at points other than their ending–points.

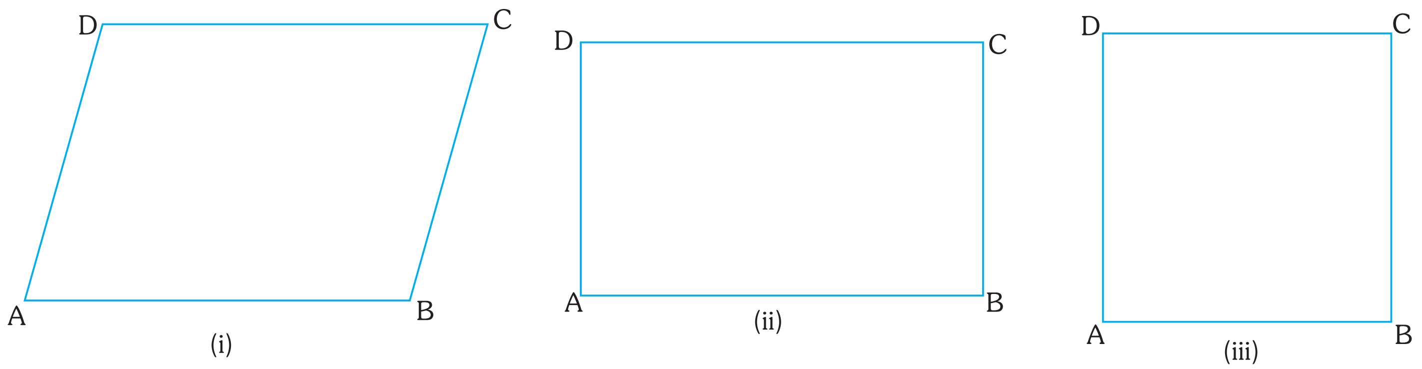
The quadrilateral with vertices A, B, C and D is generally called the quadrilateral ABCD.

Various Types of Quadrilaterals :

(i) Parallelogram : A quadrilateral in which both pairs of opposite sides are parallel and equal is called a parallelogram, written as || gm. or AB || DC.

(ii) Rectangle : A parallelogram each of whose angle is 90°,(right angle) is called a rectangle, written as AB || CD, AD || BC and $\angle A = \angle B = \angle C = \angle D = 90°$.

(iii) Square : A rectangle having all sides equal is called a square.



(iv) Trapezium : A quadrilateral in which two opposite sides are parallel and two opposite sides are non–parallel is called a trapezium.

In fig, ABCD is a trapezium in which AB || DC.

Trapezium is said to be an isosceles trapezium if its nonparallel sides are equal.

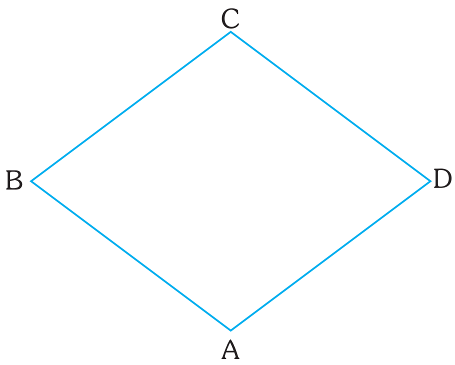
Thus, ABCD is as is isosceles trapezium if AB || DC and AD = BC.

(v) Rhombus : A parallelogram having all sides equal is called a rhombus.

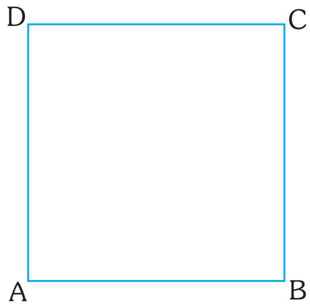
In fig, ABCD is a rhombus in which AB || DC, AD || BC and AB = BC = CD = DA.

(vi) Kite : A quadrilateral in which two pairs of adjacent sides are equal is known as kite.

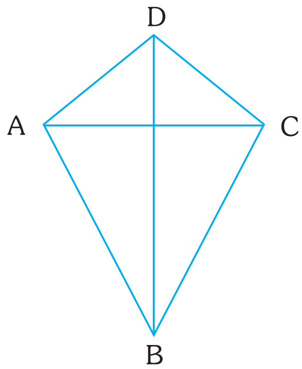
A quadrilateral ABCD is a kite, if AB = AD, BC = CD but AD ≠ BC and AB ≠ CD.



(iv)



(v)



(vi)

Result on parallelogram :

In a parallelogram

(i) the opposite sides are equal;

(ii) the opposite angles are equal;

(iii) diagonals bisect each other.

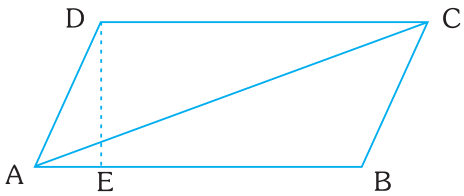
Proof : Let us consider a parallelogram ABCD. Draw its diagonal AC. Now, in triangles ABC and CDA, we have

$\angle 1 = \angle 2$ (Alternate angle)

$\angle 3 = \angle 4$ (Alternate angle)

$AC = AC$ (Common)

$\therefore \quad \Delta ABC \cong \Delta CDA$ (ASA property)



So, $AB = CD$ and $BC = DA$

Also, $\angle B = \angle D$

Similarly, by drawing the diagonal fig BD, we can prove that

$\Delta ABD \cong \Delta CDB$

From this, we get $\angle A = \angle C$

This proves (i), (ii) and (iii).

In order to prove (iv) let us consider a parallelogram ABCD. Draw its diagonals AC and BD, intersecting each other at a point O.

In triangles OAB and OCD.

We have $AB = CD$ (opposite sides of a para.)

$\angle COD = \angle AOB$ (Vertically opp. angles)

$\angle DCO = \angle OAB$ (Alternate angles)

$\therefore \quad \Delta OAB \cong \Delta OCD$

Hence, $OA = OC$ and $OB = OD$.

This shows that diagonals of a parallelogram bisect each other.

Remark 1 : A rhombus, a rectangle and a square are special types of parallelograms. So, all the properties of parallelogram are satisfied in each of them

The converse of the above properties :

(i) A quadrilateral is a parallelogram, if its opposite sides are equal.

(ii) A quadrilateral is a parallelogram if its opposite angles are equal.

(iii) A quadrilateral is a parallelogram if it has one pair of opposite sides parallel and equal.

(iv) A quadrilateral is a parallelogram if it has one pair of opposite sides parallel and equal.

Remark 2 : Since opposite sides of a parallelogram are equal, therefore perimeter = $2(l + b)$,

Where l and b are the lengths of its two adjacent sides.

Diagonal properties of rhombus : The diagonals of a rhombus bisect each other of right angles.

Proof : We have proved above that the diagonals of a parallelogram bisect each other.

But, we know that every rhombus is a parallelogram.

So, it follows that the diagonals of a rhombus bisect to each other.

Now, in order to prove that the diagonals of a rhombus are perpendicular to each other, consider a rhombus ABCD. Draw its diagonals AC and BD which intersect at a point O. Now, in triangles COD and COB, We have

CD = CB (sides of a rhombus)

CO = OC (common)

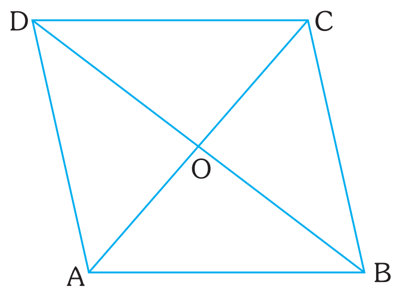
OD = OB ($\because$ O is the mid point of BD)

$\therefore \Delta COD \cong \Delta COB$

So, $\angle COB = \angle COD$

But, COB + COD = 2 right angles (linear pair)

$\therefore \angle COB = \angle COD = 1$ right angle



Hence, the diagonals of a rhombus bisect each other at right angles.

Sum of angles of a quadrilaterals is always 360°

Summary : We may summarize the properties of a rhombus as follows :

(i) All the sides of rhombus are equal.

(ii) The opposite sides of a rhombus are parallel.

(iii) The adjacent angles of a rhombus are supplementary.

(iv) The diagonals of a rhombus bisect each other at right angles.

Diagonal properties of rectangle :

Property 1 : Each angle of a rectangle is a right angle.

Property 2 : Let ABCD be a rectangle such that, $\angle A = 90°$. we have to prove that each angle of a rectangle ABCD is a right angle. for this, we have to show that $\angle B = \angle C = \angle D = 90°$

Since ABCD is a parallelogram.

AB = DC, BC = AD and $\angle A = \angle C$, $\angle B = \angle D$.

Hence, $\angle C = 90°$

Now, AB || DC and AD intersects them at A and D respectively. [$\therefore \angle A = 90°$]

$\therefore \quad \angle A + \angle D = 180°$

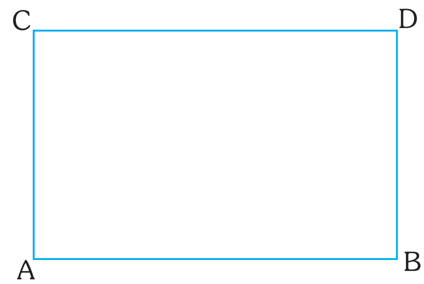
Hence $\quad 90° + \angle D = 180°$

So, $\quad \angle D = 180° - 90° = 90°$

But, $\quad \angle B = \angle D$

$\therefore \quad \angle B = 90°$

Hence, $\quad \angle A = \angle C = \angle D = 90°$

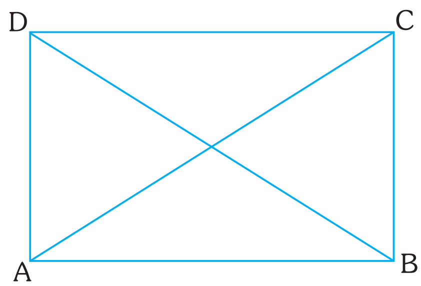


Property 2 : The diagonal of a rectangle are equal.

Proof : Given a rectangle ABCD in which AC and BD are its diagonals.

To prove AC =BD

Proof Δ ABD and Δ BAC, we have



$AB = BA$ (common)

$\angle A = \angle B$ (each equal to 90°)

$AD = BC$ (opposite side of a || gm)

$\therefore \quad \Delta ABD \cong \Delta BAC$

Hence $\cong$ $BD = AC.$

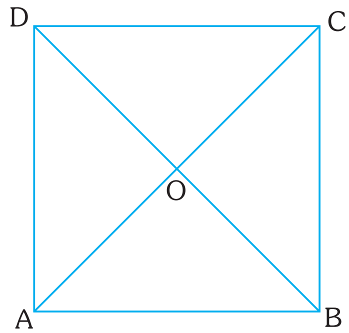
The diagonals of a rectangle are equal.

Properties of Square

Property : The diagonals of a square are equal and perpendicular to each other.

Given A square ABCD whose diagonals AC and BD intersect at O.

To prove $AC = BD$ and $AC \perp BD$.



Proof : In Δ ABC and Δ BAD, we have :

$AB = BA$ (common)

$BC = AD$ (sides of a square)

$\angle ABC = \angle BAD$ (each equal to 90°)

$\therefore \quad \Delta ABC \cong \Delta BAD$

Hence, $AC = BD$

Now, in Δ AOB and AOD, we have :

$OB = OD$ (diagonals of a || gm bisect each other)

$AB = AD$ (sides of a square)

$AO = AO$ (common)

$\therefore \quad \Delta AOB \cong \Delta AOD$

$\angle AOB = \angle AOD$

But $\quad \angle AOB + \angle AOD = 180°$

$\angle AOB = \angle AOD = 90°$

Thus, $AO \perp BD$, $AC \perp BD$

Hence $AC = BD$ and $AC \perp BD$

The diagonals of a square are equal and perpendicular to each other.

The above properties can be summarize as under :

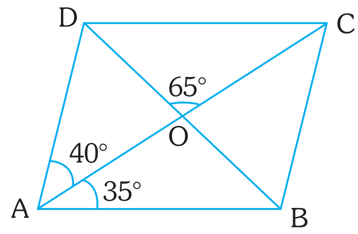
Summary : In a square :

(i) all the sides are of the same length.
(ii) each angle is right angle.
(iii) the diagonals are of equal length.
(iv) the diagonals bisect each other at right angles.

Illustrative Examples

Example 1 : In the adjacent figure, ABCD is a parallelogram in which $\angle DAO = 40°$, $\angle BAO = 35°$ and $\angle COD = 65°$ calculate :

(i) $\angle ABO$ (ii) $\angle ODC$
(iii) $\angle ACB$ (iv) $\angle CBD$



Solution : We have

$\angle AOB = COD = 65°$ [opposite pair of angles] (vertically opposite $\angle$5)

But, the sum of the angles of a triangles of 180°.

$\therefore \quad \angle ABO = 180° - (35 + 65) = 80°$

$\angle ODC = \angle ABO = 80°$ [Alternate angles]

$\angle ACB = \angle DAO = 40°$ [Alternate angles]

Now, $\angle A + \angle B = 180°$

$75° + \angle B = 180°$

So, $\angle B = 180° - 75°$

$\angle B = 105°$

$\therefore \quad \angle ABO + \angle CBD = 105°$

or $\angle CBD = (105° - 80°) = 25$ [$\therefore \ \angle ABO = 80°$]

Example 2 : Show that a cyclic parallelogram is a rectangle.

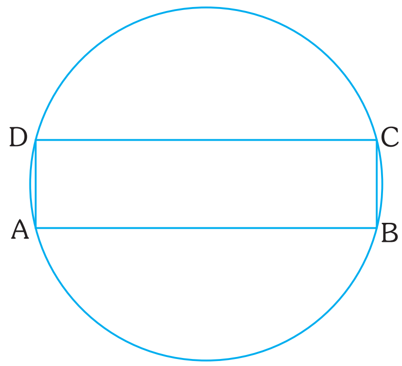
Solution : Let ABCD be a cyclic parallelogram.

We know that the sum of the opposite angles of a cyclic quadrilateral is 180°.

$\therefore \quad \angle A + \angle C = 180°$ ....(i)

We also know that the opposite angles of a parallelogram are equal.

$\therefore \quad \angle A = \angle C$ ....(ii)



From (i) and (ii) we get

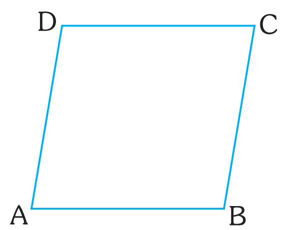
$\angle A = \angle C = 90°$

Hence, ABCD is a rectangle

Example 3 : In a parallelogram the sum of any two adjacent angles is 180° or in a parallelogram, two adjacent angles are supplementary.

Solution : Let ABCD be a parallelogram.

Then, $\angle A$, $\angle B$; $\angle B$, $\angle C$; $\angle C$, $\angle D$ and $\angle D$, $\angle A$ are four pairs of adjacent angles, we have to prove that.



$\angle A + \angle B = 180°$, $\angle B + \angle C = 180°$, $\angle C + \angle D = 180°$, $\angle D + \angle A = 180°$. In a parallelogram ABCD, we have AD || BC and transversal AB intersects them at A and B respectively. $\angle A + \angle B = 180°$.

Similarly, we can prove that $\angle B + \angle C = 180°$, $\angle C + \angle D = 180°$, $\angle D + \angle A = 180°$

Example 4 : Three angles of a quadri lateral are 60°, 75° and 100°, Find its fourth angle.

Solution : sum of three angles = $60° + 75° + 100° = 235°$

we know that sum of all angle of a quadri lateral = 360°

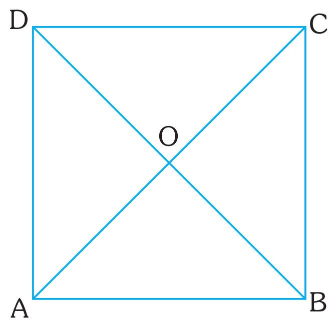
$\therefore$ Fourth angle = $360° - 235° = 125°$

Hence, the sides of the parallelogram are $3 \times 3\,m = 9\,m$ and $5 \times 3\,m = 15\,m$

Example 5 : One of the diagonals of a rhombus is equal to one of its sides find the angles of the rhombus.

Solution : Let ABCD be a rhombus such that its diagonal BD is equal to its sides.

That is, AB = BC = CD = AD = BD



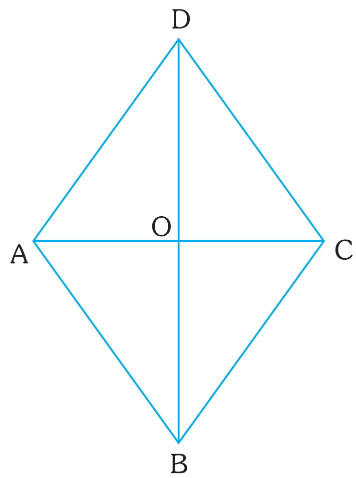
Δ ABD and BCD are equilateral

$$\angle A = \angle C = 60°$$

Now,
$$\angle A + \angle B = 180°$$
$$60° + \angle B = 180°$$
$$\angle B = 180° - 60° = 120°$$

Hence, $\angle A = 60° = \angle C$ and $\angle B = \angle D = 120°$

Example 6 : Find the length of a side of the rhombus. Whose diagonals AC and BD are of lengths 8 *cm* and 6 *cm* respectively. Let AC and BD intersect at O. Since the diagonals of a rhombus bisect each other at right angles.



$$AO = \frac{1}{2} AC = \frac{1}{2} \times 8\text{ cm} = 4\text{ cm}$$

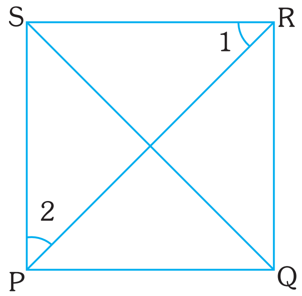
and $$BO = \frac{1}{2} BD = \frac{1}{2} \times 6\text{ cm} = 3\text{ cm}$$

Since AOB is a right triangle right angled at O.
therefore, by Pythagoras theorem
$AB^2 = OA^2 + OB^2$
$AB^2 = 4^2 + 3^2$
$AB^2 = 16 + 9$
$AB^2 = 5^2$ $\Rightarrow$ A B = 5 cm

Example 7 : PQRS is a square. PR and SQ intersect at O. State the measure of ∠POQ.

Solution : Since the diagonals of a square intersect a right angle. Therefore, ∠POQ = 90°

Example 8 : PQRS is a square determine ∠SRP.



Solution : PQRS is a square.

∴ PS = RS and ∠PSR = 90°

Now, in PSR, we have

PS = SR

∠1 = ∠2 [∴ Angle opp. to equal sides are equal]

But, ∠1 + ∠2 + PSR = 180° [∴ ∠PSR = 90°]

∴ 2∠1 + 90° = 180°

So, 2∠1 = 90°

∠1 = 45°

Exercise 11.1

1. The measure of one angle of a parallelogram is 70°. What are the measures of the remaining angle ?
2. Two adjacent angles of a parallelogram are in 1 : 2. Find the measures of all the angles of the parallelogram.
3. The sum of two opposite angles of parallelogram is 130°. Find all the angles of the parallelogram.
4. In the below figure 1, ABCD is a trapezium in which AB || DC. If ∠A = 60° and ∠B = 40°. Find the measure of its remaining two angles.
5. Show that a diagonal of a parallelogram divided it into two congruent triangles.

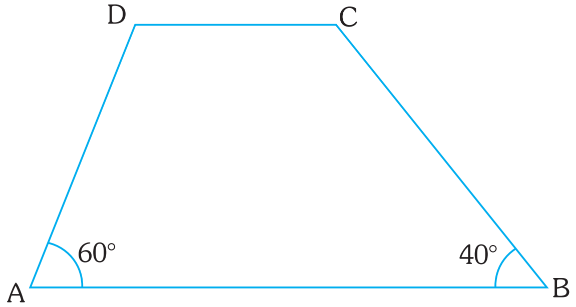


Fig. 1

6. In a parallelogram ABCD, the diagonals bisect each other at O. ∠ABC = 30°, ∠BDC = 10° and ∠CAB = 70°. Find ∠DAB, ∠ADC, ∠BCD, ∠AOD, ∠DOC, ∠BOC, ∠AOB, ∠ACD, ∠CAB, ∠ADB, ∠ACB, ∠DBC and ∠DBA.
7. In fig. 2, BDEF and DCEF are both parallelograms. Is it true that BD = DC ?

8. In fig. 2, suppose it is known that DE = DF. Then, is ABC isosceles?

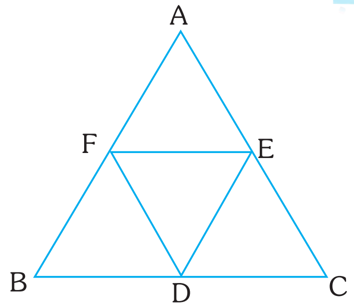


Fig. 2

9. **Diagonals of parallelogram ABCD intersect at O as shown in fig. 3, xy contains o, and x,y are point on opposite sides of the parallelogram. Give reasons for each of the following :**

(a) OB = OD

(b) $\angle$OBY = $\angle$DOX

(c) Δ BOY = Δ DOX

(d) $\angle$BOY = $\angle$DOX

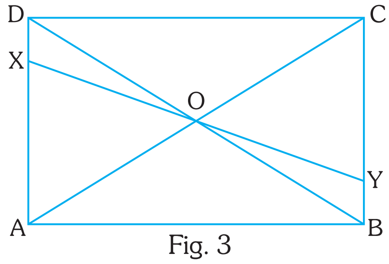


Fig. 3

10. Draw a parallelogram ABCD, in which AB = 4 *cm*, AD = 3 *cm* and $\angle$BAD = 60° measure its diagonals.

11. Draw a parallelogram ABCD, if AB = 5 *cm*, AD = 3 *cm* and BD = 4.5 *cm* measure AC.

12. The diagonals of a parallelogram are not perpendicular. Is it a rhombus ? Why or why not ?

13. ABCD is a rhombus. If $\angle$ACB = 40°, find $\angle$ADB.

14. If the diagonals of a rhombus are 12 *cm* and 16 *cm*, find the length of each side.

15. Construct a rhombus whose diagonals are of length 10 *cm* and 6 *cm*.

16. Draw a rhombus, having each side of length 3.5 *cm* and one of the angles as 40°.

17. **ABCD is rhombus and its diagonals intersect at O.**

(a) Is Δ BOC $\cong$ Δ DOC ? State the congruence condition used ?

(b) Also state it $\angle$BCO = $\angle$DCO.

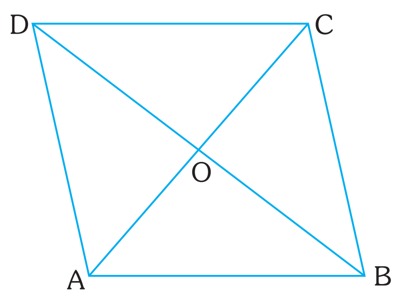
18. ABCD is a rhombus whose diagonals intersect at O. If AB = 10 *cm*, diagonal BD = 16 *cm*, find the length of diagonal AC.

19. The sides of a rectangle are in the ratio 3 : 2 and its perimeter is 20 cm. Draw the rectangle.

20. The sides of a rectangle are in the ratio 5 : 4. Find its sides if the perimeter is 90 *cm*.

21. Draw a square whose each side measure 4.8 *cm*.

22. In the adjacent figure, ABCD is a rhombus whose diagonals intersect at O. If AB = 10 *cm* and diagonal BD = 16 *cm*, find the length of diagonal AC.



[Hing OB = 8 *cm*, AB = 10 *cm* and $\angle$AOB = right angle]

$OA^2 = (AB^2 - OB^2)$ (by Pythagoras Theorem)

Now, AC = 2 OA

23. **Which of the following statements are true:**

(a) Rhombus has only two pairs of equal sides.

(b) Rectangle's diagonals are equal.

(c) Square has all its sides of equal length.

(d) Rectangle's diagonals are perpendicular.

(e) Square diagonals are equal to its sides.

(f) Rhombus has all its sides of equal length.

(g) Rectangle's diagonals are equal and bisect each other.

(h) Rhombus is a parallelogram.

Points to Remember :

- A quadrilateral is a polygon of four sides.
- In quadrilateral if each of its angles is less then 180°, then it is convex.
- The sum of measures of the angles of a quadrilateral is 360°
- The sum of the measures of exterior angles of a polygon is 360°
- A parallelogram is a quadrilateral with opposite sides parallel.
- In a parallelogram, opposite sides are equal.
- In a parallelogram, opposite angles are equal.
- In a parallelogram the diagonals bisect each other.
- In a parallelogram, two adjacent angles are supplementary.
- A quadrilateral, whose ane pair of opposite sides is parallel is called a trapezium.
- If non parallel sides of trapezium are equal, it is called isosceles trapezium.
- A parallelogram, whose all sides are equal, is called a rhombus.
- The diagonals of a rhombus bisect each other at right angles.
- If all angles of parallelogram are 90°, it is a rectangle.
- The diagonals of a rectangle are equal and bisect each other.
- If all angles of a rhombus are right angles, then it is a square.
- Diagonals of a square are equal and bisect each other at a right angles.
- If 2 pairs of adjacent sides of quadrilateral are equal, then it is a kite.

EXERCISE

1. MULTIPLE CHOICE QUESTIONS (MCQs) :

Tick (✓) the correct options.

(a) If the diagonals of a quadrilateral bisect each other, then it must be :

(i) Square (ii) Rectangle (iii) Rhombus (iv) Parallelogram

(b) The sum of angles of a quadrilateral is :

(i) 90° (ii) 180° (iii) 270° (iv) 360°

(c) If the three angles of a quadrilateral are 75^0 each, then its fourth angle must be:

(i) Acute (ii) Obtuse (iii) Right (iv) None of these

(d) The quadrilateral which is equilateral but not equiangular is :

(i) Square (ii) Rectangle (iii) Rhombus (iv) Trapezium

(e) A quadrilateral which is equiangular but not equilateral is :

(i) Rectangle (ii) Square (iii) Rhombus (iv) Trapezium

(f) A quadrilateral which is both equiangular and equilateral is :

(i) Kite (ii) Rectangle (iii) Square (iv) Rhombus

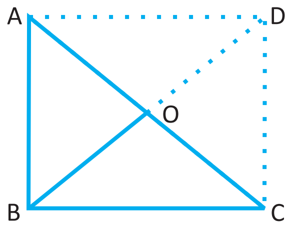
(g) The measure of each angle of a convex polygon is :

(i) More than 180° (ii) Less than 180° (iii) Equal to 180° (iv) None of these

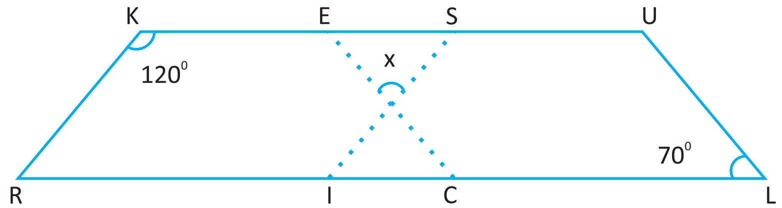
(h) Is the number of sides of a polygon same as the number of angles ?

(i) Yes (ii) No (iii) Do not know (iv) May be

2. In the adjoining figure, ABC is a right-angled triangle and O is the mid point of the side opposite to the right angle. Explain why O is equidistant from A, B and C. (The dotted lines are drawn additionally to help you)



3. In the adjoining figure, both RISK and CLUE are parallelograms. Find the value of *x*.



4. State true (T) or false (F):

(a) All parallelograms are trapezium.

(b) Every square is a rectangle as well as a rhombus.

(c) All rectangles are squares.

(d) All rhombuses are parallelograms.

(e) All squares are trapezium.

(f) All trapeziums are square.

5. Find the value of x, if the two adjacent angles of a parallelogram are $(3x–4)°$ and $(3x+16)°$. Also, find the measure of each of its angles.

HOTS

If the sides of a square are $(5a – 17)$ cm and $(2a + 14)$ cm, then find the length of its sides and diagonal.

Lab Activity

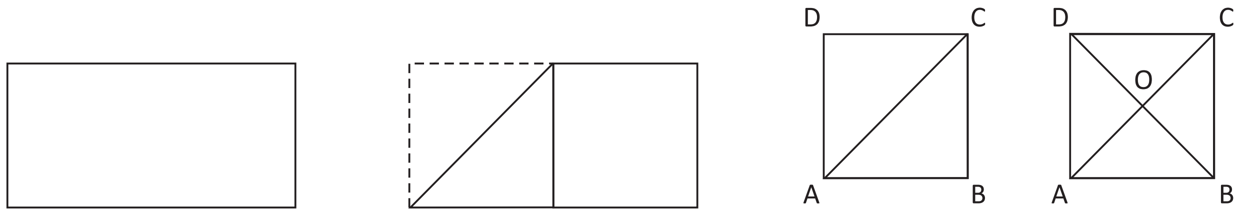
Objective : To verify the properties of a square by paper folding.

Materials Required : A sheet of paper, pencil, scissors.

Procedure :

Step 1. Take a rectangular sheet of paper.

Step 2. Fold it as shown in figure and cut off the extra portion.



(i) (ii) (iii) (iv)

Step 1. Unfold the sheet and you get a square ABCD with crease AC as diagonal.

Step 1. Fold it along BD and you get another diagonal with crease BD.

In square ABCD, you observe that:

(i) AB = BC = CD = AD

(ii) $m\angle A = m\angle B = m\angle C = m\angle D = 90°$

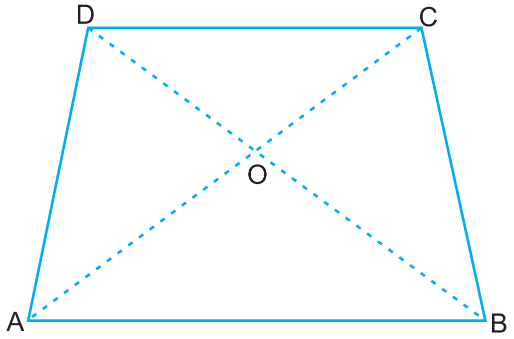
(iii) $\overline{AC} = \overline{BD}$

(iv) $\overline{OA} = \overline{OB} = \overline{OC} = \overline{OD}$ and $AC \perp BD$.

Thus, all the properties of a square stand verified.

12 Construction of Quadrilaterals

Quadrilaterals



Let A, B ,C ,D be four points in a plane such that not three of them are collinear and the line segments AB, BC, CD and DA do not intersect except at their end points.

Then the figure formed by these four lines segments is called a quadrilateral.

A quadrilateral ABCD has

(i) Four sides : AB, BC, CD and DA

(ii) Four angles : ∠A, ∠B, ∠C and ∠D

(iii) Two diagonals : AC and BD

Adjacent Sides : Two sides of a quadrilateral which have no common end point are called its adjacent sides. For Example : AB and BC, BC and CD etc.

Opposite Sides : Two sides of a quadrilateral which have a common end points are called its opposite sides. Thus AB, CD and BC are two pairs of opposite sides.

Adjacent Angles : Two angles of a quadrilateral which have a common side are called adjacent angles. For Example : ∠A and ∠B , ∠B and ∠C etc.

Opposite Angles : The angles which are not adjacent are known as opposite angles. Thus in the given figure, ∠A and ∠C, ∠B and ∠D are two opposite angles.

[The sum of all the angles of a quadrilateral is 360°]

Construction of Quadrilaterals

We know that in case of a quadrilateral the elements are its four sides, two diagonals and its four angles. Thus a quadrilateral has ten elements. It is possible to draw a convex quadrilateral if any five independent elements are given. To draw a non-convex quadrilateral, six independent elements are required.

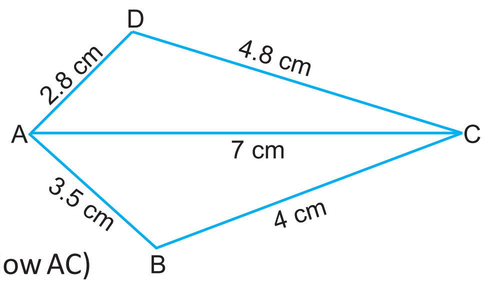
We divide the required quadrilateral into two triangles which can be easily constructed. These two triangles together will form a quadrilateral.

1. To construct a quadrilateral when four sides and one diagonal are given:

Example 1 : Construct a quadrilateral ABCD in which AB = 3.5*cm*, BC 4*cm*, CD 4.8*cm*, DA = 2.8 *cm* and AC = 7*cm*.

Solution : First we draw a rough sketch of ABCD and write down its dimensions as shown.

We may divide it into two triangles ABC and ACD.



Steps of construction :

(i) Draw AC = 7 *cm*.

(ii) With a as centre and radius 3.5*cm* draw and arc (below AC)

(iii) With B as centre and radius 4*cm* draw another arc. Cutting the previous one at B.

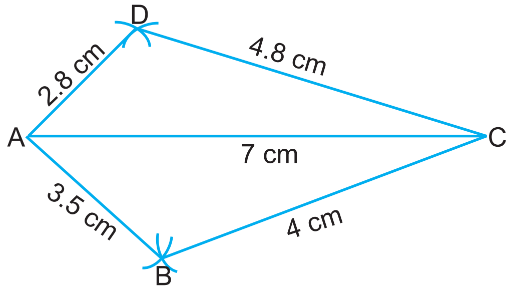
(iv) Join AB and BC.

(v) With A as centre and radius 2.8 *cm* draw an arc (above AC)

(vi) With as centre and radius equal to 4.8 *cm* draw another are. Cutting the previous one at D

(vii) Join AD and CD.

(viii) ABCD is the required quadrilateral shown is figure.

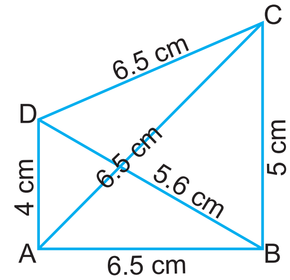


2. To construct a quadrilateral when three sides and two diagonals are given :

Example 2 : Construct a quadrilateral ABCD in which BC = 5*cm*, CA = 6.5*cm*, AD = 4.4*cm*, CD = 6.5*cm* BD = 5.6*cm*.

Solution : Let us draw a rough sketch of the required quadrilateral and write down its dimensions.We can divide the quadrilateral into two triangles DCA and DCB.

So, we construct these triangles to have the required quadrilateral.



Steps of construction :

(i) Draw DC =6.5*cm*.

(ii) With D as centre and radius equal to 4.4*cm* draw an arc.

(iii) With C as centre and equal to 5.6*cm* draw another arc to cut the previous arc at A.

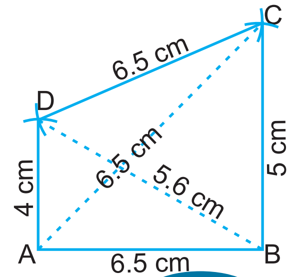
(iv) Join AD and AC.

(v) With D as centre and radius equal of 6.5*cm* draw an arc.

(vi) With C as centre and radius equal to 5*cm* draw another arc to cut the previously drawn arc at B.

(vii) Join BC, BD and AB

(viii) ABCD is the required quadrilateral shown in the figure.

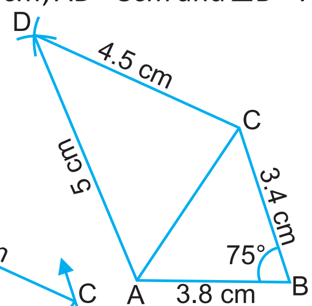


We observe, that while doing construction on quadrilateral, we construct two triangles from the given data.

3. To construct a quadrilateral when four sides and one angle are given :

Example 3 : Construct a quadrilateral ABCD in which AB = 3.8*cm*, BC = 3.4, CD = 4.5*cm*, AD = 5*cm* and ∠B = 75°

Solution : Draw a rough sketch of the required quadrilateral and write down its dimensions. We can divide it into Δ ABC and ACD.



Steps of construction :

(i) Draw AB =3.8*cm*.

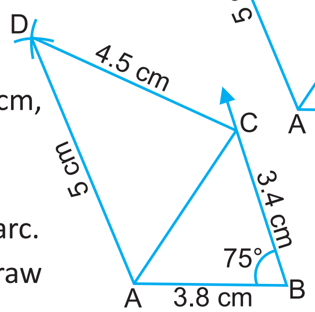
(ii) Make ∠ABX = 75°

(iii) With B as centre and equal radius equal to 3.4cm, cut off BC = 3.4*cm* along BX.

(iv) Join AC.

(v) With A as centre and radius equal to 5*cm* draw an arc.

(vi) With C as centre and radius equal to 4.5cm draw



another arc to cut the previously drawn arc at D.

(vii) Join DC and DA.

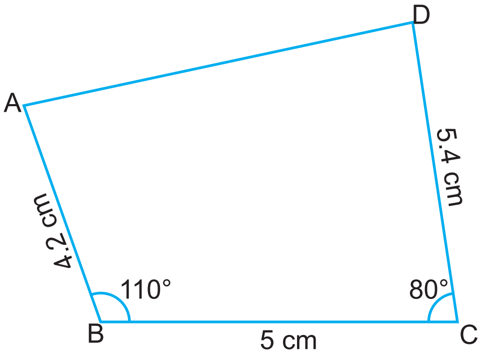
(viii) ABCD is the required quadrilateral shown in the figure.

4. To construct a quadrilateral when three sides and two included angles are given :

Example 4 : Construct quadrilateral ABCD in which AB = 4.2*cm*, BC = 5*cm*, CD = 5.4*cm*, ∠B = 110° and ∠C = 80°

Solution : Let us draw a rough sketch of the required quadrilateral and write down the given dimensions.

Steps of construction :



(i) Draw BC = 5*cm*,

(ii) Make ∠CBX = 110°

(iii) With B as centre and radius 4.2*cm*, cut out BA 4.2*cm* along BX.

(iv) Make BCY = 80°

(v) With C as centre and radius 5.4*cm* draw an arc to cut CY at D.

(vi) Join DA.

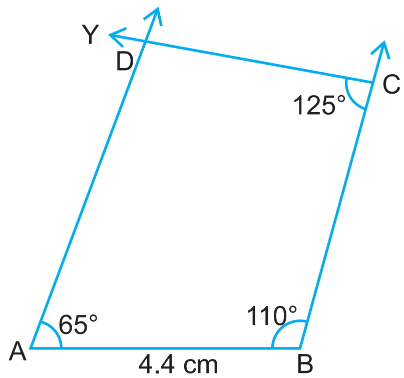
(vii) ABCD is the required quadrilateral, shown in the figure.

5. To construct a quadrilateral when three angles and their two included sides are given :

Example 5 : Construct a quadrilateral ABCD in which AB = 4.4*cm*, BC = 5*cm*, ∠A = 65°, ∠B = 110° and ∠C = 125°

Solution : Draw a rough sketch of the required quadrilateral ABCD and write down its dimensions.

Step of Construction :



(i) Draw AB = 4.4*cm*,

(ii) Make ∠ABX = 110°

(iii) With B as centre and radius 5*cm* draw an arc to cut BX at C.

(iv) Make ∠BCY = 125° and ∠BAZ = 65°

(v) Let CY and AZ intersect at D.

(vi) Then ABCD is the required quadrilateral.

Exercise 12.1

1. Construct a quadrilateral ABCD in which AB = 4.3*cm*, BC = 6.2*cm*, CD = 5*cm*, DA = 5*cm* and AC = 8.3*cm*.
2. Construct a quadrilateral PQRS in which PQ = 5.5*cm* QR = 4.8*cm*, RS = 4.3*cm* SP = 3.9*cm* and diagonal PR = 4*cm*.
3. Construct a quadrilateral PQRS in which QR = 7.5*cm*, PR = PS = 7*cm*, RS = 6*cm* and QS = 10*cm*, measure the fourth side.
4. Construct a quadrilateral ABCD in which AB = BC = 4*cm*, CD = 3.5*cm*, DA = 5.5*cm*, AC = 8*cm* and BD = 5*cm*.
5. Construct a quadrilateral ABCD in which AB = BC = 4*cm* AD = 5.5*cm* and ∠ ABC = 130°.

6. Construct a quadrilateral ABCD in which AB =3*cm* BC = 4*cm* CD = 3*cm* and DA = 3.5*cm* ∠A and ∠A = 80°.

7. Construct a quadrilateral ABCD in which AB =*cm* BC = 5*cm* CD = 4.8*cm* DA = 120°and ∠C = 70°.

8. Construct a quadrilateral PQRS in which PQ = 6*cm* QR = 5.8*cm*, RS = 3*cm* ∠Q= 45° and ∠R = 90°.

9. Construct a quadrilateral ABCD in which AB =6*cm* BC = 4.5*cm* ∠A = 60°, ∠B = 110° ∠D = 90°.

10. Construct a quadrilateral ABCD in which AB = 4*cm* AC = 5.2*cm*, AD = 5.6*cm* and ∠ABC = ∠ACD = 80°.

Construction of special types of quadrilaterals.

Example 1 : Construct a parallelogram ABCD in which AB= 3.6*cm*, BC = 4.2*cm* and AC = 6.5*cm*.

Solution : In a parallelogram opposite sides are equal. Thus we have to construct a quadrilateral ABCD in which AB= 3.6*cm* BC = 4.2*cm* CD = 3.6*cm*, AD = 4.2*cm* and AC= 6.5*cm*.

Steps of Construction :

(i) Draw AC = 6.5*cm* as shown in fig.

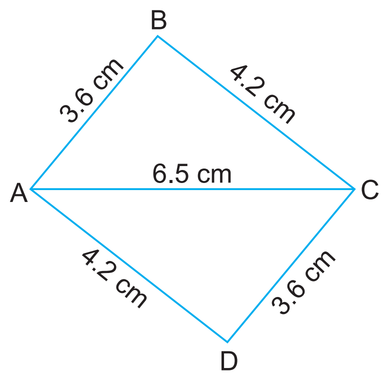
(ii) With A as centre and radius AB = 3.6*cm* draw an arc.

(iii) With C as centre and radius BC = 4.2*cm* draw and an arc, interacting the arc drawn is steps || at B.

(iv) With A as centre and radius AD = 4.2*cm* draw and arc on the side of AC opposite to that of B.

(v) With C as centre and radius CD = 3.6*cm*, draw another arc the intersect the arc drawn in step iv at.

(vi) Join AB, BC. AD and CD to obtain the required. parallelogram ABCD.



Example 2 : Construct a rectanlge ABCD in which side BC= 5*cm*, BD = 6.2*cm*.

Solution : First draw a rough sketch of the required rectangle and write down its dimensions.

Steps of Construction :

(i) Draw BC = 5*cm.*

(ii) Draw CX and BC.

(iii) With B as centre and radius 6.2*cm* draw and arc to cut CX at D.

(iv) Join BD.

(v) With D as centre and radius 5cm draw an arc.

(vi) With B as centre and radius equal to CD draw another arc. Cutting the previous arc at A.

(vii) ABCD is required rectangle.

Example 3 : Construct a rhombus with sides 5cm and one diagonal 6.5*cm*.

Solution : A rhombus is quadrilateral having all sides equal and opposite sides parallel, thus we have to construct a quadrilateral ABCD. Whose all sides are equal to 5*cm* and one diagonal. say AC = 6.5*cm*. To draw this, we follow the following steps.

Steps of Construction :

(i) Draw AC = 6.5*cm*

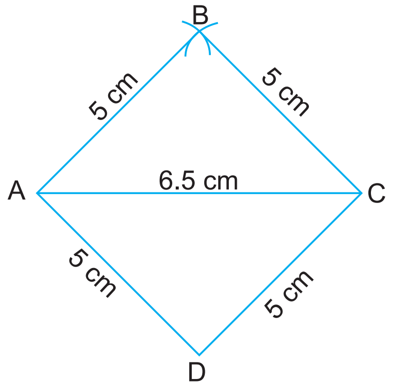
(ii) With A as centre and radius AB = 5*cm* draw an arc.

(iii) With C as centre and radius CB = 5*cm*, draw another arc intersecting the arc drawn at B.

(iv) With A as centre and radius AD = 5cm, draw an arc on the side at AC opposite to that of B.

(v) With C as centre and radius CD = 5*cm*, draw another arc, intersecting the arc at D.

(vi) Join AB, BC, CD and AD to obtain the required rhombus.



Example 4 : Construct a square ABCD, each of whose diagonal is 5.5*cm*.

Solution : We know that the diagonals of a square bisect each other at right angles so, we proceed according to the following steps.

Steps At Constructions :

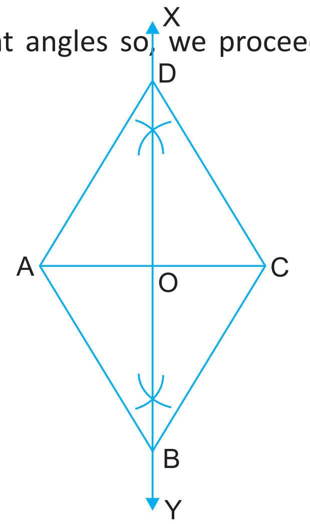
(i) Draw AC = 5.5*cm*

(ii) Draw the perpendicular XY on AC, meeting AC to O.

(iii) From O set off OB = ½ (5.5) = 2.75*cm* along OX. DO = 2.75*cm* along OX.

(iv) Join AB, BC, CD and DA.

(v) ABCD is the required square.



Exercise 12.1

1. Construct a parallelogram ABCD in which AB = 5.3*cm* BC = 4.6*cm* and AC = 7.5*cm*.

2. Construct a parallelogram ABCD in which AB = 4.3*cm* AD = 3.9 *cm* and BD = 6.7*cm*.

3. Construct a parallelogram ABCD in which BC= 6*cm* $\angle$BCD = 110° and 4.5*cm*.

4. Construct a parallelogram one of whose side is 4.3*cm* and whose diagonals are 5.7*cm* and 7.1 *cm* measure the other side.

5. Construct a parallelogram ABCD in which diagonal AC = 3.7*cm*, diagonal BD = 4.5*cm* and the angle between AC and BD is 65°.

6. Construct a rectangle ABCD whose adjacent sides are 12*cm* and 8*cm*.

7. Construct a square, each of whose diagonals measures 6*cm*.

8. Construct a square, each of whose diagonals measures 7.2*cm*.

9. Construct a rectangle PQRS in which QR = 4*cm* and diagonal PR = 6.5*cm*. Measure the other side of the rectangle.

10. Construct a rhombus whose diagonals are 5.7cm and 8.2cm.

11. Construct a rhombus whose side is 7.5cm and one angle is 70°.

12. Construct a trapezium ABCD in which AB = 5.9*cm* BC = 4.1*cm*, CD = 3.4*cm*, $\angle$B = 80° and DC||AB.

13. Draw trapezium ABCD in which AB||DC AB = 7.1*cm* BC = 54.*cm*, AD = 6.2*cm* and $\angle$B = 62°.

Points to Remember :

- A quadrilateral is a geometrical figure with four sides and four angles. It is a polygon having four sides.
- The eight elements of quadrilateral are–four sides and four angles.
- A quadrilateral can be constructed, if—
  - (a) Four sides and one diagonal have been provided
  - (b) Four sides and one angle have been provided.
  - (c) Three sides and two diagonals have been provided.
  - (d) Three sides and two included angle have been provided.
  - (e) Two adjacent sides and three angle have been provided.
- We can also construct special types of quadrilaterals, viz, parallelogram, rectangle, square and rhombus.

EXERCISE

1. MULTIPLE CHOICE QUESTIONS (MCQs) :

Tick (✓) the correct options.

(a) A quadrilateral is also a kind of—

(i) hexagon ☐ (ii) polygon ☐ (iii) 3D shape ☐ (iv) sketch ☐

(b) How many minimum elements are needed to draw any quadrilateral?

(i) 8 ☐ (ii) 6 ☐ (iii) 5 ☐ (iv) 4 ☐

(c) Three sides of a quadrilaterals have been given. what additional information do you need to draw it?

(i) fourth side ☐ (ii) any two angles ☐

(iii) two included angles between given sides ☐ (iv) none of these ☐

(d) A square has four angles, each one of them being a/an—

(i) acute angle ☐ (ii) obtuse angle ☐ (iii) right angle ☐ (iv) none of these ☐

(e) If three angles of a quadrilateral have been given, we need the measure of the following to draw it.

(i) three sides. ☐ (ii) four sides ☐ (iii) two included sides ☐ (iv) fourth angle ☐

(f) The diagonals of a rectangle are—

(i) equal ☐ (ii) unequal ☐ (iii) non-intersecting ☐ (iv) none of these ☐

(g) The sum of the interior angles of a square is—

(i) 270° ☐ (ii) 290° ☐ (iii) 360° ☐ (iv) 380° ☐

(h) Each diagonals of a parallelogram divides it into—

(i) two equal squares ☐ (ii) two equal triangles ☐

(iii) two congruent triangles ☐ (iv) none of these ☐

2. Draw a quadrilateral PQRS, whose data is as follows:

$\overline{PQ}$ = 6 cm　　$\overline{QR}$ = 5.1 cm

$\overline{RS}$ = 3.4 cm　　$\overline{PR}$ (diagonal) = 6.9 cm

Write the steps of construction, too.

3. Draw a quadrilateral ABCD, whose data is as follows:

$\overline{AB}$ (base)= 3.5 cm　　$\overline{BC}$ = 5.3 cm

$\overline{CD}$ = 7.2 cm　　$\overline{DA}$ = 3.2 cm

$\angle DAB = 125°$

Write the steps of construction too.

4. Draw a quadrilateral PQRS in which $\overline{PQ}$ (base) = 3 cm, $\overline{QR}$ = 2.8 cm, $\overline{RS}$ = 4.3 cm $\overline{PR}$ (diagonal) = 4.1 $\overline{SQ}$ (diagonal) = 5.3 cm. Write the steps of construction, too.

5. Draw a quadrilateral ABCD in which AB is the base. The vital data is as follows.

$\overline{AB}$ = 5 cm　　$\overline{BC}$ = 4.9 cm　　$\overline{CD}$ = 4 cm

$\angle DAB = 80°$　　$\angle CBA = 69°$

Draw the steps of construction, too.

6. Draw the step of quadrilateral PQRS for which the following data has been provided $\overline{PQ}$ (base) = 7.0 cm $\overline{QR}$ = 4.9 cm

$\angle SPQ = 74°$　　$\angle PQR = 95°$　　$\angle QRS = 73°$

Write the steps of construction, too.

7. Draw a quadrilateral ABCD in which $\overline{AB}$ = 4.7 cm, $\overline{BC}$ = 3.5 cm and $\overline{AC}$ = 5 cm. What type of quadrilateral is it ? Write the steps of construction, too.

8. Draw a quadrilateral PQRS in which $\overline{PQ}$ = 7 cm and diagonal $\overline{PR}$ = 8.6 cm. Write the steps of construction, too.

9. Draw a square ABCD in which $\overline{PQ}$ = 7 cm and diagonal $\overline{PR}$ = 8.6 cm. Write the steps of construction, too.

10. Draw a square PQRS, whose diagonal length is 4.5 cm.

11. Draw a quadrilateral ABCD in which AB = 6.5 cm, BC = 5 cm, ĐA = 60°, ĐB = 110° and ĐD = 85°.

HOTS

In which of the following cases, can you draw a quadrilateral ?

(a) Four sides and a diagonal are given.

(b) Four angles and one side are given.

(c) Four sides and one angle are given.

(d) Three sides and two diagonals are given.

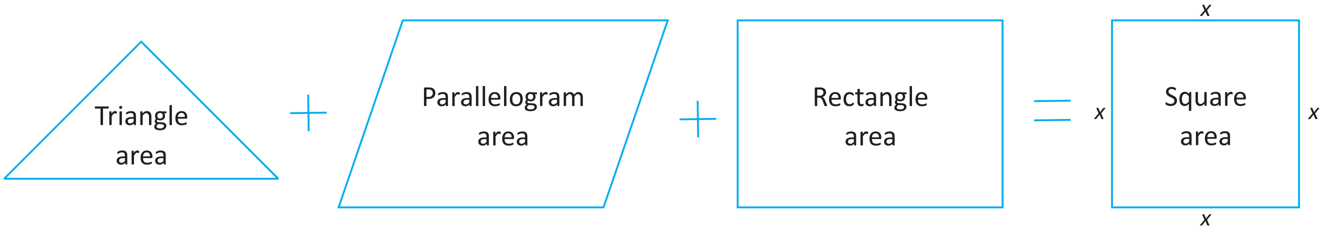
Lab Activity

Objective : To understand the shapes of various quadrilaterals.

Material Required :
1. Cardboard (two big sheets)
2. Glue
3. Cutter
4. Pencil and eraser
5. Ruler
6. Set squares and compass

Procedure

1. Draw a triangle on the cardboard sheet. Its base is 8 cm and its height is 4 cm. Cut out the sheet.
2. Draw a parallelogram with base 10 cm and height 6 cm. Use set squares and compass to do so. Cut it out of the sheet.
3. Draw a rectangle of length 8 cm and breadth 3 cm. Cut it out of the sheet.
4. Calculate the area of all these three shapes.
5. On the second sheet of cardboard, draw two boxes. They must be big enough. Look at the figure shown ahead.

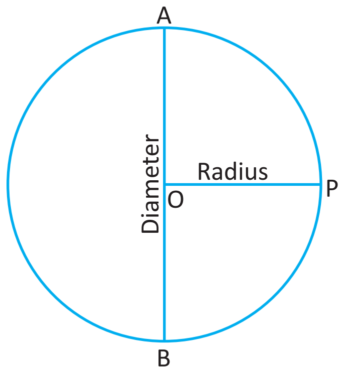


6. The area of the square is the sum of the areas of three shapes drawn earlier. Calculate the length of its sides and draw it on the sheet.
7. Paste the triangle, parallelogram and rectangle on the left hand box.
8. Paste the square on the right hand box.
9. Fill in the blank boxes below.

Sides of the square:	____________
Area of the square:	____________

13 Circle and its Properties

We have learnt about circles and some of their properties in the previous class. Let us recall some definitions.



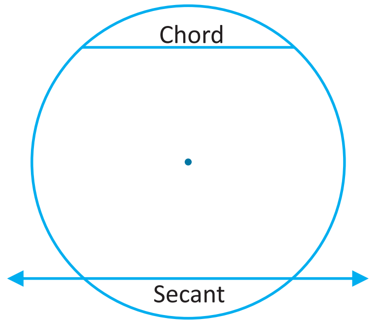
Circle : A circle is a set of those points in a plane that are at a given constant distance from a given fixed point in the plane.

Radius : A line segment with one end point at the centre of the circle and the other on the boundary of the circle is called a radius of the circle. All radii of a circle are equal.

Diameter : A line segment passing through the centre of the circle and having its end points on the boundary of the circle is called a diameter of the circle. In fig AB is a diameter of the circle.

Clearly, AB = 2× op

i.e. diameter = 2× radius.



Chord: A line segment joining any two points on a circle is called a chord of the circle.

Clearly, a diameter is the longest chord of the circle.

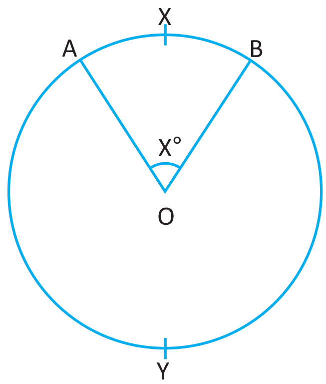
Semicircle : A diameter of a circle divides the circle into two equal parts, each part is called a semicircle.

Secant: A secant is a line segment that intersects a circle at two points.

Circumference of a circle : The perimeter of a circle is called its circumference.

If r is the radius of a circle, then its circumference C is given by the formula.

$C = 2\pi r$.



Arc : An arc is a part of a circle included between two points on the circumference of the circle.

Minor Arc: An arc of a circle is called minor arc if its length is less than the length of the semicircle.

Major Arc : An arc of a circle is called a major arc if its length is greater than the length of the semicircle.

In fig we have minor arc $\overset{\frown}{AB} = \overset{\frown}{AXB}$,

Major arc $\overset{\frown}{AB} = \overset{\frown}{AYB}$

Degree measure of an arc: The degree measure of an arc is the measure of its central angle of degree.

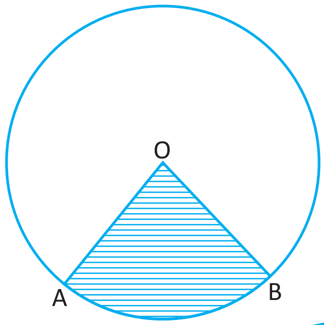
Thus in the given figure, $\overset{\frown}{AB}$ is an arc of a circle with centre O ∠AOB is the central angle and the measure of ∠AOB in degrees is called the degree measure or simply the measure of arc AB, written as m (AB) = x°.

Congruent arcs : Two arcs of a circle are congruent if either of them can be superposed on the other so as to cover it exactly. The degree measures of the two arcs are the same.

The lengths of the congruent arcs are always equal.

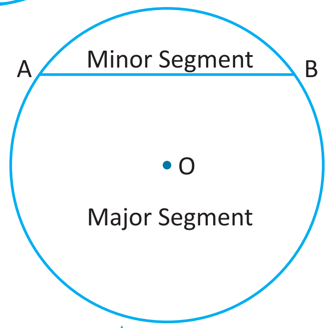
Sector of a circle : A sector of a circle is the region enclosed by an arc of the circle and the two radii of its end points.

Thus in the given figure, region OAB is a sector.



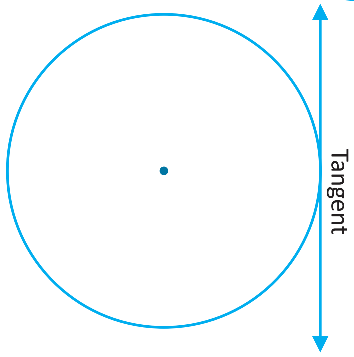
Segment of circle : A segment of a circle is the region bounded by an arc of the circle and the chord.

In fig. AB is a chord of the circle with centre O. This chord divides the circular region into two parts. Each part is called a segment of a circle. The region containing the centre of the circle is called the major segment and the part which does not contain the centre is called minor segment.



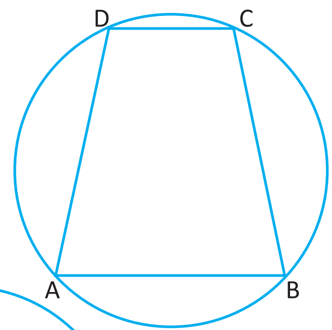
Tangent of a circle : A line that intersects a circle in exactly one point is called tangent to the circle.

The point at which the tangent intersects the circle is called the point of contact.



Cyclic quadrilateral : If all the four vertices of a quadrilateral lies on a circle then such a quadrilateral is called cyclic quadrilateral.

Thus in the given figure, ABCD is a cyclic quadrilateral and the points ABCD are concyclic.



Properties of Chords of A Circle

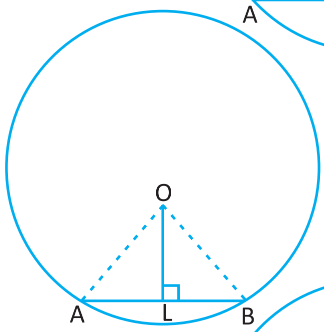
Property 1 : The perpendicular from the centre of a circle to a chord bisects the chord.

Proof : Let AB be a chord of a circle with centre O and let OL ⊥ AB. Join OA and OB.

In right triangles OLA and OLB, we have

	OA =	OB	(Radii of the same circle)
	OL =	OL	(Common line segment)
and,	∠OLA =	∠OLB	(Each equal to 90°)
So,	AL =	LB	[∴ Congruent parts of congruent triangles are equal]

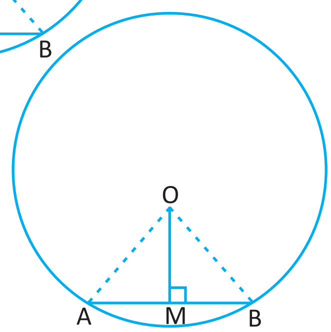
Hence, L bisects AB.



Property 2 : (Converse) The line joining the centre of a circle to the mid point of a chord is perpendicular to the chord.

Proof : Let AB be a chord of a circle with centre O and let M be the mid point of AB. Join OM, AO and BO. Then we have to prove that

OM ⊥ AB.



In Δ OMA and OMB, we have

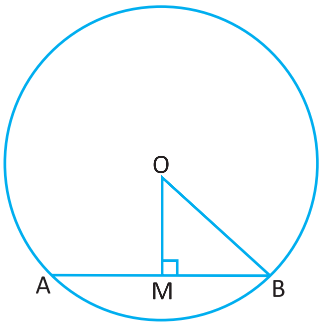
	OA	=	OB	(radii of the same circle)
	OM	=	OM	(Common)
	AM	=	BM	($\therefore$ M is the mid point of AB)
$\therefore$	ΔOMA	$\cong$	ΔOMB	(sss property)
So,	$\angle$OMA	=	$\angle$OMB	(congruent parts of congruent AS)
But,	$\angle$OMA	+	$\angle$OMB = 180°	(linear pair)
$\therefore$	$\angle$OMA	=	$\angle$OMB = 90°	

Hence, OM $\perp$ AB.

Illustrative Examples

Example 1 : In a circle with centre O and radius equal to 13cm AB is a chord which is 24cm long. Find the distance of the chord from the centre.

Solution : $\therefore$ Draw OM $\perp$ AB.



Since the perpendicular from the centre of a circle bisects the chord, we have AM = MB = 12cm

Now in right angles Δ OMB we have

$OM^2 = (OB^2 - MB^2) = [(13)^2 - (12)^2] = 25.$

$\therefore$ $OM = \sqrt{25} = 5$cm.

Hence, the distance of the chord from the centre = 5cm.

Example 2 : In a circle of radius 7cm, find the distance of a chord of length 9cm from the centre.

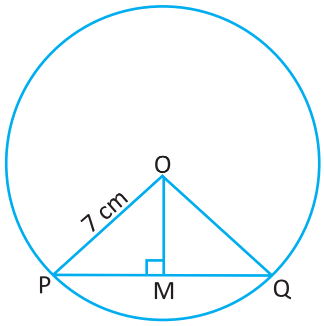
Solution : Let PQ be a chord of a circle with centre O and radius OP = 7cm such that PQ = 9cm

Let OM $\perp$ PQ.

Since perpendicular from the centre of the circle on a chord bisects the chord.

Therefore, M is the mid point of PQ i.e.

$PM = MQ = \frac{9}{2}$ cm



Thus, in right triangle OMP, we have

$OP = 7cm, PM = \frac{9}{2}$ cmand $\angle OMP = 90°$

$\therefore$ By Pythagoras theorem,

$OP^2 = OM^2 + PM^2$

$OM^2 = OP^2 - PM^2$

$OM = \sqrt{OP^2 - OM^2}$

$$OM = \sqrt{7^2 - \left[\frac{9}{2}\right]^2} = \sqrt{49 - \frac{81}{4}} = \sqrt{\frac{196 - 81}{4}} = \sqrt{\frac{115}{4}}$$

$OM = \frac{1}{2}\sqrt{115}$ cm.

Hence, the distance of the chord from the centre is $\frac{\sqrt{115}}{2}$ cm.

Example 3 : Two circles of radii 10 cm and 8cm intersect each other and the length of the common chord is 12cm. Find the distance.

Solution : Let O and O' be the centres of the circles of radii 10cm and 8cm respectively and let PQ be their common chord. We have OP=10cm, O'P= 8cm and PQ = 12cm

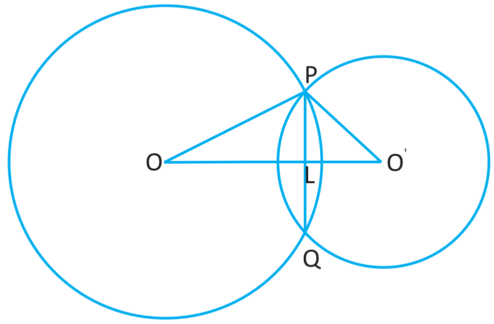
$\therefore$ PL = 1/2 PQ = 6cm

In right triangle OLP, we have

$$OP^2 = OL^2 + LP^2$$

$$OL = \sqrt{OP^2 - LP^2} = \sqrt{10^2 - 6^2}$$

$$= \sqrt{64\text{ cm}} = 8\text{ cm}$$



In right triangle O'LP we have

$$O'P^2 = O'L^2 + LP^2$$

$$O'L^2 = \sqrt{OP^2 - LP^2} = \sqrt{8^2 - 6^2} = \sqrt{28\text{ cm}} = 5.29\text{ cm}$$

$\therefore$ O'O = OL + LO = (8+5.29) cm = 13.29 cm

Exercise 13.1

1. In a circle of radius 13 cm, a chord is drawn at a distance of 12 cm from the centre. Find the length of the chord.

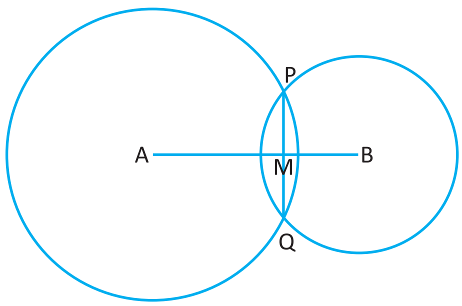
2. A chord of a circle is 20 cm in length and its distance from the centre is 24 cm. Find the radius of the circle.

3. In a circle with a radius 10 cm what will be the length of a chord 6 cm away from the centre of the circle ?

4. In a circle of radius 10 cm, a chord is placed 6 cm from its centre. Another chord with same length is placed 15cm away in another circle. Find the radius of the second circle.

5. A chord of a circle is 16 cm in length and its distance from the centre is 6 cm. Find the length of a chord of the same circle at a distance of 8 cm from the centre.

6. Two circles with centres A and B intersects in P and Q and M is the mid point of PQ. AM,BM are joined. Find $\angle$AMP and $\angle$BMP. Also, show that the points A,M,B are collinear.



[**Hint :** $\angle$AMP and $\angle$BMP form a linear pair.

7. Give a method to find the centre of the given circle.

8. PA and PB are two chords of a circle with centre O. If diametre POQ bisects $\angle$APB< prove that PA= PB.

9. AB is a diameter of a circle and CD is a chord of the circle which intersects AB at x at right angles. Given that AX = 2 cm and XB =18 cm. Find CD.

Equal chords of a circle and angles subtended to the centre :

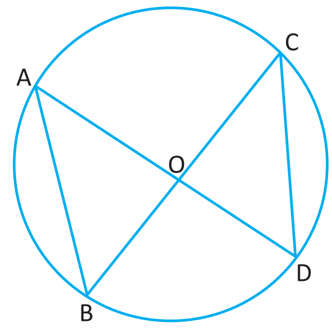
Property 1 : In a circle, equal chords subtend equal angles at the centre.

We can verify the above result by an experiment given below.

Example 1 : Draw a circle with centre and any two equal chords of a circle and any radius *r*.

Then we have to prove that

$\angle AOB = \angle COD.$



In Δ AOB and COD, we have

AB = CD (Given)

OA = OC (Each equal to radius r)

OB = OD (Each equal to radius r)

So, by sss condition of congruency we have

$\Delta AOB \cong \Delta COD$

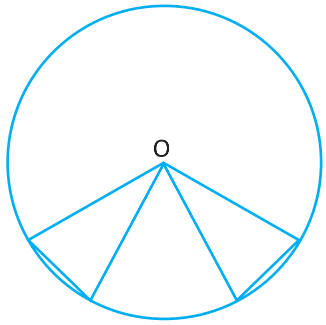
$\angle AOB = \angle COD$ (Corresponding parts of congruent triangles are equal)

Repeat the experiment with two more different radii of the circle.

In each case, you will find that equal chords subtend equal angles at the centre.

Property 2 : In a circle, chords which subtend equal angles at the centre are equal. We can verify the above result by an experiment given below.

Example 2 : Let AB and CD be two chords of a circle with centre O such that they subtend equal angles at the centre i.e. $\angle AOB = \angle COD$. Then, we have to prove that AB=CD.



OA = OC (Each equal to radius)

$\angle AOB = \angle COD$ (given)

OB = OD (Each equal to radius)

So, by SAS criterion of congruence, we have

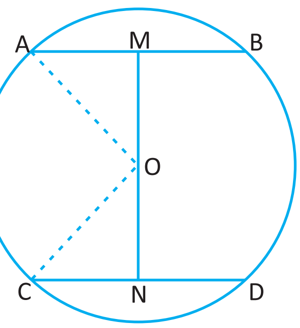
$\Delta AOB \cong \Delta COD$

AB = CD

Repeat the experiment with two more chords of diffent radii. In each case, you will find the chords which subtend equal angles at the centre are equal.

Property 3 (Converse) : Chords of a circle equidistant from the centre are equal.

Example 3 : Let us draw a circle with centre O and any radius.



Let AB and CD be any two chords such that OM = ON, join OA and OC.

In right angled triangle OMA and ONC, we have

OA = OC (radii of the same circle)

OM = ON (given)

$\therefore$ $\Delta OMA \cong \Delta ONC$ (RHS property)

So, AM = CN.

But, we know that the perpendicular from the centre of a circle bisects the chord.

$\therefore$ AN = ½ AB and CN = ½ CD.

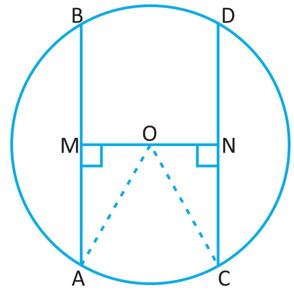
So, AM = $\frac{1}{2}$ CNAB = ½ CD

AB = CD

Hence, the chord AB and CD are equal.

Property 4 : Equal chords of a circle are equidistant from the centre.

Proof : Let us draw AB and CD be two equal chords of a circle with centre O. join OA and OC. Draw OM $\perp$ AB and ON $\perp$ CD, we have to prove that OM=ON.



AM = ½ AB and CN = ½ CD.

But, AB = $\frac{1}{2}$ CDAB = ½ CD

AM = CN

Now, in right angled triangles OMA and ONC, we have

OA = OC (radii of the same circle)

AM = CN (proved above)

$\therefore$ ΔOMA $\cong$ ΔONC (RHS property)

So, OM = ON (congruent parts of congruent triangles)

Angles in the same segment of a Circle (or angles in the same arc of a circle) are equal.

Illustrative Examples

Example 1 : Find the regular number of sides of a regular polygon inscribed in a circle if each side of it subtends an angle of 60° at the centre.

Solution : Let the number of sides be n

Then, $n \times 60° = 360$, $n = \frac{360}{60} = 6$

$\therefore$ The number of sides of the polygon = 6.

Example 2 : A regular hexagon of side 5 cm is inscribed in a circle. What is the radius of the circle ?

Solution : A regular hexagon has 6 equal sides.

These 6 sides become the six equal chords of the circle.

$\therefore$ Each side of the hexagon

Subtends an angle = $\left[\frac{360}{60}\right]^{\circ}$ = 60°.

$\therefore$ $\angle$ AOB = 60°.

In AOB, we have

OA = OB

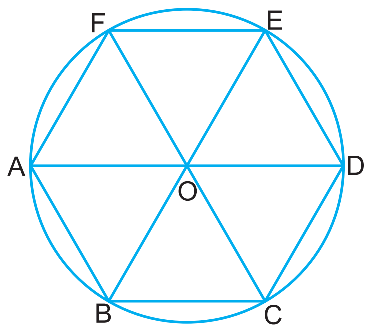
$\angle$OAB = $\angle$OBA

But $\angle$AOB + $\angle$OAB+$\angle$OBA = 180°

60° + 2$\angle$OAB = 180°

2$\angle$OAB = 180° – 60°

OAB = $\frac{120°}{2}$ = 60°



Thus, in ΔOAB, we have

$\angle OAB = \angle OBA = \angle AOB = 60°$

So, the triangle is equilateral

Hence, OA = OB = AB   OA = 8cm

Example 3 : What is the size of the angle subtended by the side of a regular triangle inscribed in a circle at the centre ?

Solution : We know that the angle subtended by the side of a regular *n* sided triangle inscribed in a circle at its centre is given by—

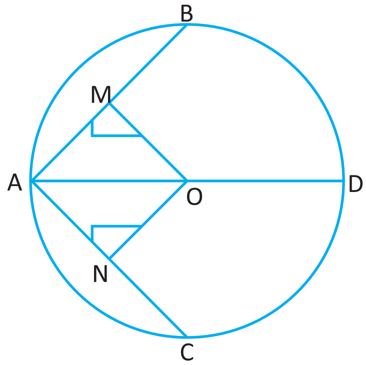
Here $n = 3$ $\left[\frac{360}{n}\right]^{\circ} = \left[\frac{360}{3}\right]^{\circ} = 120°$

Example 4 : If two chords are equally inclined to the diameter through their point of intersection, prove that the chords are equal.

Solution : Let AB and AC be two chords of a circle with centre O. Such that AB and AC are equally inclined to the diametre $\angle AOD$ i.e. $\angle DAB = \angle OAC$. We have to prove that AB= AC

Draw $OM \perp AB$ and $ON \perp AC$.

Now in right angled Δ OMA and ONA, we have



	OA	=	OA	(common)
	$\angle MAO$	=	$\angle NAO$	(given)
	$\angle AOM$	=	$\angle AON$	$[\because 90° - \angle BAO = 90° - \angle CAO]$
$\therefore$	ΔOMA	$\cong$	ΔONA	
So,	OM	=	ON	

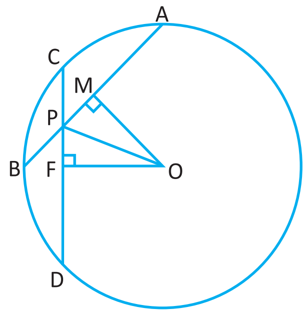
This shows that the chords AB and AC are equidistant from the centre.

But, the chords equidistant from the centre are equal.

$\therefore$ AB = AC

Example 5 : In the given figure, two chords AB and CD of a circle with centre O intersects at a point P inside the circle; $OE \perp AB$, $OF \perp CD$ and OP is joined. If $\angle OPE = \angle OPF$, prove that (i)$\Delta OMP \cong \Delta ONP$ (ii) OE = OF (iii) AB = CD

Solution : In Δ OEP and OFP we have



	$\angle OPE$	=	$\angle OPF$	(given)
	$\angle POE$	=	$\angle POF$	$[\because 90° - \angle OPE = 90° - \angle OPF]$
	OP	=	OP	(common)
$\therefore$	ΔOEP	$\cong$	ΔOEP	
So,	OE	=	OF	(congruent parts of congruent triangles)

This shows that the chords AB and CD are equivalent from the centre.

But the cords equidistant from the centre are equal.

$\therefore$ AB = CD

Exercise 13.2

1. A regular octagon is inscribed in a circle. What angle does each side of the octagon subtend at the centre ?
2. A square ABCD is inscribed in a circle with centre. What angle does each side of the square subtend at the centre O?
3. A regular polygon is inscribed in a circle. If a side subtends an angle of 72° at the centre, what is the number of sides of the polygon?
4. An equilateral triangle ABC in inscribed in a circle with centre O. Find $\angle$BOC.
5. In fig (a) AX and CX are equal and are secants to the circle with centre O. AX cuts the circle at A and B and CX cuts the Circle at C and D. Prove that AB = CD.
6. A ΔABC is inscribed in a circle with centre O and the centre is equidistant from the sides as shown in fig (b). Prove that ABC is an equilateral triangle. [Hint : chords AB and BC are equidistant from O. So, they are equal.]

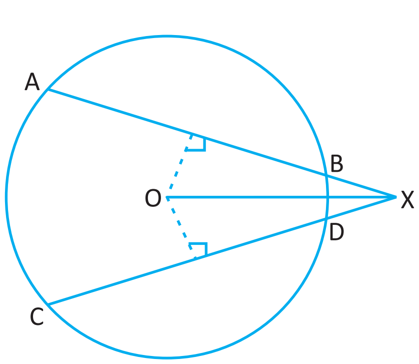


fig. (a)

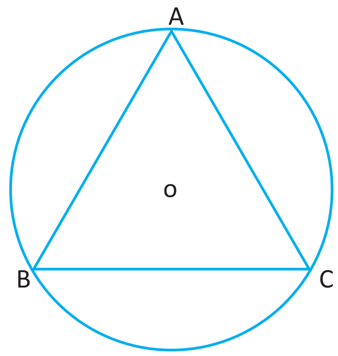


fig. (b)

7. Chords AB and CD of a circle with centre O intersect at a point inside the circle. OM $\perp$ AB and ON $\perp$ CD meet AB and CD in M and N respectively as shown in fig. (c). If $\angle$OSN = $\angle$OSM, then prove that :
   (i) ΔOSM $\cong$ ΔOSN (ii) OM = ON (iii) AB = CD
8. In fig. (d) AOB is a diametre of a circle with centre O. Chords XY and WZ are parallel and equal. Show that AC = DB.
9. If the radius of a circle is 12cm. Find the side of a regular hexagon inscribed in it.

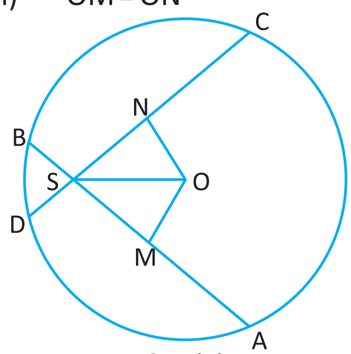


fig. (c)

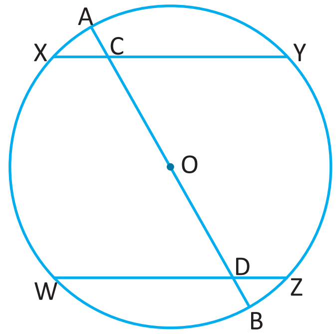


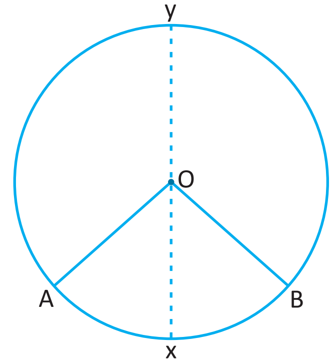
fig. (d)

Points to Remember :

- A circle is a set of those points in a plane that are at a given constant distant from a given fixed point in the plane. The fixed point is the centre and the given constant distance is the radius.
- In a circle, the line joining the centre to the mid point of a chord is perpendicular to that chord.
- Chords of a circle which are equidistant from the centre are equal.
- Each side of a regular polygon inscribed in a circle subtends the same angle at the centre.
- The sum of all the angles subtended by a regular polygon inscribed in a circle at its centre is 360°.
- An arc of a circle is called a major arc if it is greater than the semi circle.
- An arc of a circle is called minor arc if its length is less than the length of the semi circle.

Some Angle Properties of The Circle

Degree measure of an arc of a circle : In a circle with centre O, the degree of a minor arc AB is the measure of the centre angle AOB and intended by the symbol m (AB).



Thus in fig m (AB) = ∠AOB.

m (major arc AYB) = 360°–∠m (minor AB)

= Ritles ∠AOB

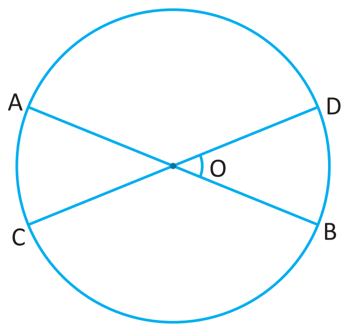
m (minor arc A x B) = ∠AOB

so, m (minor AB) + m (major AB)= 360°.

In circle with centre O, the degree measure of a semi circle is 180° as shown in figure.

Thus, m (semi circle) = 180°

Example 1 : Diameter AB and CD of a circle intersect at O are shown.



Solution : If m (BD) = 60°. Find m (AO), m (AC) and m (BC).

We have,

m(BD) = 60°

∠BOD = 60°

Since ∠AOD and ∠BOD form a linear pair.

∴ ∠AOD + ∠BOD = 180°

∠AOD = 180° – ∠BOD

∠AOD = 180°–60° = 120°

∴ m(AD) = ∠AOD = 120°

Now, m (AC) = ∠AOC

= ∠BOD = 60°

We have, m (BC) = ∠BOC

= ∠AOD

= m(AD)= 120°

Example 2 : In a circle the measure of major arc is three times the measure of minor arc, find the measure of each arc.

Solution : Let the measure of minor arc = $x°$ Then, measure of major arc = (360° –$x°$) It is given that :

Measure of Major arc = 3 (measure of minor arc)

$$(360° – x°) = 3x°$$

$$360 – x = 3x$$

$$360 = 3x + x$$

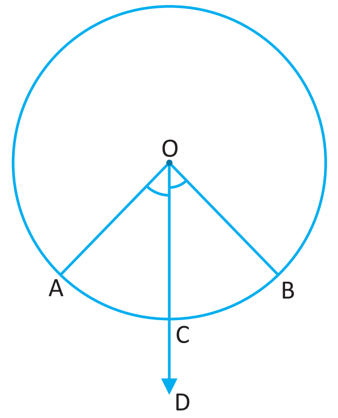
$$4x = 360$$

$$x = \frac{360}{4} = 90°$$

Hence, measure of major arc = (360°–90°) = 270°

measure of minor arc = $x°$ = 90°

Example 3 : In the adjoining figure, AB is a minor arc of a circle with centre O such that m (AB) = 80°. Bisect AB.



Solution : Since m(AB) = 80°, we have AOB = 80°.

Now we have to find a point COB

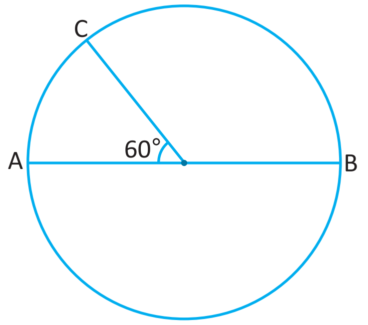
AB such that m (AC) = m (CB) = 40°

Draw a ray OD such that AOD = 40°

Let ray OD meet arc AB at C.

Then clearly C bisects AB.

Example 4 : In the given figure, AB is a diameter and OC is a radius of a circle with centre O. If ∠AOC = 60°, find :



(i) m(ac) (ii) m(BC) (iii) m(ABC)

Solution : (i) m(AC) = angle subtended by AC at O = ∠AOC = 60°.

(ii) m(BC) = angle subtended by BC at O = ∠BOC= 180°–60° = 120°.

(iii) m(ABC) = 360° –m(AC) = 360° – 60° = 300°

Exercise 13.3

1. In the given fig.-A, AB and CD are two diametres of a circle with centre O. If m (AD) = 140°, final m (AC) m(BD) and m (BC).

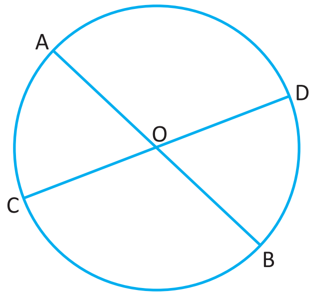


fig.-A

2. **Draw a circle with centre O and radius 2 cm. Mark points A,B and C (in the order) on the circle, such that**
   (a) m (AB) = 60° and
   (b) m (BC) = 120° what say about m (ABC) ?

3. **In fig.-C OA, OB and OC are radii of a circle such that OA ⊥ OB and ∠BOC = 110°. What is the measure of**
   (a) Minor arc AB ?
   (b) Major arc AB ?
   (c) Minor arc AC ?
   (d) Major arc AC ?
   (e) Major arc BC ?
   (f) Minor arc BC ?

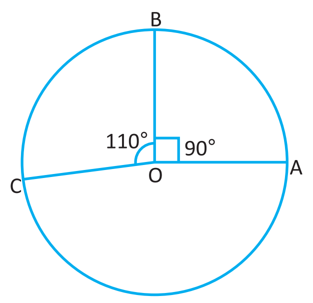


fig.-C

4. **In the fig.-D, AB is a diameter and OC is a radius of a circle with centre O. If $\angle BOC = 50°$, find the measure of—**

(a) Minor arc BC

(b) Major arc BC

(c) Minor arc AC

(d) Major arc AC

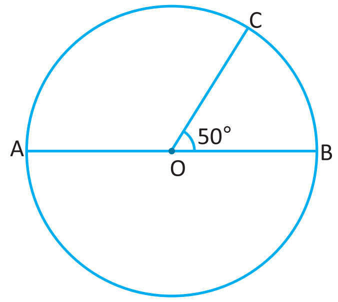


fig.-D

5. Draw a circle with centre O and radius 4.2 *cm*. Mark points A,B,C (In this order) on the circle such that m (AB) = 75° and *m* (BC) = 95°. Find *m* (ABC) and m (ACB).

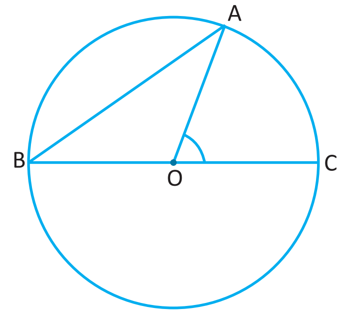


fig.-E

6. Two points on a circle determine a minor arc and a major arc to be in the ratio 4 : 5, Find the measure of each arc.

7. **In fig.-E BC is a diameter. Find the value at $\angle AOC$, $\angle A + \angle B$, $\angle A$ and $\angle B$ if**

(a) $m\,(AC) = 70°$ (b) $m\,(AC) = 2x°$

8. In fig.-F AB and CD are two chords of a circle with centre *O*. If *m*(minor AB) = *m* (minor CD) prove that chord AB = chord CD.

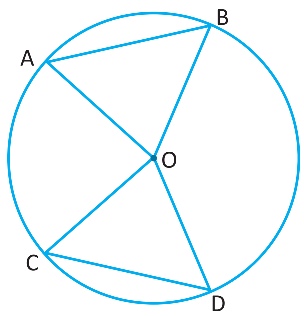


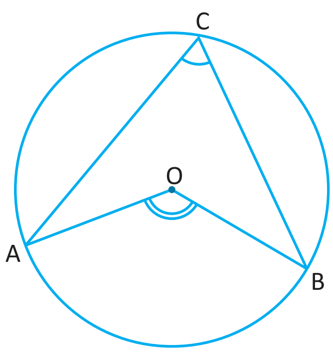
fig.-F

Relation between an inscribed angle and measure of its intercepted arc.

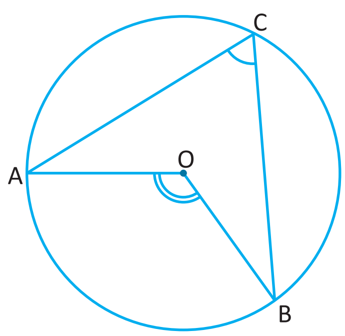
We shall learn about an important relation between an inscribed angle in a circle and the measure of it intercepted arc. We state the said relation as a result as under.

Result 1 : In a circle, the measure of an inscribed angle is half the measure of its intercepted arc.

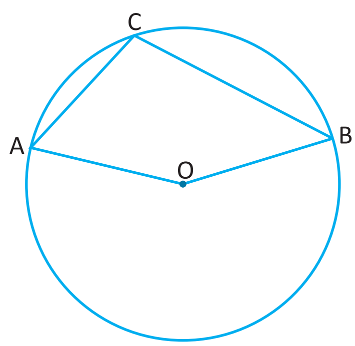
Example 1 : Draw three circles as shown in fig. and label them as (i) (ii) and (iii). Label the centre of each circle as O.



(i)



(ii)



(iii)

Solution : In the circle (i), take an arc AXB and a point C an the remaining part of the circle. Join OA, OB, AC and BC.

Mesure $\angle$ACB and $\angle$AOB.

Repeat the above procedure with circle (ii) and (iii).

Circle No.	$\angle$ACB	$\angle$AOB	M (A × B)	m(A × B) – 2 $\angle$ACB or m$\angle$AOB – 2$\angle$ACB

In each case, you will find that that $\angle$AOB – 2 $\angle$ACB is zero (or so small that it may be ignored). Thus, in all cases, we have—

$\angle AOB - 2\angle ACB = 0$

$\angle AOB = 2\angle ACD$

Result 2 : Angle in a semi-circle is a right angle.

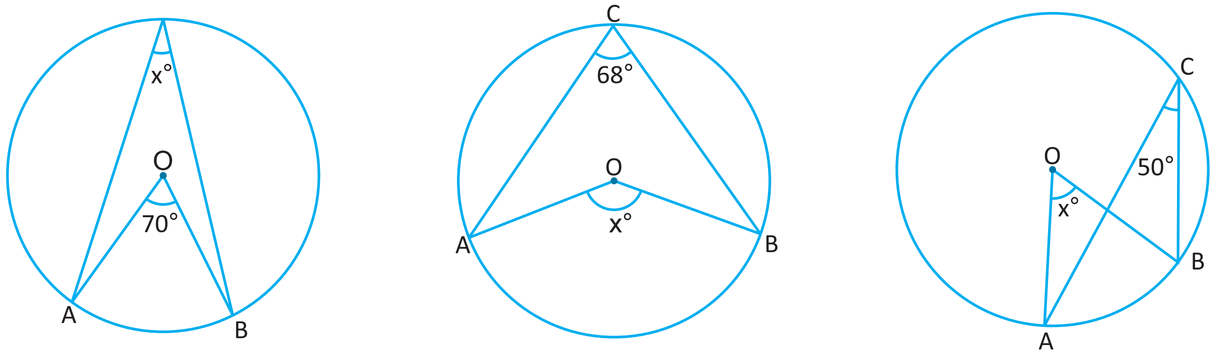
Proof : Let AB be a diametre of a circle with centre O.

Let C be a point on the semi-circle, since the angle formed by are AB at the centre is double the angle formed at C.

$\therefore\ 2\angle ACB = \angle AOB = 180°, \angle ACB = \frac{180°}{2}$

or ACB = 90° = 1 right angle.

Example 1 : In each of the following figures point O is the centre of the circle. Find the value of *x*.

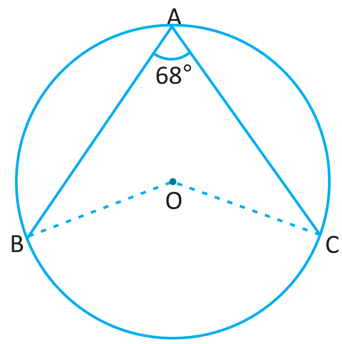


Solution : Using the properties we know that in a circle the angle subtended by it at any point on the remaining part of the circle, we have

(i) $\angle AOB = 2\angle ACB$ or $2x = 70°$ or $x = 35°$.

(ii) $\angle AOB = 2\angle ACB$ or $x = 2 \times 68°$ or $x = 136°$.

(iii) $\angle AOB = 2\angle ACB$ or $x = 2 \times 50° = 100°$.

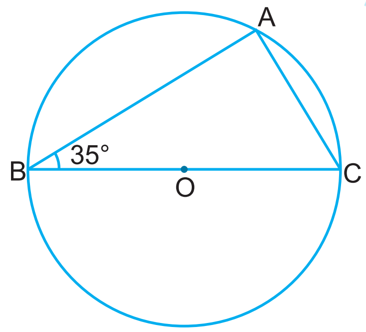


Example 2 : In the adjoining figure, ADC is an equilateral triangle inscribe in a circle with centre O. Find $\angle BOC$.

Solution : Since ΔABC is an equilateral triangle each of its angles measure 60°.

$\therefore \quad \angle BAC = 60^\circ$

Now $\quad \angle BOC = 2\angle BAC = 2 \times 60^\circ = 120^\circ$



Example 3 : In the given figure Δ ABC is inscribed in a circle with centre O and BC is a diameter, If $\angle ABC = 35^\circ$, Find $\angle ACB$.

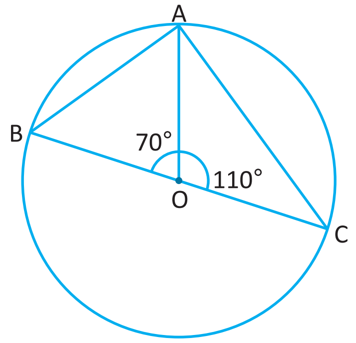
Solution : We know that the angle in a semicircle is right angle.

$\therefore \quad \angle BAC = 90^\circ$

Also, $\quad \angle ABC = 35^\circ$ (given)

$\therefore \quad \angle ACB = 180^\circ - (90^\circ + 35^\circ)$

$= 55^\circ$



Example 4 : In figure A, B, C are three point on a circle such that the angles subtended by the chords AB and AC at the centre O are 70° and 110° respectively. Determine $\angle BAC$ and the degree measure of arc $\angle BPC$.

Solution : We have, since arc BPC makes $\angle BOC$ at the centre and $\angle BAC$ at a point on the remaining part of the circle.

$\angle BAC = \angle ½\, BOC$

Now $\angle BOC = 360^\circ - (110^\circ - 70^\circ) = 180^\circ$.

m (BPC) = 180° and $\angle BAC = ½\,(\angle BOC)$

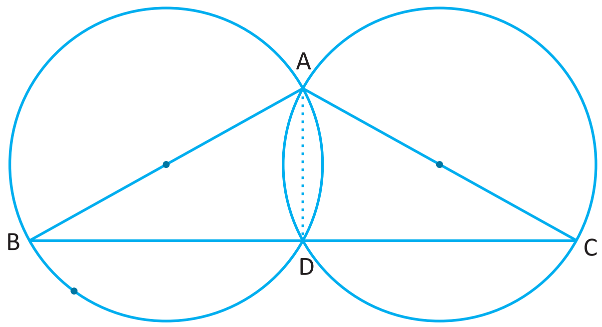
$= ½ \times 180^\circ \times 90^\circ$

Example 5 : In two circles arc down with sides AB, AV of a triangle ABC as diametres. Two circles intersect at a point D. Prove that D lies on BC.

Solution : Join AD since angle in a semicircle is a right angle.

Therefore ,

$\angle ADB = 90^\circ$ and $\angle ADC = 90^\circ$



$\angle ADB + \angle ADC = 90^\circ + 90^\circ$

$\angle ADB + \angle ADC = 180^\circ$

BDC is a straight line, D lies on BC.

Example 6 : In figure chords CA and DB are both perpendicular to AB. Show that CA = DB.

Solution : Join BC and AD.

Now, in Δ ABC, we have

$\angle A = 90^\circ$

BC is a diameter of the circle. Thus in ABD,

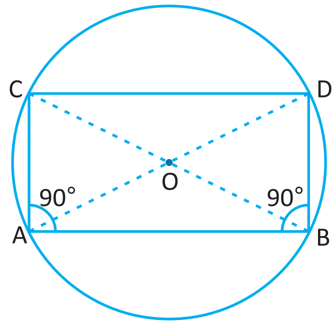
We have ∠90°

∠AD is a diameter of the circle.

Thus AD and BC intersect at O, the centre of the circle.

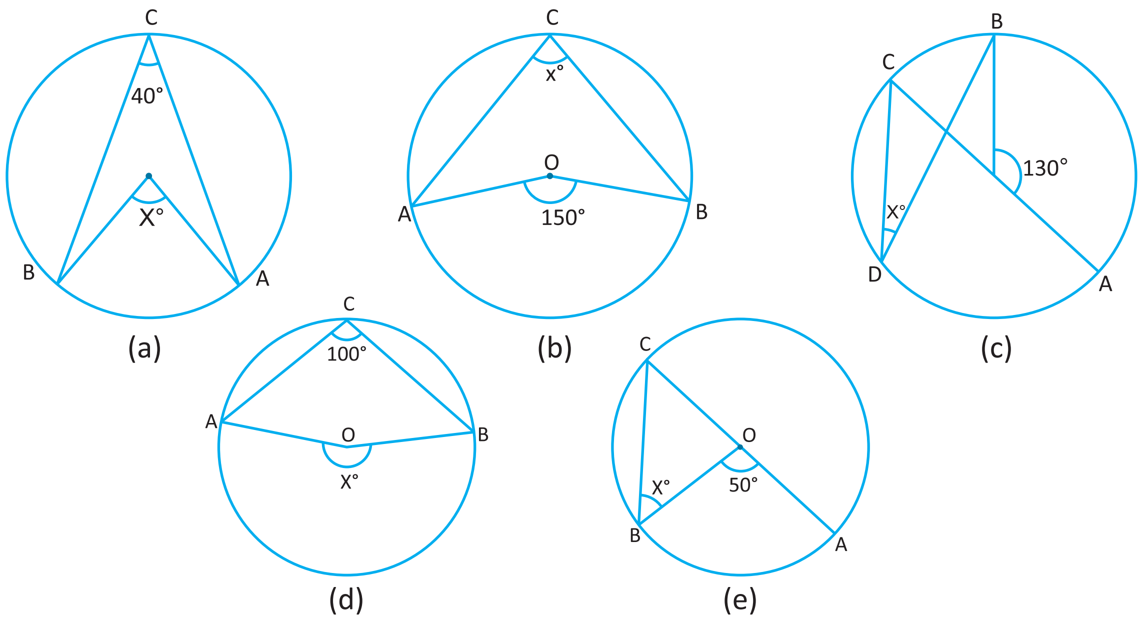
∠AOC = ∠BOD

Chord AC = Chord BD.

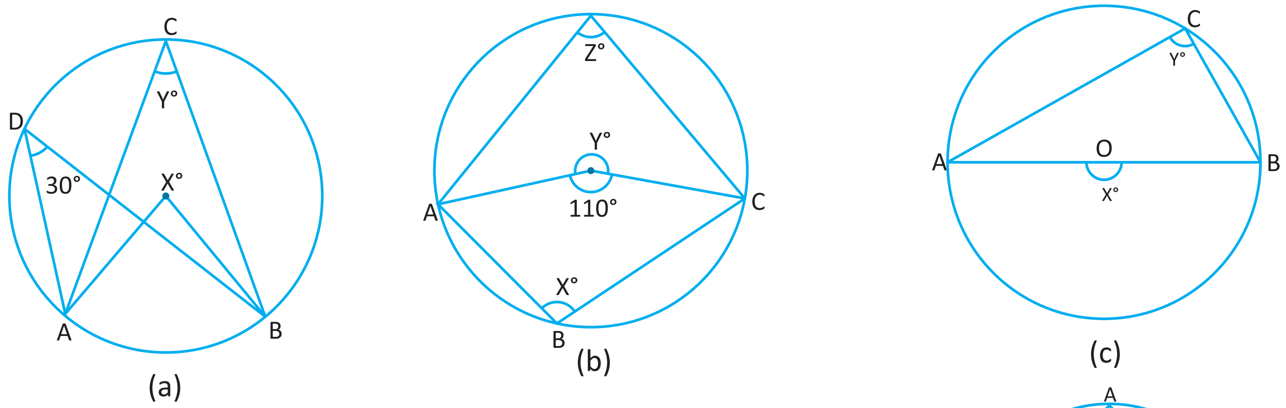


Exercise 13.4

1. In each of the following figures, point O is the circumcentre of the circle. Find the value of *x*.



2. Find the value of *x*, *y*, *z* in figure (i), (ii), (iii).



3. In the adjoining fig. (a), Δ ABC is inscribed in a circle with centre O and BC is a diameter. If ∠BCA = 48°, find ∠ABC.

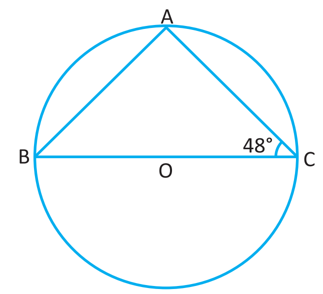


fig. (a)

4. In the fig. (b), two circles intersect at P and Q. AP and PB are the diametres. Show that AQB is a straight line.

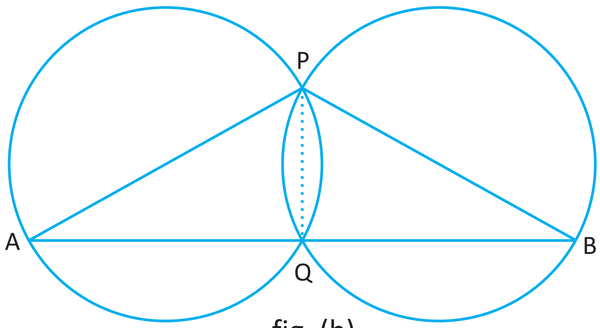


fig. (b)

5. In the fig. (c), O is the centre of the circle. If chord BC = radius OB = 4*cm* and A is any point on the circle, find $\angle$BAC.

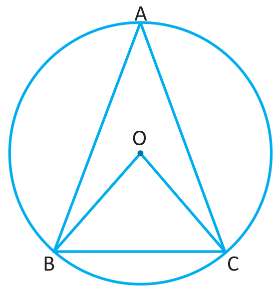


fig. (c)

6. In fig. (d) ACB is semicircle and CD AB. Show that CD^2 = AD × BD.

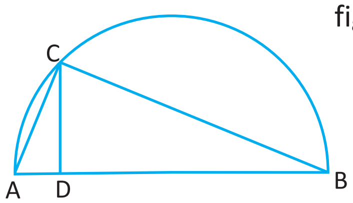


fig. (d)

7. The measures of the arcs of two sides of triangles are 130° and 80°. Find the angles of the triangle.

8. Two angles of a triangle inscribed in a circle are 60° and 70°. Find the measure of the arcs of the three sides.

9. The sides of an inscribed triangle have arcs measuring $(2x - 24)°$, and $(2x + 12)°$ and $(3x + 36)°$. What kind of a triangle is it?

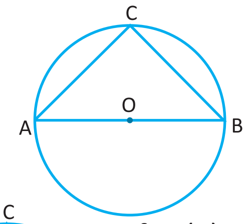


fig. (e)

10. In fig. (e), if AOB is a diameter of the circle and AC = BC, then what is the measure of $\angle$CAB?

11. A circle is divided into three parts in the ratio 3:4:5. Find the angles of the triangle formed by joining the points of division.

12. In fig. (f), $\angle$OAB = 40°. What is the measure of $\angle$ACB ?

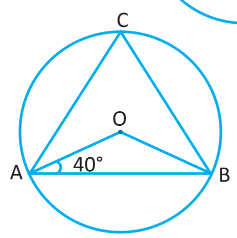


fig. (f)

Points to Remember :

- The measure of an arc AB of a circle with centre O is AOB.
- An arc a circle is intercepted by an angle if–
  (i) The end point of the arc is on the arms of the angle.
  (ii) Each arm of the angle contains at least one end - point, and
  (iii) Except for the end-points, the arc lies in the interior of an angle.
- An angel with its vertex at the centre of a circle is called a central angle of the circle.
- In a circle, if two central angles are equal, then intercepted arcs are congruent.
- The angle in a semicircle is 1 right angle.
- A quadrilateral is called a cyclic quadrilateral if all vertices lie on circle.
- In a cyclic quadrilateral, the sum of each pair of opposite angles is 180°.

EXERCISE

1. MULTIPLE CHOICE QUESTIONS (MCQs) :

Tick (✓) the correct options.

(a) Which one is the circumference of the circle?

(i) 2π (ii) $2r$ (iii) $2\pi r$ (iv) πr

(b) Which divides circle into two equal parts?

(i) Radius (ii) Chord (iii) Arc (iv) Diameter

(c) A line segment passing through the centre is ....

(i) Chord (ii) Diameter (iii) Radius (iv) Arc

(d) If OP = 13 cm and PM = 12 cm in the fig. (a) then OM = ?

(i) 5 cm (ii) 25 cm

(iii) 10 cm (iv) 7 cm

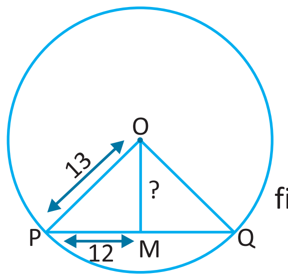


fig. (a)

(e) An arc of a circle is greater than the semicircle called ....

(i) Minor arc (ii) Major arc (iii) Major chord (iv) Minor chord

(f) In the fig. (b) $\angle BOC = 60°$, $\angle BAC = ?$

(i) 120° (ii) 30°

(iii) 90° (iv) 45°

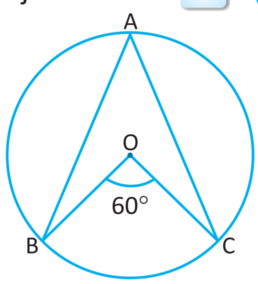


fig. (b)

2. In a circle with a radius 10 cm what will be the length of a chord 6 cm away from the centre of the circle ?

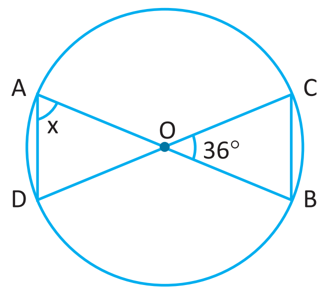
3. In a circle of radius 10 cm, a chord is placed 6 cm from its centre. Another chord with same length is placed 15cm away in another circle. Find the radius of the second circle.

4. If the radius of a circle is 12cm. Find the side of a regular hexagon inscribed in it.

5. The measures of the arcs of two sides of triangles are 130° and 80°. Find the angles of the triangle.

HOTS

1. In the figure O is the centre of the circle. If AB and CD are two diameters, $\angle BOC = 36°$ then find the measure of $\angle x$.



Lab Activity

Objective : To verify that angle in a semicircle is a right angle.

Materials Required : Chart paper, pencil, compass, ruler, a pair of scissors, fevistick/glue.

Procedure :

Step 1. Draw a circle of convenient radius with O as centre on the chart paper (Fig. 1).

Step 2. Draw a diameter and name it as AB. AB divides the circle into two semicircles (Fig. 1).

Step 3. Mark any point P on one of the semicircles and join P to A and B as shown in Fig 1.

Step 4. Cut out triangular portion APB as shown in Fig 2.

Step 5. Cut out another triangular portion A'P'B' congruent to APB obtained in Step 4 above as shown in Fig 3. Shade congruent angles P and P' as shown.

Step 6. Draw a line XY on the chart paper and mark a point M on it.

Paste $\angle$P and $\angle$P' side by side in such a way that both vertices P and P' fall on M and the sides PB and P'B' coincide with each other as shown in Fig. 4.

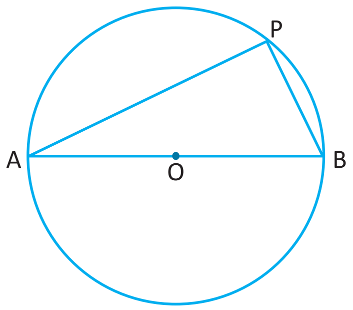


Fig. 1

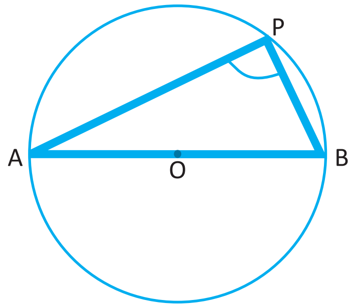


Fig. 2

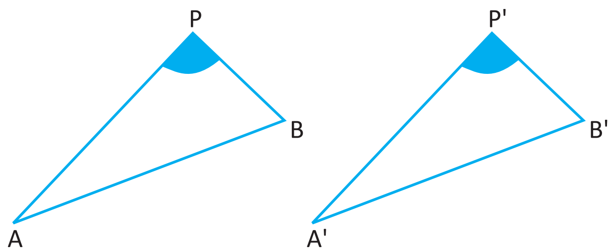


Fig. 3

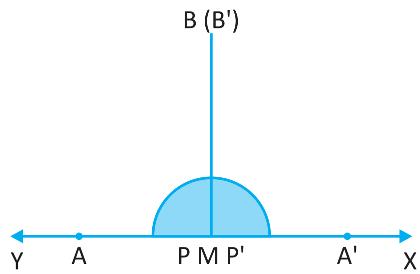


Fig. 4

You will notice that side PA and P'A' of $\angle$P and $\angle$P' fall on the line XY.

It shows that $\angle APB + \angle A'P'B' = 180^\circ$

But $\quad \angle APB = \angle A'P'B' \quad (\Delta APB \cong \Delta A'P'B')$

$\therefore \quad 2\angle APB = 180^\circ$

$\Rightarrow \quad \angle APB = 90^\circ$

Hence, angle in a semicircle is a right angle.

14 Visualising Solid Shapes

We are familiar with the different types of shapes. In order to study these shapes, we can divide them into two categories — plane shapes and solid shapes.

Plane shapes have two measurements like length and breadth and therefore, they are called two-dimensional shapes or 2D shapes. Whereas a solid object has three measurements like length, breadth, height or depth. Hence, they are called three-dimensional shapes or 3D shapes. Also, a solid object occupies some space.

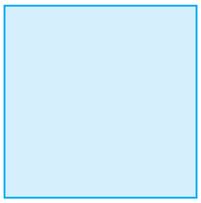
What is the relationship between the number of edges and vertices of any pyramid.

Number of Faces   Number of vertices   Number of edges

(Well know as Euler's formula)

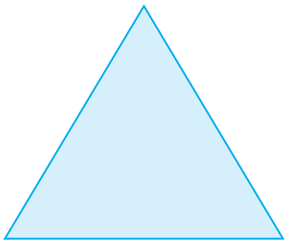
LET US HAVE A LOOK ON THE FOLLOWING 2D AND 3D SHAPES.

2D SHAPES



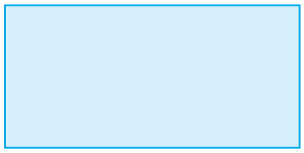
Square

....



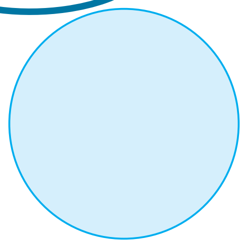
Triangle

....



Rectangle

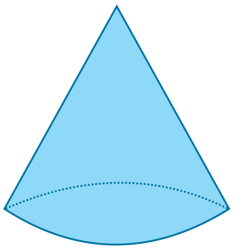
....



Circle

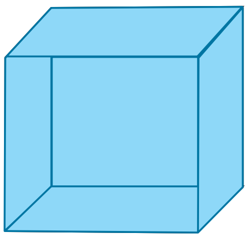
....

3D SHAPES



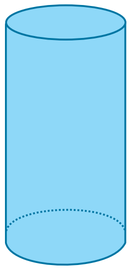
Cone

....



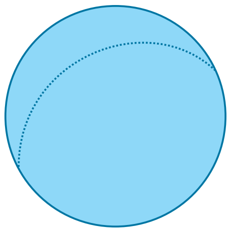
Cube

....



Cylinder

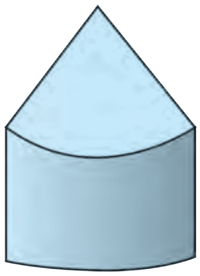
....



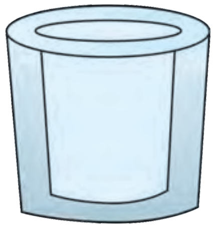
Sphere

....

Many a times, we come across combinations of different shapes. For example—

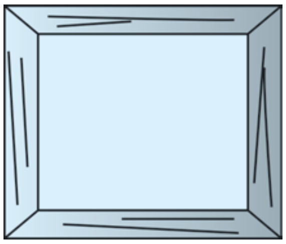


A tent
A cone surmounted
on a cylinder

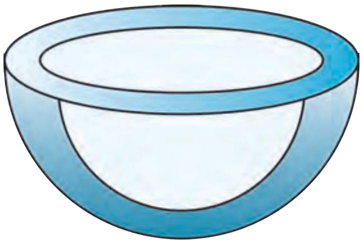


A tin
A cylindrical shell

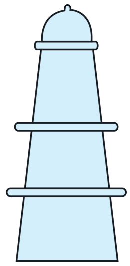
Softy (ice-cream)
A cone surmounted by a
hemisphere



A photoframe
A rectangular path



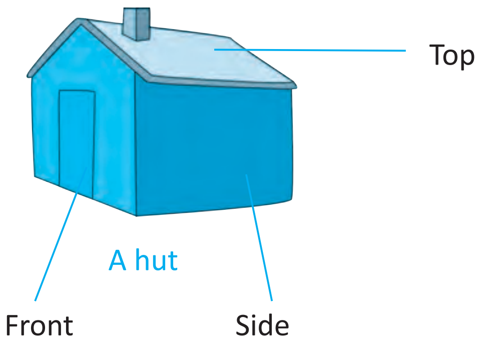
A bowl
A hemispherical shell



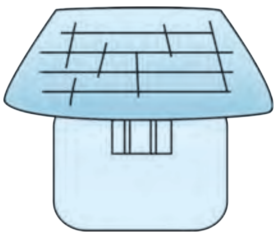
Tomb on a pillar
Cylinder surmounted
by a hemisphere

Views of 3d-Shapes

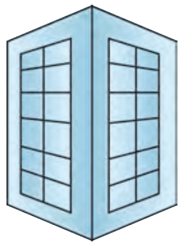
You know that a 3-dimensional object looks differently from different positions so we can draw it from different perspectives. For example, a given hut can have the following views.



Front view



Side view

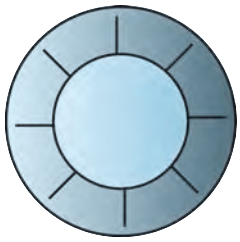


Top view

Similarly, a glass can have the following views

A glass

Side view

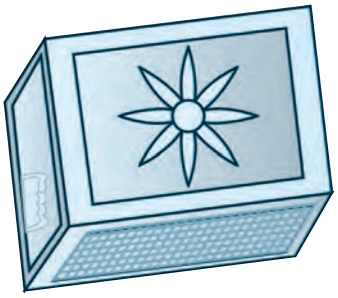
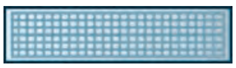


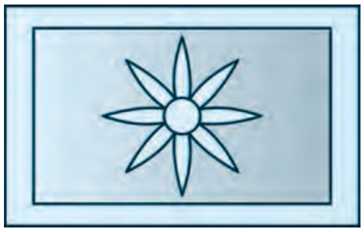
Top view

Exercise 14.1

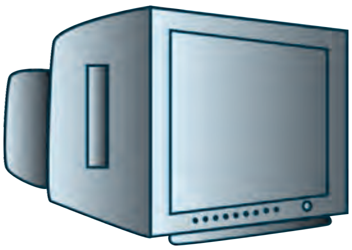
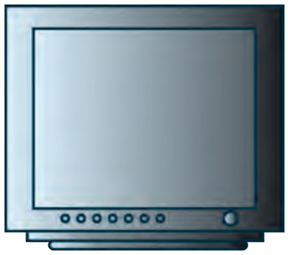
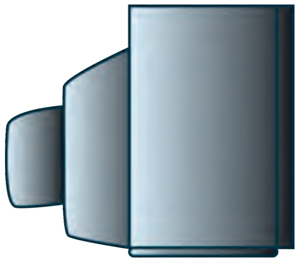
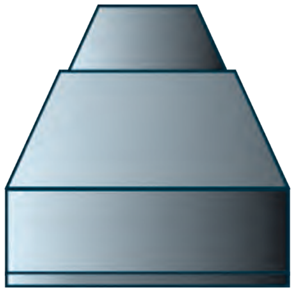
1. The three views are given for each of the given solids. Identify for each solid the corresponding top, front and side views.

(a) A Match Box (i) (ii) (iii)

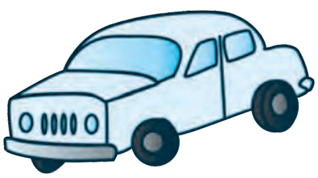



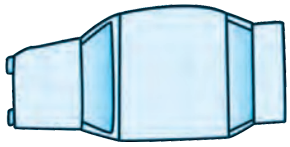


(b) A Television

(c) A Car



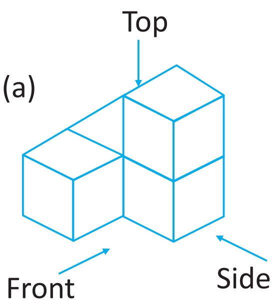



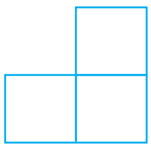
(d) An almirah



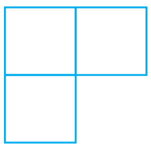

2. Identify the top view, front view and side view for each of the given solids.

(a)

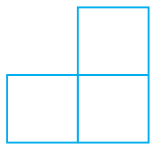




(i)

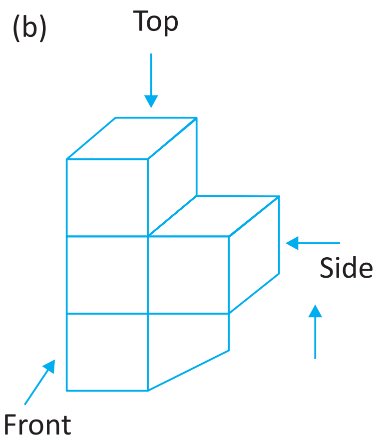


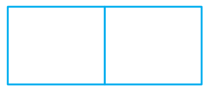
(ii)



(iii)

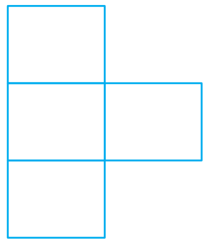
(b)





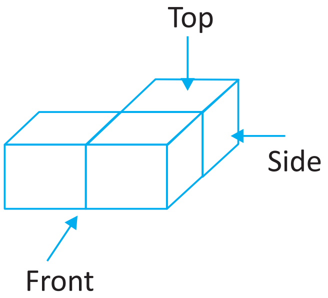
(i)

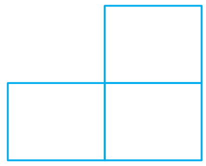
(ii)



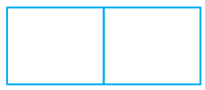
(iii)

(c)

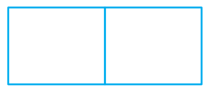




(i)

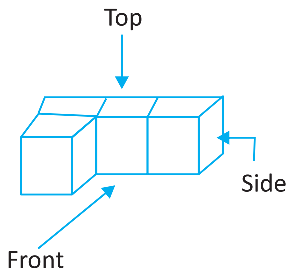


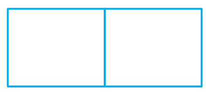
(ii)



(iii)

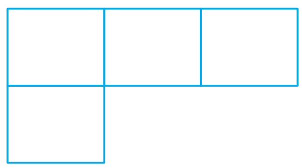
(d)





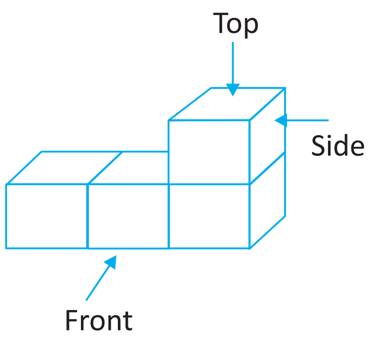
(i)

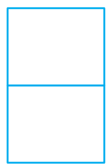
(ii)



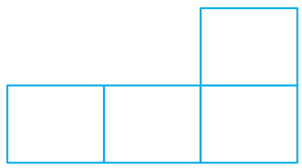
(iii)

(e)

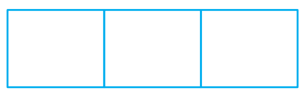




(i)



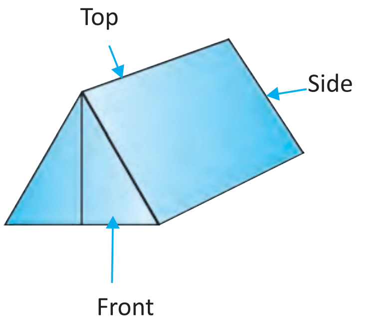
(ii)



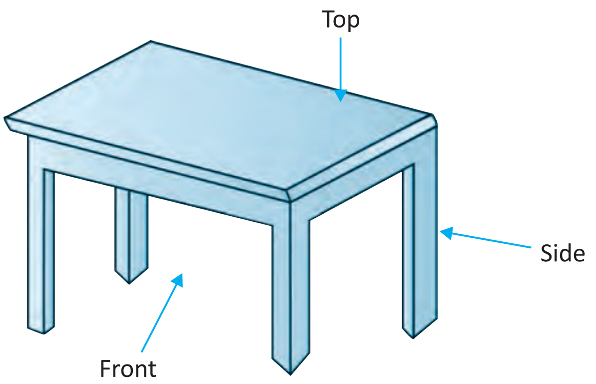
(iii)

3. Draw the front view, side view and top view of the given objects.

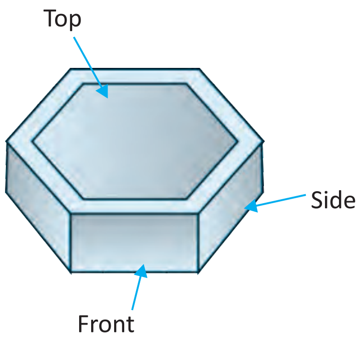
(a) A military tent



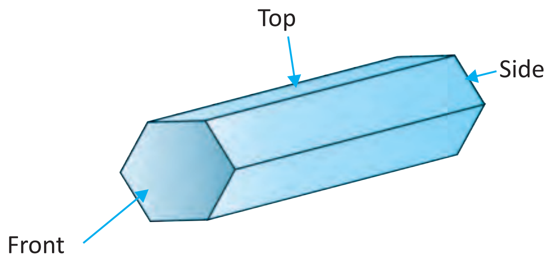
(b) A table



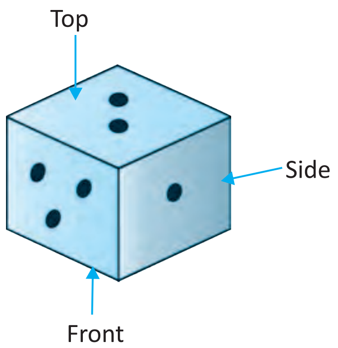
(c) A hut



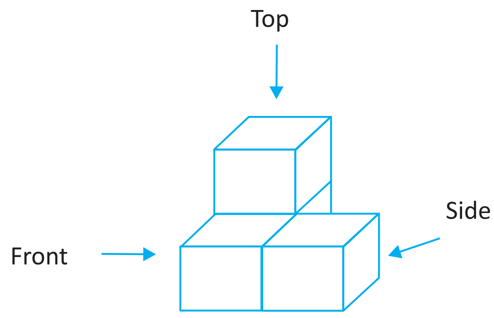
(d) A hexagonal block



(e) A dice



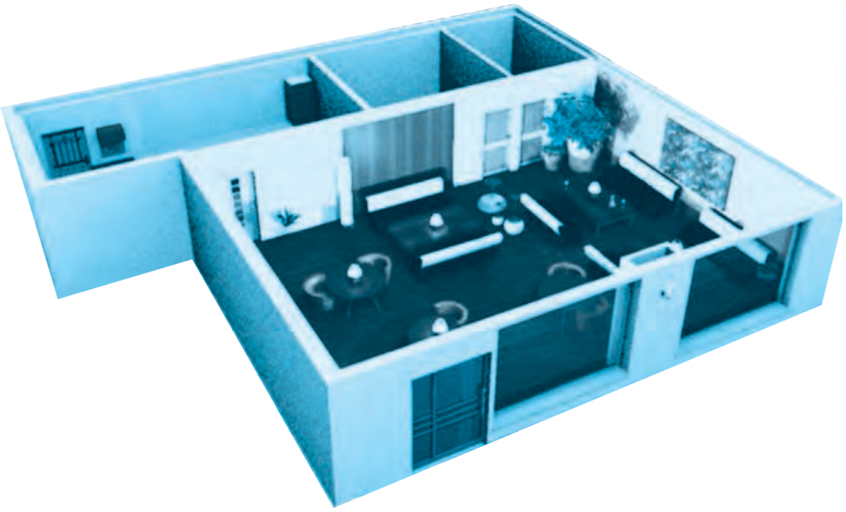
(f) A solid

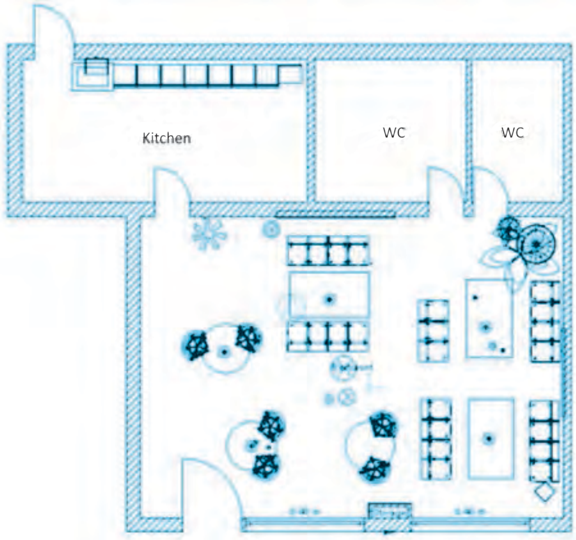


Mapping Space Arounds Us

A map is a drawing of the earth or a part of it on the paper. You must have gone through various maps while reading the subjects of history and geography.

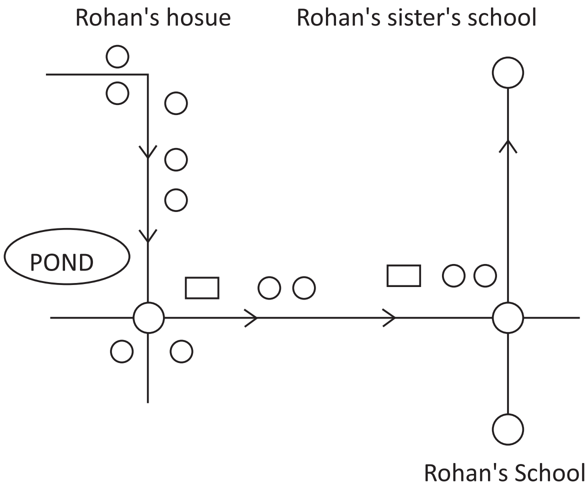
How do we read maps? What can we conclude and understand while reading a map? What information does a map have and what it does not have? Is it different from a picture? In this section, we will try to find answers to some of these questions. Look at the map of a house whose picture is given alongside.





When we draw a picture, we attempt to represent reality as it is seen with all its details, a map depicts only the location of an object. Secondly, different persons give pictures completely different from others. It is not true in a map. The map of the house remains the same irrespective of the position of the observer. In other words, **view is very important for drawing a picture but it is not relevant for map.**

Now, look at the map. It is drawn to show the route from Rohan's house to his school.



From this map, can you tell —

(i) The distance of Rohan's school from his house ?

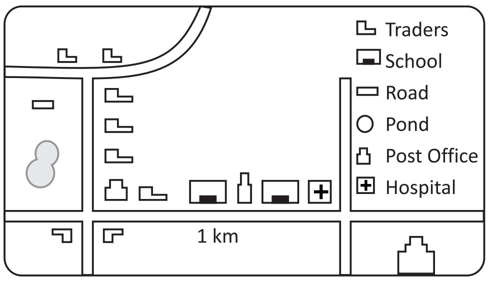
(ii) Does every circle in the map appear a round about ?

(iii) Whose school is near to the house, Rohan's or his sister's ?

It is very difficult to answer the above questions on the basis of the given map.

The reason is that we do not know if the distances have been drawn properly or whether the circles drawn are roundabouts or represent something else.

Another map is drawn to show the route from the house to Rohan's sister school.



This map is different from the earlier maps. Here, different symbols have been used to mark different landmarks. Secondly, longer line segments have been drawn for longer distances and shorter line segments have been drawn for shorter distances. Thus, it is a map to a scale.

Now, following questions arise :

⇒ How far is Rohan's school from his residence ?

⇒ Whose school is nearer to the house, Rohan's or his sister's ?

⇒ Which are the important landmarks on the route ?

Thus we realise that, use of certain symbols and mentioning of distances has helped us read the map easily. The distances shown on the map are proportional to the actual distances on the ground. This is done by considering proper scale. While drawing a map, one must know, to what scale it has to be drawn, i.e., how much of actual distance is denoted by 1 mm or 1 cm in the map. This means, that if one draws a map, he/she has to decide that 1 cm of space in that map shows a certain fixed distance, say 1 km or 10 km. This scale can vary from map to map but not within a map.

Have a look at the map of India and the map of Delhi. In the map of Delhi 1 cm of space is representing smaller distances as compared to the distances in the map of India.

The larger the place and smaller the size of the map drawn, the greater is the distance represented by 1 cm.

Thus, we can conclude that—

(a) A map shows the location of a particular object/place in relation to other objects/places.

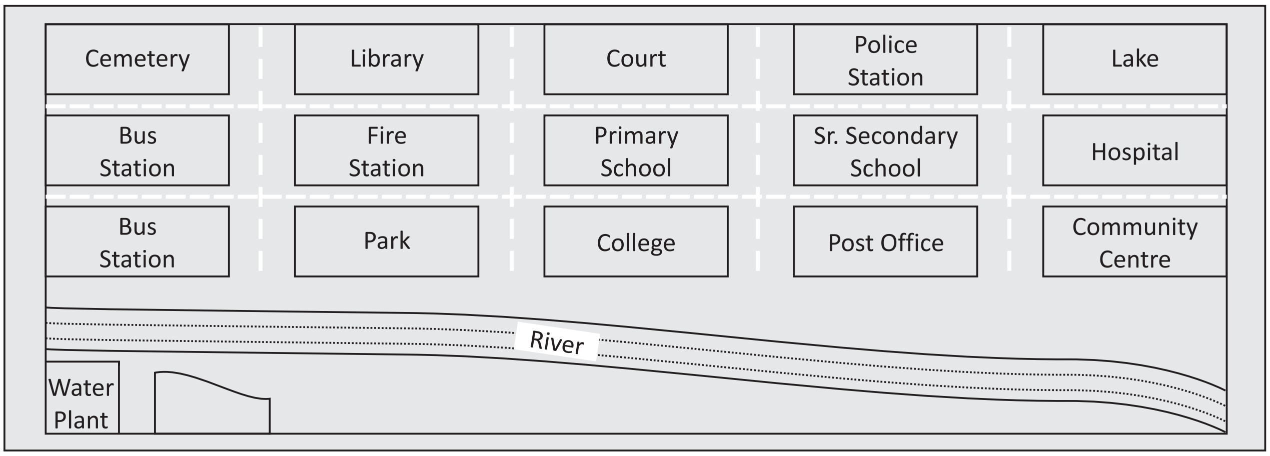
(b) Signs are used to show the different objects/places.

(c) There is no reference or perspective in map, i.e., objects that are closer to the observer are shown to be of the same size as those that are farther away. For example, look at the following illustrations.

(d) Maps use a scale which is fixed for a particular map. It reduces the real distances proportionately to distances on the paper.

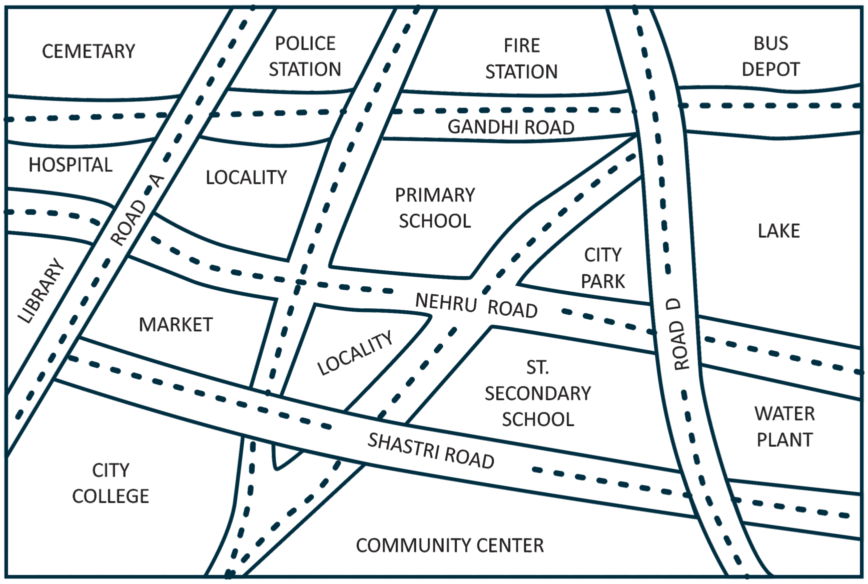
Exercise 14.2

1. Look at the following map of a city.



Colour the map as follows: Blue-Water, Red-Fire station, Pink-Library, Dark green-Schools, Green-Park, Brown– Community Centre, Purple-Hospital, Orange - Cemetery.

2. Look at the given map of a city.



Answer the following :

(a) Colour the map as follows : Blue-Water, Red-fire station, Yellow-Library, Dark green-School, Green-Park, Pink-College, Purple-Hospital, Orange-Cemetery.

(b) Mark a green 'X' at the intersection of Road 'C' and Nehru Road, Green 'Y' at the intersection of Gandhi Road and Road A.

(c) Which is further east, the city park or the market ?

(d) Which is further south, the primary school or the Sr. Secondary School ?

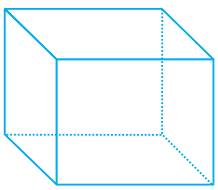
3. Sketch a map of your classroom using proper scale and symbols for different objects.

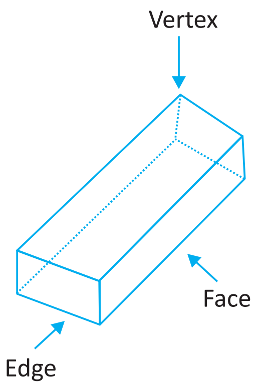
4. Sketch a map of your school compound using proper scale and symbols for various features like playground, main building, garden etc.

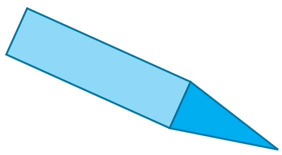
5. Sketch a map giving instructions to your friend so that she reaches your house without any difficulty.

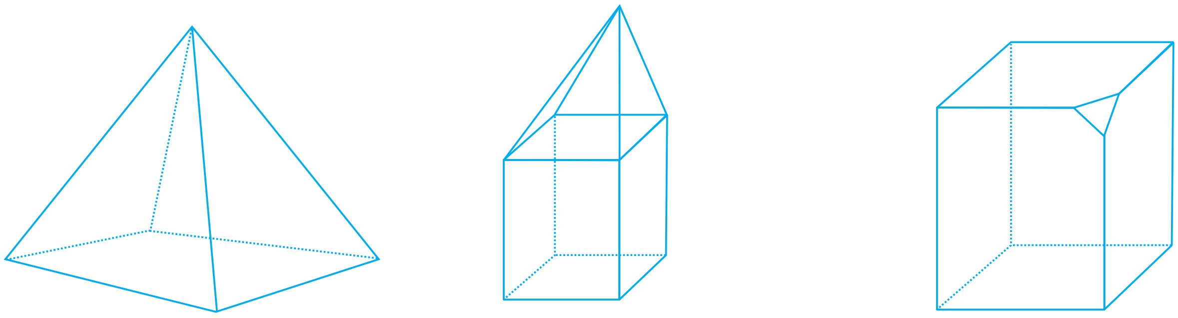
Faces, Edges And Vertices

Look at the following solids.

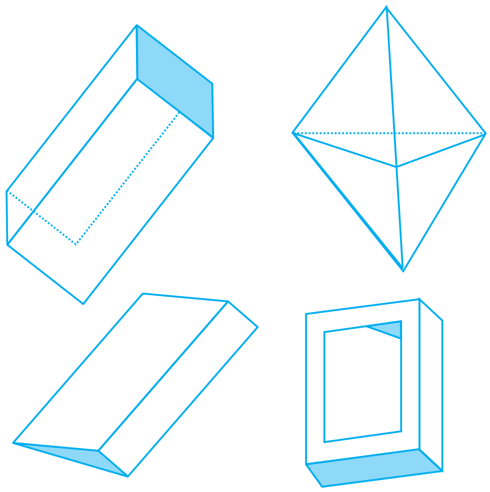




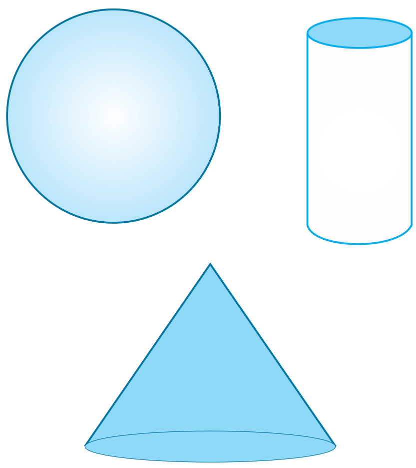




Each of these solids is made up of polygonal regions which are called its faces. These faces meet at edges which are line segments; and the edges meet at vertices which are points. Such solids are called polyhedrons.

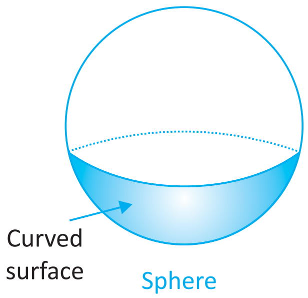


These solids are polyhedrons

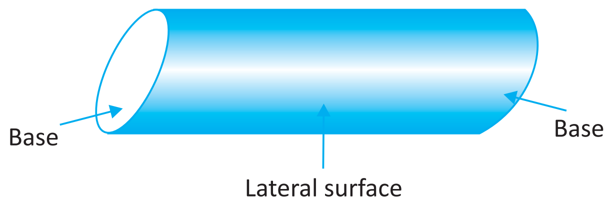


These solids are not polyhedrons

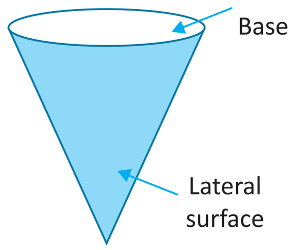
How are the polyhedrons different from the non-polyhedrons ? Study the following figures carefully.



Sphere

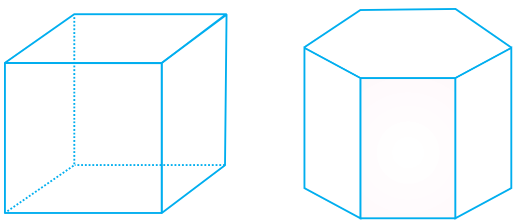


Cylinder

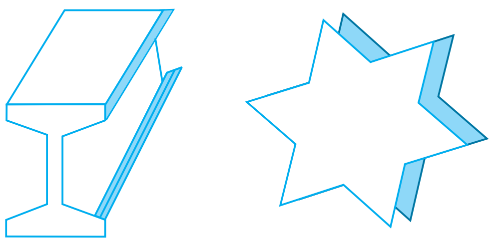


Cone

Convex polyhedrons : You will recall the concept of convex polygons. The idea of convex polyhedron is similar.



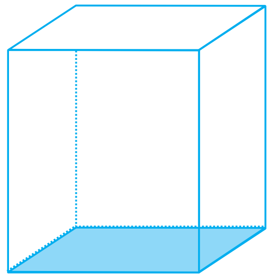
These are convex polyhedrons

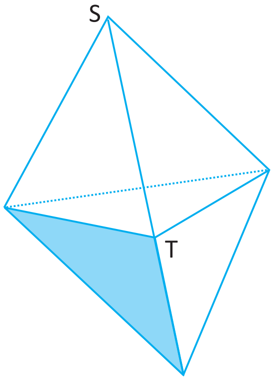


These are not convex polyhedrons

Regular polyhedrons : A polyhedron is said to be regular if its faces are made up of regular polygons and the same number of faces meet at each vertex.

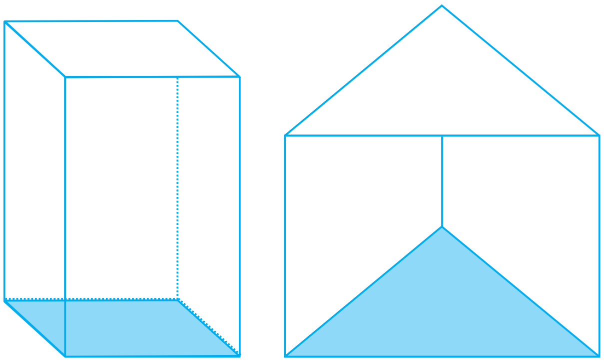
This polyhedron is regular.
Its faces are congruent, regular polygons. Vertices are formed by the same number of faces.



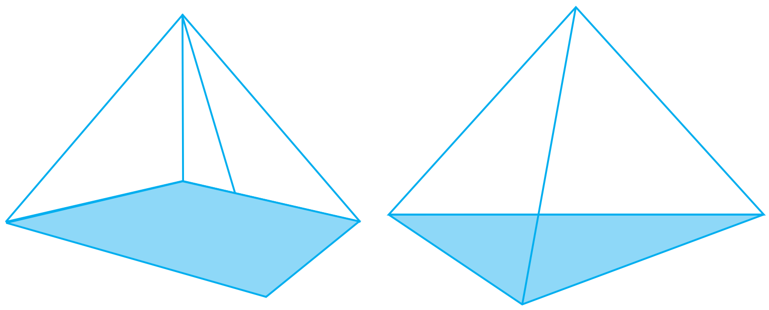


This polyhedron is not regular. All the sides are congruent; but the vertices are not formed by the same number of faces.
3 faces meet at S, but 4 faces meet at T.

Two important members of polyhedron family around are prisms and pyramids.



Prisms



Pyramids

A polyhedron whose base and top are congruent polygons and whose other faces, i.e., lateral faces are parallelograms in shape is called a prism.

On the other hand, a polyhedron whose base is a polygon (of any number of sides) and whose lateral faces are triangles with a common vertex is called a pyramid. (If you join all the corner of a polygon to a point not in its plane, you get a model for pyramid). A prism or a pyramid is named after its base. Thus a hexagonal prism has a hexagon as its base, and a triangular pyramid has a triangle as its base. What, then, is a rectangular prism ? What is a square pyramid ? Clearly their bases are rectangle and square respectively.

For any polyhedron,

$F + V - E = 2$ Number of faces + Number of vertices = Number of edges + 2

Where, 'F' stands for number of faces, 'V' stands for number of vertices, 'E' stands for number of edges. This relationship is called Euler's formula.

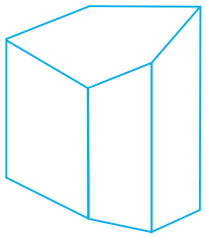
Exercise 14.3

1. (a) How are prism and cylinders alike ?

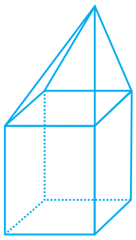
(b) How are pyramids and cones alike ?

2. Verify Euler's formula for these solids.

(a)



(b)



3. Can a polyhedron have for its faces –

(a) 3 triangles ? (b) 4 triangles? (c) a square and four triangles?

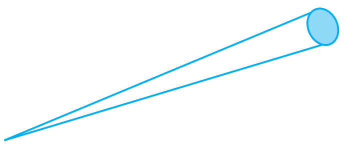
4. Is a square prism same as a cube ? Explain.

5. Is it possible to have a polyhedron with any given number of faces ? (**Hints :** Think of a pyramid).

6. Can a polyhedron have 10 faces, 20 edges and 15 vertices ?

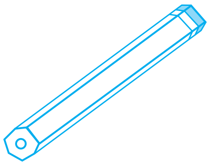
7. Which are prisms among the following ?

(a)



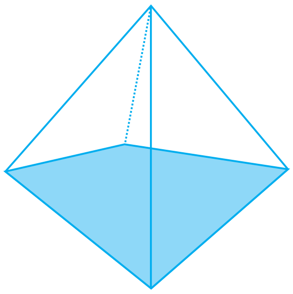
A nail

(b)



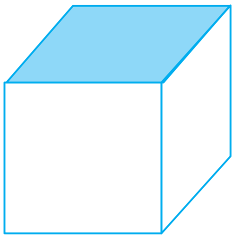
Unsharpened pencil

(c)



A table weight

(d)



A Box

Points to Remember :

- A cuboid is a solid bounded by six rectangular faces. It has 6 faces, 12 edges and 8 vertices.
- A cuboid is called a cube when the length, breath and height of a cuboid are equal.
- A prism is a solid whose side faces are rectangles and whose ends are identical polygons in parallel planes.
- A pyramid is a solid whose base is a plane rectilinear figure and whose side faces are triangles having a common vertex.
- A pyramid with a triangular base is called tetrahedron. It has 6 faces, 6 edges and 4 vertices.

EXERCISE

1. MULTIPLE CHOICE QUESTIONS (MCQs) :
Tick (✓) the correct options.

(a) The number of edges in a cuboid is—

(i) 8 ☐ (ii) 10 ☐ (iii) 12 ☐ (iv) 14 ☐

(b) A solid figure bounded by six rectangular faces is called a—

(i) cube ☐ (ii) cuboid ☐ (iii) cylinder ☐ (iv) prism ☐

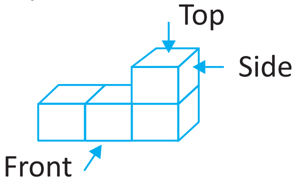
(c) A pyramid with a triangular base is called a—

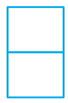
(i) pyramid ☐ (ii) tetrahedron ☐ (iii) prism ☐ (iv) cube ☐

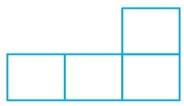
(d) Which of the following is a polyhedron?

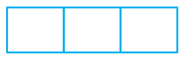
(i) cube ☐ (ii) cone ☐ (iii) sphere ☐ (iv) cylinder ☐

2. Identify the top view, front view and side view of the given figure.

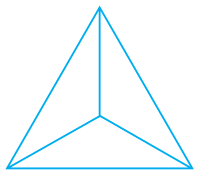


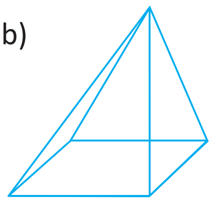
(a) 
....

(b) 
....

(c) 
....

3. Verify Euler's formula for these solids.

(a) 

(b) 

4. Can a polyhedron have for its faces –

(a) 3 triangles ? (b) 4 triangles? (c) a square and four triangles?

5. What is a map?

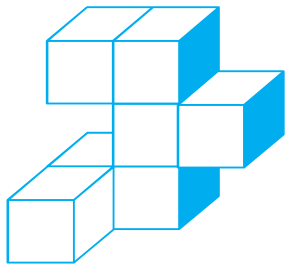
6. What are 3-D shapes?

7. Are the faces of a regular polyhedron made up of regular polygons?

8. Sketch a map giving instructions to your friend so that she reaches your house without any difficulty.

HOTS

1. Look at the given picture. Seven cubes are glued together.
Can you find the total number of faces glued together?

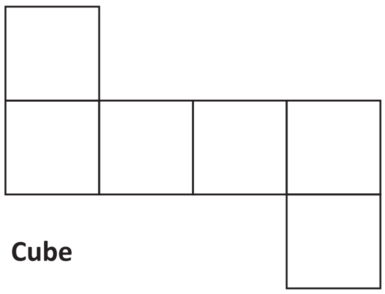


Lab Activity

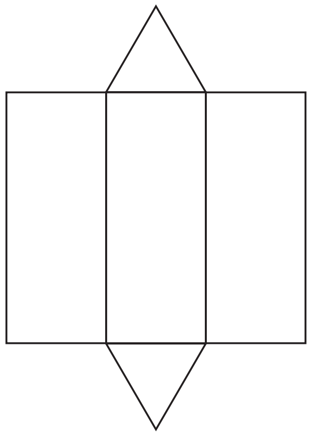
Objective : To visualise and understand 3-D figures

Materials Required : Chart paper, pencil, ruler and a pair of scissors.

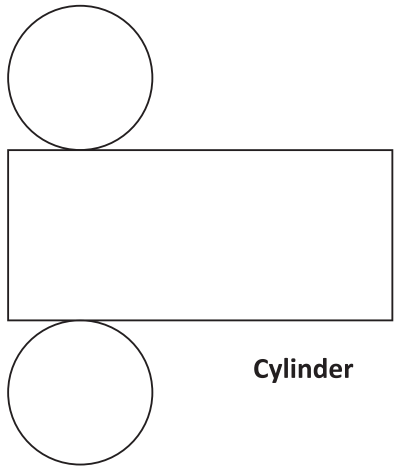
Procedure : Draw the net of 3-D figures on a chart paper. You can draw the diagram of the net of 3-D figures (as shown below) on a large scale. Use scissors to cut along the lines. Fold the chart paper to form the solid figures. Now in the figures so formed use different coloured chart paper to decorate your solid shapes with interesting patterns.



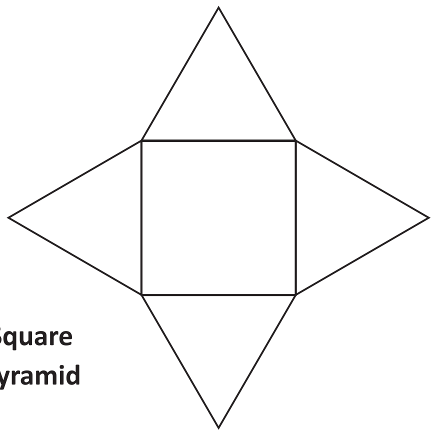
Cube



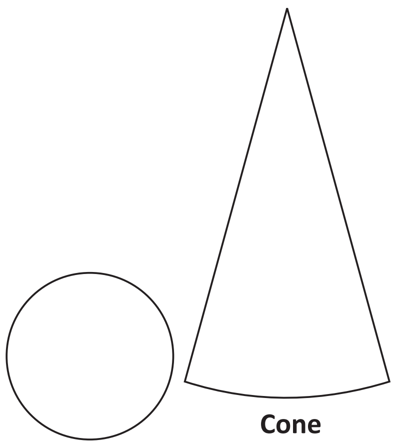
Triangular prism



Cylinder



Square pyramid



Cone

15 Area of Triangle and Parallelogram

Introduction

We have learnt about the areas of rectangles and squares upto class VII. In this section, we shall learn to find the areas of some more rectilinear figures, a parallelogram, a triangle and a trapezium.

Some Definitions and Properties

Rectilinear figure

A figure made up of some line segment is called a rectilinear figure the line segments forming the figure are known as the side of the figure. A rectilinear figure is said to be a closed rectilinear figure if it has no open ends. A rectilinear figure is said to be simple if no two sides of it intersect except at a common end point.

Clearly, a rectangle, a square, a rhombus, a triangle etc. are simple rectilinear figures.

Region

The part of the plane enclosed by a simple closed rectilinear figure is called the region enclosed by it.

Area

The magnitude of the plane region enclosed by a simple closed figure is called its area.

Units of measurement of area

A square centimetre (cm^2 or sq. cm) is a standard unit of area and is defined as follows.

A square centimetre is the area of the region formed by square of side 1 cm.

Other standard units of area are square metre (cm^2) sq – decimetre (dm^2), sq-decametre (dam^2) or an arc, square hectometre (hm^2) or hectare square kilometre (km^2) etc.

Other standard units of area and their relations are

	$100\ cm^2$	=	$10 \times 10\ cm^2$	=	$1\ dm^2$
	$100\ dm^2$	=	$10 \times 10\ dm$	=	$1\ m^2$
	$1\ m^2$	=	$100 \times 100\ cm^2$	=	$10000\ cm^2$
	$100\ m^2$	=	$1\ dam^2$		
or,	$100\ m^2$	=	$1\ dam^2$	=	1 arc
	$10000\ m^2$	=	$1\ hm^2$		
or	$10000\ m^2$	=	1 hectare		
Also	100 ares	=	1 hectare		
	$10^6\ m^2$	=	$1\ km^2$		
Also	100 hectares	=	$1\ km^2$		

Area of a square and a rectangle

In the earlier class, we have learnt how to find the area of a rectangle and a square. We shall review the formula and problems related to them.

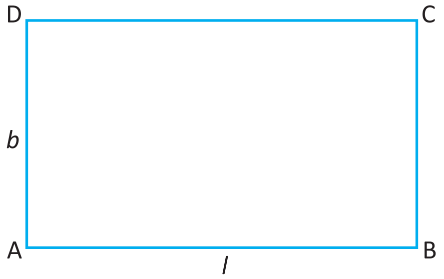
Area of rectangle

Let ABCD be a rectangle of length = AB = l units and breadth = AD = b units. Then area of the rectangle = ($l \times b$) sq. units. Then—

Length = $\left[\dfrac{Area}{Breadth}\right]$ units.

Breadth = $\left[\dfrac{Area}{Length}\right]$ units.

Diagonal = $\sqrt{l^2 + b^2}$ units.



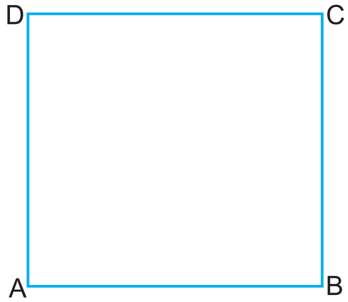
2. **Area of the four walls of a room :** Let there be a room with length = *l* units, breadth = *b* units and height = *h* units, Then—

   (i) Area of the 4 walls = [2 (*l* + *b*) × *h*] sq. units

   (ii) Diagonal of the room = $\sqrt{l^2 + b^2 + h^2}$ units

3. **Area and perimeter of a square :**

   Let ABCD be a square each of whose sides measures a unit, Then—



   (i) Area of the square = a^2 sq. units.

   (ii) Diagonal of the square = $(\sqrt{2}a)$ units

   (iii) Perimeter of the square = 4 *a* units

   (iv) Area of the square = [½ (diagonals)^2]

Illustrative Examples

Example 1 : A rectangular grassy lawn measuring 38 m by 25 m had been surrounded externally by a 2.5 m wide bath. Calculate the cost gravelling the path at the rate of ` 6.50 per sq. metre.

Solution : Let ABCD be the grassy lawn and let EFGH be the external boundary of the path around the lawn.

Then the area of the path = (The area of rec. EFGH) – (The area of rect. ABCD)

Thus, AB = 38 m and BC = 25 m.

The area of rect. ABCD = (AB × BC)

= $(38 \times 25)m^2 = 950\ m^2$

Also, EF = (2.5m + 38m + 2.5m) = 43m

FG = (2.5m + 25m + 2.5m) = 30m.

The area of rect. EFGH = (EF × FG)

= $(43 \times 30)m^2 = 1290m^2$

The area of the path = (The area of rect. EFGH) – (The area of rect. ABCD)

= $(1290 - 950)$m2 = $340m^2$

The cost of gravelling the path = ₹ (340 × 6.50) = ₹ 2210

Example 2 : The length of a rectangle is twice its breadth. Find the dimensions of the rectangle if its area is 288 m2.

Solution : Let the length of the given rectangle be x cm. then,

Breadth $=$ $2x$ cm [$\therefore$ Breadth $= 2 \times$ length (given)]

Area at the rectangle $=$ $(2x \times x)$ cm$^2 = 2x^2$ cm^2

But area is given as 288 cm^2

$\therefore$ $2x^2 = 288$

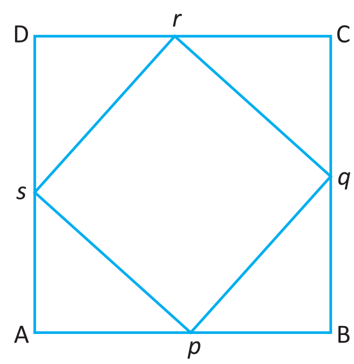
$x^2 = \frac{288}{2} = 144$

$x^2 = \sqrt{144}$

$x = 12$ cm

Hence, length of the rectangle = 24 cm, and breadth of the rectangle = 12 cm.

Example 3 : Find the area of a rectangular plot, one side of which measure 35 metres and the diagonal is 37 metres.



Solution : Let the other side be x metres.

Then $(35)^2 + x^2 = (37)2$

or $x^2 = [(37)^2 - (35)^2] = 144$

$\therefore$ $x = 12$

Thus, the other side of the rectangle = 12 metres.

The area at the rectangle = $(35 \times 12)\text{m}^2 = 420$ m

Example 4 : Find the area of a square, the length of whose diagonal is 3 metres.

Solution : Area of the square $=$ ½ × (diagonal)

$= [½ \times 3 \times 3]$ m^2

$= 4.5$ m^2

Example 5 : Find the area of the square joining the mid-points of the sides. If the area of square is 16 cm^2.

Solution : We have,

Area of square ABCD $=$ 16 cm^2

$\therefore$ Each side of square $= \sqrt{16 \text{ cm}^2} = 4$cm

In DAPS, we have

AP = ½ AB = 2 cm and AS = ½ AD = 2 cm.

Also, PS2 = AP2 + AS2 (using Pythagoras theorem)

$\text{PS} = \sqrt{4^2 + 4^2} = \sqrt{32} = 4\sqrt{2}$

Thus, each side of square PQRS is of length $4\sqrt{2}$ cm.

$\therefore$ Area of the square = PQRS = $(4\sqrt{2} \text{ cm})^2 = 32\text{cm}^2$.

Example 6 : A room is 9 metres long, 8 metres broad and 6.5 metres high. It has one door 2×1.5 m and three windows each of whose dimensions are 1.5 m × 1 m. Find the cost of white washing the walls at ₹3.75 per m^2.

Solution : Length = 9 m, breadth = 8 m, height = 6.5 m.

∴	Area of room (4 walls)	=	$2(l + b) \times h$
		=	$2(9 + 8) \times 6.5 = 221\ m^2$
	The area of 1 door	=	$(1.5 \times 2)\ m^2 = 3.0\ m^2$.
	The area of 3 windows	=	$3 \times (1.5)\ m^2 = 4.5\ m^2$.
∴	The area of 1 door and 3 windows	=	$(3.0 + 4.5)\ m^2 = 7.5\ m^2$.
So,	The area to be white washed	=	$(221 - 7.5)\ m^2 = 213.5\ m^2$.
∴	Cost of white washing	=	₹(213.5 3.75) = ₹800.63.

Exercise 15.1

1. A rectangular hall 12 metres long 10 metres broad is surrounded by a verandah 3 metres wide. Find the area of the verandah.

2. A table clothes 5m × 3m, is spread on a meeting table. If 25 cm of the table cover is hanging all around the table, find the area of the table top.

3. A 115 m long and 64 m broad lawn has two crossroads at right angles, one 2m wide, running parallel to its length and the other 2.5m wide, running parallel to its breadth. Find the cost of the roads at ₹4.60 per m^2.

4. The length and breadth of a rectangular park are in the ratio 5 : 2. A 2.5 m wide path running all around the outside of the park has an area of 305 m^2. Find the dimensions of the park.

5. Find the area of a rectangular plot one side of which measures 35m and diagonal 37m.

6. If the perimeters of two squares are in the ratio $a : b$, prove that their areas are in the ratio $a^2 : b^2$.

7. A hall is 36 m long and 24 m broad. Allowing 40 square metres for doors and windows, the cost of papering the walls at ₹ 8.40 per square metre is ₹ 4704. Find the height of the hall.

8. The area of 4 walls of a room is 168 m^2. The breadth and height of the room are 10 m and 4 m respectively. Find the length of the room.

9. A room is 8.5 m long, 6.5 m broad and 3.4 m high. It has two doors, each measuring 1.5 m by 1 m. And two windows, each measuring 2 m by 1 m. Find the cost of painting its four walls at ₹4.60 per square metre.

10. The perimeters of two squares are 3.36 m and 7.48 m respectively. Find the perimeter of square whose area is equal to the sum of the areas of these two squares.

11. Find the length of the largest pole that can be placed in a hall that is 10 m long, 10 m broad and 4 m high.

    [**Hint :** length of the diagonal of the room $= \sqrt{l^2 + b^2 + h^2}$]

12. The area of 4 walls of a hall is 320 m^2. The length and breadth of the hall are 12.5 m and 7.5 m respectively. Find the height of the hall.

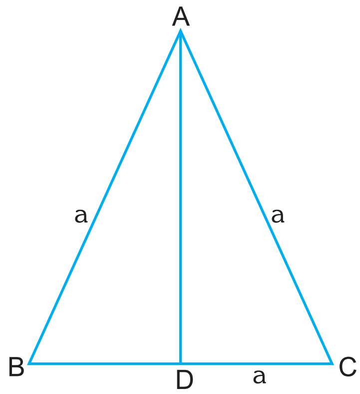
Formula for the Area of a Parallelogram and Triangle

We have learnt in previous class, how the area of a triangle can be obtained when its base and the corresponding altitude are given and the measurement of the region enclosed by the four sides of a parallelogram is its area.

In this section, we will learn to find the area of an equilateral triangle and the area of a parallelogram whose sides are given. Let us consider the following experiment.

Formula 1 : Area of equilateral triangle :

Let ABC be an equilateral triangle whose each side is unit in length. Let applying Pythagoras theorem in DABD, we have



AB2 = AD2 + BD2

AD2 = AB2 – BD2

$$AD = \sqrt{a^2 - \frac{a^2}{4}} = \sqrt{\frac{3a^2}{4}} = \frac{a}{2} \times \sqrt{3} \text{ units}$$

Area of DABC = ½ × (Base × Height)

= ½ × BC × AD

$$= \left[\frac{1}{2} \times a \times \sqrt{3}\ \frac{a}{2}\right] \text{ sq. units.}$$

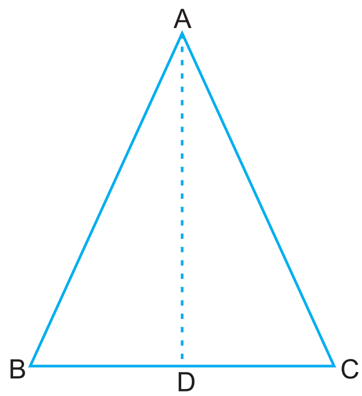
$$= \left[\frac{\sqrt{3}}{4} \times (\text{side})^2\right] \text{ SQ. UNITS.}$$

Thus, area of an equilateral triangle = $\left[\frac{\sqrt{3}}{4} \times (\text{side})^2\right]$ sq. units.

Area of an isosceles triangle

Let ABC be an isosceles triangle such that AB = AC = *b* units and BC = *a* units. Draw AB = BC. Then, BD = DC = *a*/2.

Applying Pythagoras theorem in DABC, we have



$AB^2 = AD^2 + BD$

$b2 = AD^2 + \left[\frac{a}{2}\right]^2$

$AD^2 = b^2 - \frac{a^2}{4}$

$AD = \sqrt{b^2 - \frac{a^2}{4}}$

$= \frac{1}{2} \times a \times \sqrt{b^2 - \frac{a^2}{4}} = \frac{1}{2} \times \text{base} \times \sqrt{(\text{equal side})^2 - (\text{base})^2}$

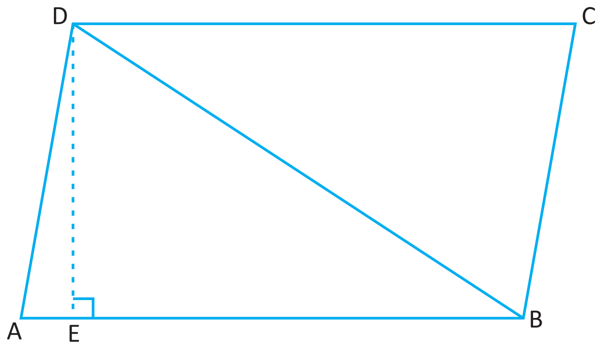
Heron's Formula

Let a, b, c be the lengths of the sides of a given triangle. Then, $s = ½ (a + b + c)$ is called the semiperimeter of the triangle.

A Greek mathematician, Heron, gave the formula for the area of a triangle as

Area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$ sq units.

Let ABCD is a parallelogram. Take AB as the base of the parallelogram. Clearly, the diagonal BD divides their parallelogram into two equal triangles.



Let DE $\perp$ AB.

Area of parallelogram ABCD

= 2 × (area of DABD)

= 2 × (½ × AB × DE) sq. units

= (AB × DE) sq. units

= (base × height) sq. units.

Illustrative Examples

Example 1 : Find the altitude of a parallelogram whose area is 2.25 m^2 and base is 25 dm.

Solution : We have,

Area of the given parallelogram = 2.25 m^2

Base of the given parallelogram = $25 dm = \frac{25}{10} m = 2.5 m$

Altitude of the given parallelogram = $\frac{Area}{Base} = \frac{2.25}{2.5} m$

= 0.9 m = (0.9 × 10) dm

= 9 dm

Example 2 : Find the area of a parallelogram with base 5 cm and altitude 4.2 cm.

Solution : We have,

Base = 5 cm and altitude = 4.2 cm

Area of the parallelogram = Base × Height

= $(5 \times 4.2)\ cm^2 = 21\ cm^2$

Example 3 : The base of parallelogram is twice its height. If the area is 512 cm^2, find the base and its height.

Solution : Let the height be x cm

∴ Then, the base = $2x$ cm

The area of the parallelogram = $(2x \times x)\ cm^2 = 2x2\ cm^2$

∴ $x = \sqrt{256} = 16$

Thus, the height is 16 cm and the base is 32 cm.

Example 4 : Find the altitude of a triangle whose base is 20 cm and area is 150 cm^2.

Solution : We have altitude of a triangle $= \dfrac{2\times \text{Area}}{\text{Base}}$

Here, base = 20 cm and area = 150 cm altitude $= \dfrac{2\times 150}{20} = 15\text{cm}$

Example 5 : Find the area of an equilateral triangle, each of whose sides is 10 metres long.

Solution : The area of the triangle $= \left[\dfrac{\sqrt{3}}{4}\times 10\times 10\right]\text{m}^2$

$= \left[\dfrac{1.73\times 10\times 10}{4}\right]\text{m}^2 \quad \left[\therefore \sqrt{3} = 1.73\right] = 43.25\text{ m}^2$

Example 6 : Find the area of an equilateral triangle having each side 4 cm.

Solution : We know that the area of an equilateral triangle is equal to $\left[\dfrac{\sqrt{3}}{4}(\text{side})^2\right]$ sq. units

Here, side = 4 cm

Area of the given triangle $= \left[\dfrac{\sqrt{3}}{4}\times 4^2\right]\text{cm}^2$

$= 4\sqrt{3}\text{ cm}^2$

Example 7 : The base of an isosceles triangle is 12 cm and its perimeter is 32 cm. Find its area.

We have, base = 12 cm and perimeter = 32 cm

Solution : Let the length of each equal sides = 6 cm [\ base=12cm given]

$\therefore$ Perimeter = 32 cm

$2b + 12 = 32$ cm

$2b = (32 - 12)$ cm

b = 10 cm : Sum of equal sides = (10 + 10) = 20 cm

We have

Base = 12 cm and equal side = 10 cm $\dfrac{1}{2}\times \text{Base}\times \sqrt{(\text{equal side})^2 - (\text{base})^2}$

$\therefore$ Area of the triangle $= \dfrac{1}{2}\times 12\times \sqrt{(20)^2 - (12)^2}$

$= 6\times \sqrt{400 - 144}$

$= 6\times \sqrt{256} = 6\times 16$

$= 96\text{ cm}^2$

Example 8 : The area of a triangle is equal to that of a square whose each side measures 60 metres. Find the side of the triangle whose corresponding altitude is 90 metres.

Solution : We have,

Area of the square $= (60\times 60)\text{ m}^2$

Area of the square $= 3600\text{ M}^2$

Altitude of the triangle $= 90$ m

Side of the triangle $= \dfrac{2 \times \text{Area}}{\text{corresponding altitude}}$

$= \left[\dfrac{2 \times 3600}{90}\right]\text{m} = 80\text{ m.}$

Exercise 15.2

1. Find the area in square centimetre of the triangle whose base and altitude are as under :

(a) Base = 15 cm, Altitude = 8 cm (b) Base = 1.5 cm, Altitude = 8 cm

(c) Base = 32 cm, Altitude = 105 cm

2. Find the area of a triangle, the length of whose sides are 78 m 50 m and 112 m.

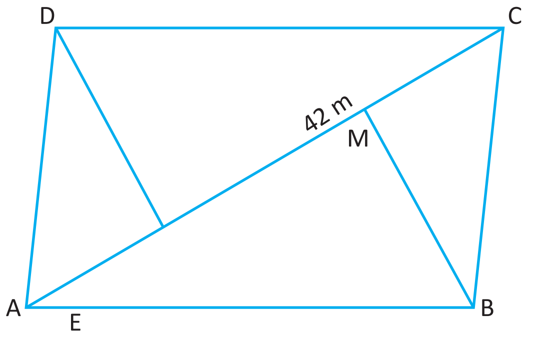
3. The cost of painting the top surface of a triangular board at 80 paise per square metre is ₹ 176.40. If the height of the board measures 24.5 m, find its base.

4. Find the area of the equilateral triangle, each of whose sides measures.

(a) 18 m (b) 20 m
(c) 11 cm

5. Find the area of a right angled triangle with hypotenuse 25 cm and base 7 cm.

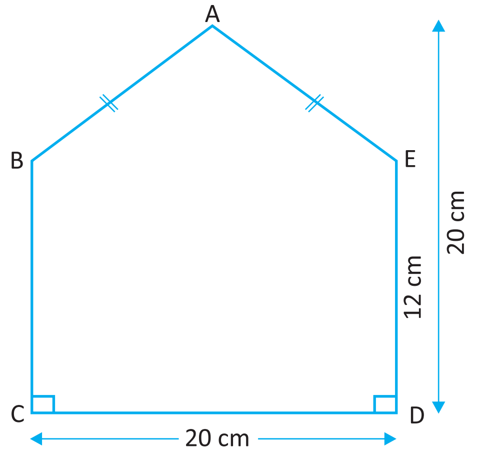
6. A field in the form of a parallelogram has one of its diagonals 42 m long and the perpendicular distance of this diagonal form either of the outing vertices is 10.8 m. Find the area of the field.



7. A field is in the form of a triangle. If its area be 2.5 m^2 and the length of its base be 250 cm, find its altitude.

8. A rectangular field is 48 m long and 20 m wide. How many right triangular flower beds, whose sides containing the right angle measure 12 m and 5 m can be laid in this field ?

9. Calculate the area of the pentagon ABCDE where AB = AE and with dimensions as shown in figure.



10. Find the area in square metre of the parallelogram whose base and altitudes are as under :

(a) Base = 10 dm, Altitude = 4.6 dm

(b) Base = 2 m, 20 cm, Altitude = 60 cm

(c) Base = 6.4 dm, Altitude = 25 cm

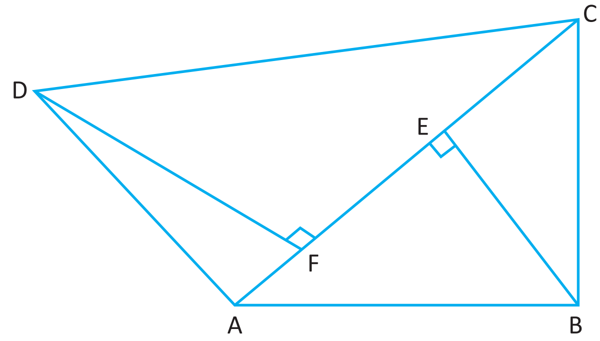
11. Find the altitude of a parallelogram whose area is 2.25 m^2 and base is 25 dm.

12. The adjacent sides of a parallelogram are 10 m and 8 m. If the distance between the longer sides is 4 m, find the distance between the shorter sides.

Area of A Quadrilateral, A Rhombus and A Trapezium

Area of a quadrilateral

Let us consider a quadrilateral ABCD whose one diagonal, say AC, and the lengths of the perpendiculars to AC from the opposite vertices B and D are given.



Let BE ⊥ AC and DF ⊥ AC.

Then, the area of quadrilateral ABCD

= Area of DACD + Area of DABC

= ½ × AC ×BE + ½ AC × DF

= ½ × AC × (BE + DF) sq. units.

Example 1 : A diagonal of a quadrilateral is 30 m in length and the perpendicular to it form the opposite vertices are 6.8 m and 9.6 m. Find the area of the quadrilateral.

Solution : Let ABCD be the given quadrilateral in which BE ⊥ AC and DF ⊥ AC

Let AC = 30m, BE = 6.8m and DF = 9.6m

Now, the area of the quadrilateral ABCD = (Area of ABC) + (Area of ACD)

= (½ × AC × BE) + (½ × AC × DF)

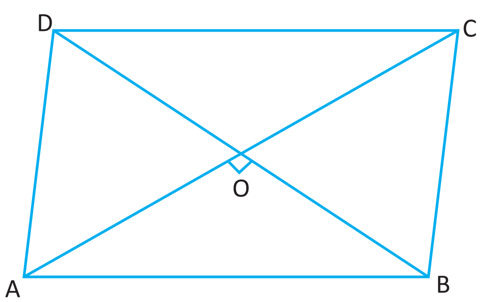
= [(½ × 30 × 6.8) + (½ × 30 × 9.6)]m^2

= (102 + 144)m2 = 246 m^2

Area of a rhombus

A rhombus is parallelogram in which all the sides are equal. We also know that the diagonals of a rhombus bisect each other at right angles.

Consider a rhombus ABCD whose diagonals AC and BD intersect at O.



Area = 4 × (½ × OA × OB)

= 2 × OA × OB

= 2 × ½ AC × ½ BD

= ½ AC × BD

= ½ (AC × BD)

Hence, area of a rhombus = ½ (product of diagonals)

Example 1 : Find the altitude of a rhombus whose area is 36m2 and perimeter is 36 m.

Solution : We have perimeter of the rhombus = 36 m

and, area of the rhombus = $36m^2$

Now, Side of the rhombus = $\frac{\text{Perimetre}}{\text{No. of sides}} = \frac{36}{4} = 9m$

∴ Altitude of the rhombus = $\frac{\text{Area}}{\text{Side}} = \frac{36}{9} = 4m$

Example 2 : Find the area of a rhombus, the lengths of whose diagonals area 36 cm and 22.5 cm.

Solution : The area of the rhombus = ½ × (product of diagonals)

= $(½ \times 36 \times 22.5)\ cm^2 = 405\ cm^2$

Example 3 : The area of rhombus is 72 cm^2. If one of the diagonals is 18 cm long, find the length of the other diagonal.

Solution : We have

Area of the rhombus = 72 cm^2

Length of one diagonal = 18 cm

Now, Area of the rhombus = 72 cm^2

½ × 18 × Length of the other diagonal = 72

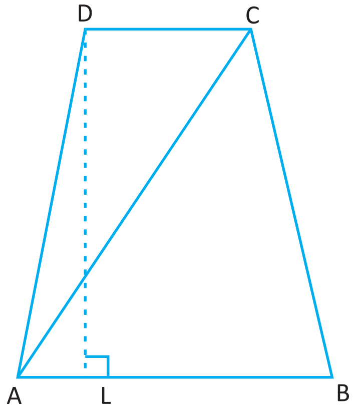
9 × Length of the other diagonal = 72

Length of the other diagonal = 72/9 cm = 8 cm

Area of trapezium

We have learnt that trapezium is a quadrilateral whose one pair of opposite sides are parallel. If two non parallel sides of a trapezium are equal. It is called an isosceles trapezium.

Let *h* be the height of the trapezium ABCD. Then, DL = *h* join AC, clearly, AC divides the trapezium ABCD into two triangle ABC and ACD.



∴ Area of trapezium ABCD = Area of DABD + area of DACD

Since *h* is the altitude of trapezium ABCD.

Therefore, it is also the altitude of DABC and DACD.

∴ Area of DABC = ½ × AB × *h*

and area of ACD = ½ × DC × *h*

Substituting these values in equation we get Area of trapezium

ABCD = ½ × AB × h + ½ × CD × *h*

= ½ × (AB + DC) × *H*

= ½ × (sum of the parallel sides) × (distance between parallel sides)

Hence, the area of a trapezium equals half the sum of parallel sides multiplied by the altitude.

Example 1 : Find the altitude of a trapezium, the sum of the length of whose bases is 6.5 cm and whose area is 26 cm2.

Solution : Let the altitude of the trapezium be *h* cm we have, Area of the trapezium = 26 cm^2

= ½ × (sum of the bases) × Altitude = 26

= ½ × 6.5 × Altitude = 26

Altitude = $\frac{26 \times 2}{6.5}$ = 8 cm

Hence, the altitude of the trapezium is 8 cm.

Example 2 : The area of a trapezium is 352 cm^2. The distance between the parallel sides is 16 cm. If one of the parallel sides is 25 cm, find the other.

Solution : Let the required side = *x* cm.

Then, the area of the trapezium = $[½ \times (25 + x) \times 16] \text{ cm}^2$

∴ = $(200 + 8x) \text{ cm}^2$

But, the area of the trapezium = 352 cm^2

∴ $200 + 8x = 352$

or, $8x = (352 - 200) = 152$

$x = \left[\frac{152}{8}\right] = 19$

Hence, the other side = 19 cm.

Example 3 : Find the area of a trapezium whose parallel sides are 57 cm and 39 cm and the distance between them is 28 cm.

Solution : The area of the trapezium = ½ × (sum of parallel sides) × (distance between them)

= $[½ \times (57 + 39) \times 28] \text{cm}^2 = 1344 \text{ cm}^2$

Exercise 15.3

1. Find the area of a rhombus whose each side is of length 5 m and one of the diagonals is of length 8 m.
2. If the area of a rhombus be 48 m2 and one of its diagonal is 12 cm. Find its altitude.
3. A diagonal of a quadrilateral is 26 cm and the perpendiculars drawn to it from the opposite vertices are 12.8 cm and 11.2 cm. Find the area of the quadrilateral.
4. **Find the area of the rhombuses whose dimensions are—**
   (a) Side = 7.5 cm, Altitude = 12 cm (b) Side = 12.6 cm, Altitude = 2 dm
5. **Find the area, in square metres, of the trapezium whose bases and altitude are as under—**
   (a) Base = 20 dm and 12 dm, Altitude = 10 dm.
   (b) Base = 20 cm and 3 dm, Altitude = 25 dm.
   (c) Base = 150 cm and 30 dm, Altitude = 9 dm.
6. The area of a rhombus is 216 cm^2. If one diagonal is 18 cm, find the other.
7. The area of a rhombus is 119 cm^2 and its perimeter is 56 cm. Find its altitude.

8. Find the area of trapezium, whose parallel sides are of length 16 dm and 22 dm and whose height is 12 dm.

9. Find the height of trapezium, the sum of the length of whose bases (parallel sides) is 60 cm and whose area is 600 cm^2.

10. The area of a trapezium is 1586 cm^2 and the distance between its parallel sides is 26 cm. If one of the parallel sides is 84 cm, find the other.

11. The area of a trapezium is 1080 cm^2. If the lengths of its parallel sides are 34.4 cm and 65.7 cm, find the distance between them.

12. The parallel sides of a trapezium are 25 cm and 13 cm; its non-parallel sides are equal, each being 10 cm. Find the area of the trapezium.

13. The cross section of a canal is a trapezium in shape. If the canal is 10 m wide at the top 6 m wide at the bottom and the area of cross section is 72 m^2. Determine its depth.

14. Find the sum of the lengths of the bases of a trapezium whose area is 4.2 m^2 and whose height is 280 cm.

15. Find the area of fig. (a) as the sum of the areas of two trapezium and a rectangle.

16. A garden is in the form of a rhombus whose side is 30 metres and the corresponding altitude is 16 m. Find the cost of levelling the garden at the rate of ₹ 2 per m^2.

17. The area of a trapezium is 180 cm^2 and its height is 9 cm. If one of the parallel sides longer than the other by 6 cm find the two parallel sides.

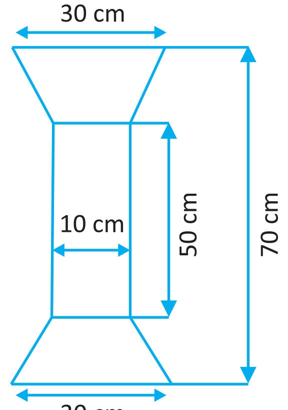


fig. (a)

Points to Remember :

- A plane figure together with its interior is called the 'region' enclosed by the plane figure.
- (a) Area of a rectangle = (length × breadth)

  (b) Lengths = $\left[\dfrac{\text{area}}{\text{breadth}}\right]$: breadth = $\left[\dfrac{\text{area}}{\text{length}}\right]$

  (c) Diagonal = $\sqrt{(\text{length})^2 + (\text{breath})^2}$
- Area of a parallelogram = base × height
- Base = $\dfrac{\text{area}}{\text{height}}$; Height = $\dfrac{\text{area}}{\text{base}}$
- Area of triangle = ½ × base × height
- Height of triangle = $\dfrac{2 \times \text{area}}{\text{base}}$ Base of triangle = $\dfrac{2 \times \text{area}}{\text{height}}$
- Area of a quadrilateral = [diagonal × sum of offsets on it]
- Area of trapezium = ½ (sum of the parallel sides) × distance between the parallel side)
- Standard unit of measurement of area is cm^2.

EXERCISE

1. MULTIPLE CHOICE QUESTIONS (MCQs) :

Tick (✓) the correct options.

(a) The area of a ____________ equals half the sum of its parallel sides multiplied by its altitude.

(i) circle ☐ (ii) trapezium ☐ (iii) triangle ☐ (iv) rectangle ☐

(b) If the length of the side of a square is 5 cm, then the perimeter of the square will be—

(i) 25 cm ☐ (ii) 15 cm ☐ (iii) 20 cm ☐ (iv) 30 cm ☐

(c) ____________ is the quadrilateral in which one pair of opposite sides are parallel to each other.

(i) trapezium ☐ (b) rectangle ☐ (iii) square ☐ (iv) circle ☐

(d) The area of a parallelogram with base 5 cm and altitude 6 cm is—

(i) $\frac{\sqrt{3}}{2} \times (\text{side})^2$ ☐ (ii) $\frac{\sqrt{3}}{4} \times \text{side}$ ☐ (iii) $\frac{\sqrt{3}}{4} \times (\text{side})^2$ ☐ (iv) (base × height) ☐

2. The parallel sides of a trapezium are 12 cm and 8 cm and the distance between them is 6 cm. Find the area of the trapezium.

3. Find the area of a triangle whose base and height are 6 cm and 12 cm respectively.

4. The diagonal of a rhombus are 16 cm and 12 cm. Find its area. Also find its perimeter.

5. If the area of a rhombus be 48 cm^2 and one of its diagonal is 12 cm. Find its another diagonal.

6. The area of a rhombus is 119 cm^2 and its perimeter is 56 cm. Find its altitude.

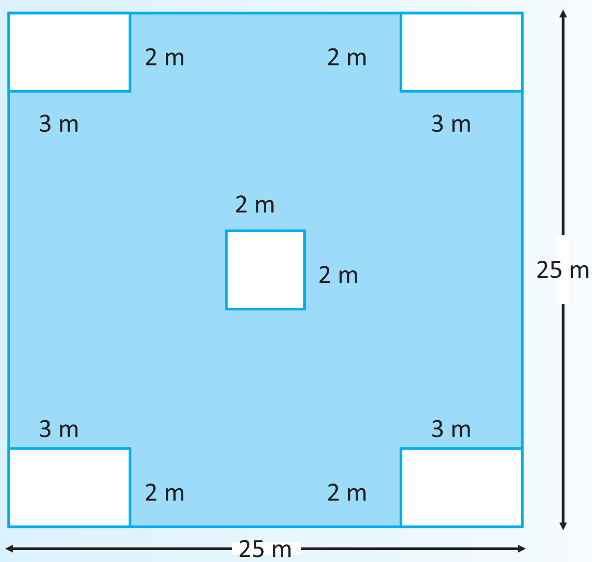
7. The cost of painting the top surface of a triangular board at 80 paise per square metre is ₹ 176.40. If the height of the board measures 24.5 m, find its base.

8. Find the length of the largest pole that can be placed in a hall that is 10 m long, 10 m broad and 5 m high.

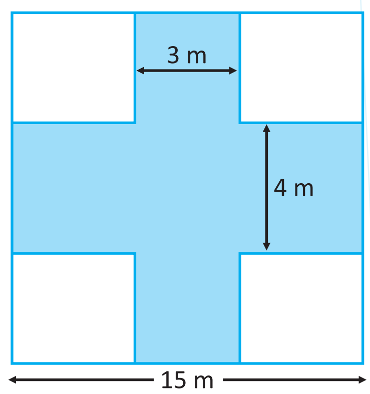
HOTS

Calculate the area of the shaded region as shown in the figure.

(a)



(b)



Lab Activity

Objective : To find area of a trapezium by paper activity.

Materials Required : Square sheet of paper, thick white sheet, marker pen, a pair of scissors, geometry box, fevistick.

Procedure :

(i) Draw a trapezium with parallel sides *a* and *b* on a squared sheet of paper.

(ii) Cut two congruent trapeziums. Name them as ABCD and EFGH. Colour both with different colours [see Fig. (a)].

(iii) Arrange the congruent trapezia in such a way that they form a parallelogram as shown in Fig. (*b*).

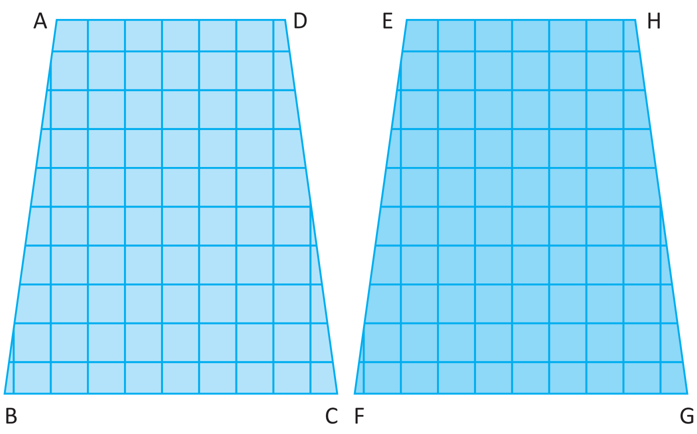


Fig. (*a*)

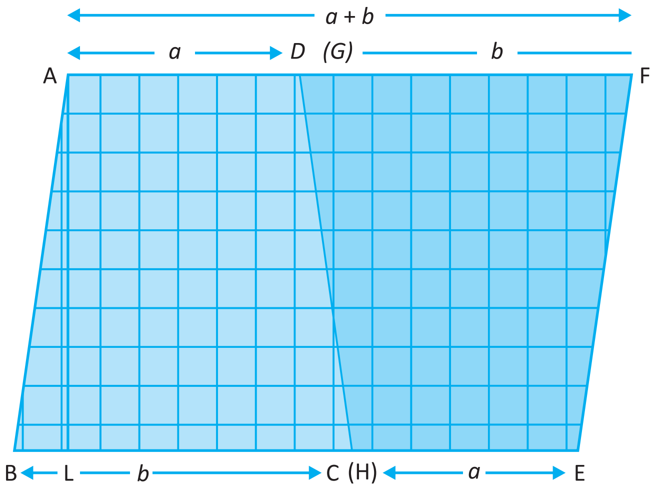


Fig. (*b*)

(iv) Draw AL ⊥ BE, let AL = h

Area of parallelogram ABEF = area of trapezium ABCD + area of trapezium EFGH

= area of trapezium ABCD + area of trapezium ABCD

= 2 (area of trapezium ABCD) {congruent figures are equal in area}

or, Area of trapezium ABCD = $\frac{1}{2}$ area of parallelogram

$\frac{1}{2}\text{BE} \times \text{AL} = \frac{1}{2}(\text{BC} + \text{CE}) \times \text{AL} = \frac{1}{2}(a+b)\times \text{h}$

Thus, area of a trapezium = $\frac{1}{2}$ sum of the parallel sides × perpendicular distance between them.

16 Surface Area and Volume

Introduction

We have learnt in previous class how to find the volume and surface area of a cuboid. We shall learn in this chapter about the volume and surface area of a right circular cylinder.

Formula

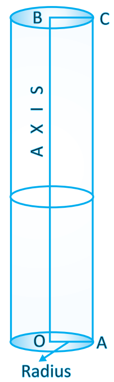
1. **Cuboid :** If l, b and h are respectively the length, breadth and height of a cuboid, then—
   (i) Volume of cuboid = $(l \times b \times h)$ cubic units.
   (ii) Total surface area of the cuboid = $2\,(lb + bh + lh)$ sq. units.
   (iii) Lateral surface area of the cuboid = $[2(l+b) \times h]$ sq. units.
   (iv) Diagonal of the cuboid = $\sqrt{l^2 + b^2 + h^2}$ units
2. **Cube :** If a unit is the length of each edge of a cube, then—
   (i) Volume of cuboid = a^3 cubic units
   (ii) Total surface area of the cube = $6a^2$ sq. units.
   (iii) Lateral surface area of the cube = $4a^2$ sq. units.
   (iv) Diagonal of the cuboid = $\sqrt{3a}$ units.
3. **Standard unit of volume :** The standard unit of volume is 1 cube centimetre, written as 1cu cm or 1 cm^3.

The volume of a cube at side 1*cm* is cm^2 other standard units of volume and their relations are :

$$1000\,mm^3 = 1cm^3$$
$$1000\,cm^3 = 1\,dm^3$$
$$1000\,dm^3 = 1m^3$$

Capacity of a vessel is expressed in litres.

$$1cm^3 = 1ml$$
$$1000cm^3 = 1000ml = 1 \text{ litre}$$
$$1m^3 = 1000 \text{ lit.} = 1kl$$

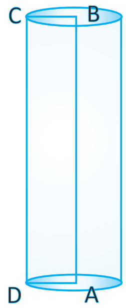


Right circular cylinder : In our daily life we see around us many solids like measuring jars, storage tank, a circular pillar, a garden roller, circular pencil, gas cylinder etc. such solids are right circular cylinder.

A right circular cylinder has two plane ends. Each plane end is circular in shape and the two plane ends are parallel; that is, they lie parallel planes. Each of the plane end is called a base of the cylinder.

An alternative definition of R.C.C. : A solid generated by the revolution of a rectangle about one of its sides is a right circular cylinder.

In the figure rectangle ABCD revolve about its side AB and completes a full round as shown in the figure, AB is called the axis of the cylinder and DA is its radius.



Volume of R.C.C. (Right Circular Cylinder): Condiser a right ciurlar cylinder of raidus r and height h.

We know that the volume of a cuboid = Area of the base × height

$= [(\pi r^2) \times h]$ cubic units

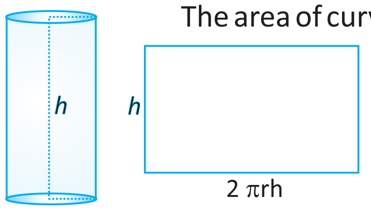
$= [(\pi r^2) \times h]$ cubic units

Surface area of R. C. C. (Right Circular Cylinder) : A right cylinder of radius r and height h is shown in the figure .

Now take a strip of paper of width h.

Wrap the strip around the cylinder, till you reach again. Now cut off the strip. Remove the piece of the strip so cut off and speed it on a plane suface. We will find that strip is a rectangle of length $2\pi r$ *and* breadth h.

∴



The area of curved surface of the cylinder = Area of the rectangle strip of paper.

$= (2\,\pi rh)$ sq. units

Total surface area $= (2\,\pi rh + (2\,\pi r^2)$ sq.

$= (2\,\pi rh + 2\,\pi r^2)$ sq. units

Volume and surface area of follow of hollow cylinder :

Let r_2 and r_1 be the external and internal raidii of a hollow cylinder and h be its height as shown in figure.

We have

(i) Each base Surface Area $= \pi (r_2^2 - r_1^2)$ sq. units

(ii) Curved Surface Area $= 2\,\pi r_1 h + 2\,\pi r_2 h$

$= 2\,\pi h\,(r_1 + r_2)$ sq. units

(iii) Total Surface Area $= 2\,\pi r_1 h + 2\,\pi r_2 h + 2\pi\,(r_2^2 - r_1^2)$

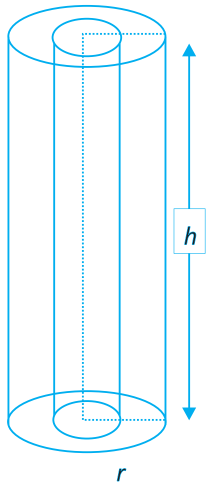
$= 2\,\pi r_1 h + 2\,\pi r_2 h + 2\,\pi\,(r_2 - r_1)(r_2 + r_1)$

$= 2\,\pi\,[(r_1 + r_2)\,h + (r_2 - r_1)(r_2 + r_1)]$ sq. units

or $= 2\,\pi\,[(r_1 + r_2)\,h + (r_2^2 - r_1^2)]$ sq. units

(iv) Volume of the material $= \pi r_2^2 h - \pi r_1^2 h$

$= \pi h\,(r_2^2 - r_1^2)$ cu. units



Illustrative Examples

Example 1 : Find the volume of a right circular cylinder, if the radius (r) of its base and height (h) are 7cm and 15 cm respectively.

Solution : Volume of a cylinder $= \pi r^2 h$

Here r = 7 *cm* and *h* = 15 *cm* $\left[\because \pi = \frac{22}{7}\right]$

Volume of the cylinder $= \frac{22}{7} \times (7)^2 \times 15$ cm^3

$= 22 \times 7 \times 15$ cm^3

$= 2310$ cm^3

Example 2. An iron pipe is 21 *cm* long and its exterior diameter is 8 *cm*. If the thickness of the pipe is 1 *cm* and iron weight is 8g/cm^3, find the weight of the pipe.

Solution :

The external radius of the pipe = 4 *cm*

The internal radius of the pipe = (4 – 1) *cm* = 3 *cm*

$\therefore$ The external volume = $[\frac{22}{7} \times 4 \times 4 \times 21]$ cm^3

= 1056 cm^3

$\therefore$ The internal volume = $\frac{22}{7} \times 3 \times 3 \times 21]$ cm^3

= 594 cm^3

$\therefore$ The volume of the metal = (1056 – 594)cm^3

= 462cm^3

The weight of the pipe = $\frac{462 \times 8}{1000}$ kg. = 3.69 kg.

Example 3 : A rectangle piece of paper of dimensions 22 *cm* by 12 *cm*. is rolled along its length to from a cylinder. Find the volume of the cylinder so formed.

Solution : The height of the cylinder is 12*cm* and the circumference of its base is 22*cm.*

Let r be the radius of the cylinder.

$$\text{Then } 2\pi r = 22 \text{ or } r = \left[22 \times \frac{7}{22 \times 2}\right] = \frac{7}{2}\text{cm}$$

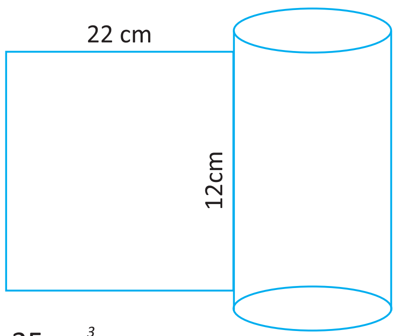
$$r = \frac{7}{2}\ cm \text{ and } h = 12cm$$

So, the volume of the cylinder = $\pi r^2 h$

$$= \left[\frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 12\right] = 462\,\text{cm}^3$$

Example 4 : The thickness of a hollow wooden cylinder is 2*cm.* It is 35*cm* long and its inner radius *is* 12*cm.* Find the volume of the wood required to make the cylinder assuring it is open at either end.

Solution : *We have*



r = inner radius of the cylinder = 12 *cm*

Thickness of the cylinder = 2 *cm*

$\therefore$ R = outer radius of the cylinder = (12 +2) = 14 *cm*

h = height of the cylinder = 35 cm

$\therefore$ Volume of the wood = $\pi(R^2 - r^2)h$

$= \frac{22}{7} \times [(14)^2 - (12)^2] \times 35\ cm^3$

$= \frac{22}{7} \times (14 + 12) \times (14 - 12) \times 35\ cm^3$

$= \left(\frac{22}{7} \times 26 \times 2 \times 35\right)$ cm^3

$= (22 \times 26 \times 2 \times 5)$ cm^3 = 5720cm^3

Example 5 : A cylindrical road roller made of iron is 1*m* wide. Its inner diameter is 54*m* and thickness of the iron sheet rolled in to the road roller is 9 *cm*. Find the weight of the roller if 1 c.c. of iron weights 8 *gm.*

Solution : The width of the road roller is 1 m = 100*cm*

So, Height (length of the cylinder = 100*cm*)

Inner radius of the cylinder = r = 54/2 *cm* = 27 *cm*

Thickness of the iron sheet = 9*cm*

Outer radius of the cylinder = R = (27 + 9) *cm*

= 36*cm*

Thus, volume of the iron sheet used = $(\pi R^2 h - \pi r^2 h)\ cm^3$

$= \pi(R^2 - r^2) \times h\ cm^3$

$= [3.14 \times (36 + 27)(36 - 27)100]\ cm^3$

$= \frac{314}{100} \times 63 \times 9 \times 100\ cm^3$

= 178038 cm3

$= \frac{178038 \times 8}{1000}$ kgs.

= 1424.304kgs.

Example 6 : The radius and height of a cylinder are in the ration 5 : 7 and its volume is 550 cm^3.

Find its radius. (π = 22/7)

Solution : Let the radius of the base and height of the cylinder be 5 *x cm* and 7 *x cm* then,

Volume = 550 cm^3

$\pi r^2 h = 550$

$\frac{22}{7} \times (5x)^2 \times 7x = 550$

$\frac{22}{7} \times 25x^2 \times 7x = 550$

$22\ x 25 x^3 = 550$

$550x^3 = 550$

$x^3 = 1 \quad x = 1\ cm$

Hence, radius of the cylinder = 5*x cm* (5 × 1) *cm* = 5 *cm*.

Example 7 : The volume of a cylinder is 448 πcm^3 and height 7*cm*. Find its lateral surface area and total surface area.

Solution : Let the radius of the base and height of the cylinder be r cm and h cm,

Then, h = 7*cm* (given)

Now, volume = $448\pi\ cm^3$

$\pi r^2 h = 448\pi$

$r^2 = \frac{448}{7} = 64$

$r = 8$ cm

$\therefore$ Lateral surface area $= 2\pi rh\ cm^2$

$= 2 \times \frac{22}{7} \times 8 \times 7\ cm^2 = 352 cm^2$

Total surface area $= (2\pi rh + 2\pi r^2)\ cm^2$

$= 2\pi r(h + r)\ cm^2$

$= 2 \times \frac{22}{7} \times 8 \times (7 + 8)\ cm^2$

$= \frac{5280}{7}\ cm^2 = 754.28\ cm^2$

Example 8 : The diameter of a garden roller is 1.4 m and it is 2 m long. How much area will it cover in 5 revolutions ($\pi = 22/7$) ?

Solution : Area covered = curved surface × No. of revolution

Here, $r = \frac{1.4}{2} = .7$m and h = 2 m

$\therefore$ Curved surface $= 2\pi r\ hm^2 = 2 \times \frac{22}{7} .7 \times 2 = 8.8\ m^2$

Area covered = curved surface × No. of revolution

$= (8.8 \times 5)\ m^2 = 44\ m^2$

Example 9 : How many cubic metres of earth be dug out to sink a well 22.5 m deep and of diameter 7 m? Also, find the must cost of plastering the inner curved surface at ₹ 3 per square metre.

Solution : Volume of earth to be dug out = Volume of the well

$= \left[2 \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 22.5\right] m^3$

$= 866.25\ m^3$

Area of the inner curved surface $= 2\pi rh$

$= \left[2X \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 22.5\right] m^2$

$= 495\ m^2$

Cost of plastering the inner curved surface = ₹ (495 × 3)

= ₹ 1485.

Exercise 16.1

1. The area of the base of a right circular cylinder is 154 cm^2 and its height is 15cm. Find the volume of the cylinder.
2. A closed metallic cylindrical box is 1.25 m high and it has a base whose radius is 35 *cm*. If the sheet of metal costs ₹ 80 per m^2, find the cost of the metal of the box.
3. If the radius of the base of a right circular cylinder is halved keeping the height same what is the ratio of the volume of the reduced cylinder to that of the original.

4. Find the number of coins, 1.5 *cm* in diameter and 0.2 cm thick to be melted to form a right circular cylinder of height 10 *cm* and diameter 4.5 *cm.*

5. A solid iron rectangular block of dimensions 4.4*m, 2.6cm* and 1 m is casted into a hollow cylindrical pipe of internal radius 30 cm and thickness 5 *cm*. Find the length of the pipe.

6. A solid cylinder has total surface area of 462 sq. *cm*. Its curved surface are one-third of its total surface area. Find the volume of the cylinder.

7. A rectangular vessel 22 *cm* by 16 *cm* by 14 *cm* is full of water. If the water is poured into an empty cylindrical vessel of radius of *8cm*. Find the height of water in the cylindrical vessel.

8. The volume of a 1 metre long circular iron rod is 3850 cm^3. Find its diameter.

9. The cost of painting the total outside surface of a closed cylindrical oil tank at 60 paise per sq. dm is ₹ 237.60. The height of the tank is 6 times the radius of the base of the tank. Find the volume correct to two decimal places.

10. Water is flowing at the rate of 3 km/hr through a circular pipe of 20 cm inter hall diameter into a circular cistern of diameter 10 *m* and depth 2 *m*. In how much time will the cistern be filled?

11. An iron pipe 20 *cm* long has exterior diameter equal to 25 *cm*. If the thickness of the pipe is 1 cm. Find the whole surface area of the pipe.

12. A cylindrical tube, open at both ends is made of metal. The internal diameter of the tube is 10.4 cm and its length is 25 *cm*. The thickness of the metal is 8 *mm* every where. Calculate the volume of the metal.

13. Find the thickness of the cylinder. The total surface area of the hollow cylinder which is open from both sides in 4609 sq *cm,* area of base ring is 115.5 sq. cm and height 7 cm.

14. Find the ratio between the total surface area of a cylinder to its curved surface area, given that its height and radius are 7.5 *cm* and 3.5 *cm.*

15. When 1 cubic an of copper weights 8.4 gm. Find the length of 13.2 kg of copper wire of diameter 4*mm.*

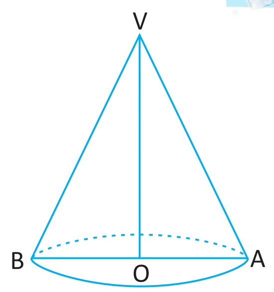
16. Two circular cylinders of equal volumes have their heights in the ratio 1 : 2. Find the ratio of their radii.

17. The inner diameter of a circular well is 3.5 *m*. It is 10 *m* deep. Find the cost of plastering its inner curve at ₹ 4 per sq. meter.

Volume and Surface Area of Right Circular Cone

Introduction :

The formula for the volume and surface area of right circular cone are very useful in our every day life, since we come across conical figures almost on every step. We see around us such as conical tomb, birthday cap, conical vessel etc.



Right circular cone : A right circular cone is a solid generated by revolving a line segment which passes through a fixed point and which makes a constant angle with a fixed line.

In the figure V is a fixed point, VA is the revolving line with VO. VO is a fixed line. When the VA revolves around the fixed line VO such $\angle$OVA remains same in very position of A, A right circular cone is generated.

The fixed point V is called the vertex of the cone.

The fixed line VO is called the axis of the cone.

A right circular cone has a plane end which is in circular shape. This is called the base of the cone.

The length of the line segment joining the vertex to the centre of the base is called the height of the cone. VO is the height of the cone.

The radius OA of the base circle is called the radius of the cone.

Volume of a right circular cone :

Experiment : Take a conical cup of radius r and h. Also take a cylindrical jar of radius r and height h. Fill the cup with water to the brim and transfer the water to the jar, repeat the process two times more. We will find that 3 cup full to brim will fill the jar completely. Thus, we conclude that –

3 (Volume of a cone of radius r and height h)

= (Volume of a cylinder of radius r and height h)

= $\pi r^2 h$ cubic units.

$\therefore$ Volume of a cone of radius r and heigh h = $\left[\frac{1}{3}\pi r^2 h\right]$ cubic units.

Also, Volume of the cone of radius r and h.

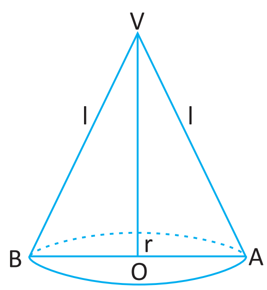
$= \frac{1}{3} \times (\pi r^2 h) \times h$

$= \frac{1}{3} \times$ (Area of the base) $\times$ height

Surface area of a right circular cone :

Experiment : Let the hollow right circular cone of radius r, height h and slant height l as show in figure. The base of the cone is circle of radius r.

Thus,



Length of circular edge = $2\pi r$ and , Area of the plane end = πr^2 cut the cone along the slant height VA and spread out it on a plane surface. You will find that the spread out fig is a sector of a circle of radius equal to the slant height l of the cone and whose arc is equal to the circumference of the base of the cone.

$\therefore$ Curved surface are of the cone

= Area of the sector VAB

$= \frac{1}{2} \times$ (arc length) $\times$ (radius)

$\frac{1}{2} \times 2\pi rl = \pi rl$

The area of the curved surface of a right circular cone of radius r and slant height l is given by,

$$S = \pi rl$$

$$S = \frac{1}{2} \times 2\pi rl$$

Also, $S = \pi rl$

$= (\text{circumference of base}) \times (\text{slant height})$

Total surface area of the cone

= curved surface area + area of the base

$= \pi rl + \pi rl^2 = \pi r(l+r)$

The curved surface area of a cone is also called the lateral surface area.

Illustrative Examples

Example 1 : Find the volume of a right circular cone 1.02 m high, if the radius of its base is 28 *cm*.

Solution : We know that the volume V of a right circular cone of radius r and height h is given by,

$$V = \frac{1}{3}\pi r^2 h$$

Here, r = 28 cm and h = 1.02m = 102 cm

$$\therefore V = \left(\frac{1}{3} \times \frac{22}{7} \times 28 \times 28 \times 10^2\right) cm^3$$

$$= 83776\ cm^3$$

Example 2 : The volume of a cone is 18480 cm^3. If the height of the cone is 40 cm. Find the radius of its base.

Solution : Let the radius of the cone be r cm.

We have h = 40 cm

and V = 18480 cm^3

$$\therefore \frac{1}{3} \times \frac{22}{7} \times r^2 \times 40 = 18480$$

$$r^2 = \frac{18480 \times 3 \times 7}{22 \times 40} = 441$$

$$r = \sqrt{441}\ cm = 21\ cm$$

Example 3 : The base radii of two right circular cones of the same height are in the ratio 3 : 5 find the ratio of their volumes.

Solution : Let r_1 and r_2 be the radii of two cones and v_1 and v_2 be their volumes let h be the height of two cones.

Then $v_1 = \frac{1}{3}\pi r_1^2 h$, $v_2 = \frac{1}{3}\pi r_2^2 h$,

$$\therefore \frac{V_1}{V_2} = \frac{\frac{1}{3}\pi r_1^2 h}{\frac{1}{3}\pi r_2^2 h} = \frac{r_1^2}{r_2^2} = \frac{9}{25} \qquad \therefore = \frac{r_1}{r_2} = \frac{3}{5}$$

Example 4 : A conical tank is 3 m deep and its circular top has radius 1.75m. Find the capacity of the tank in kilometers.

Solution : We have, r = 1.75 m and h = 3 m.

Capacity of the tank $= \frac{1}{3}\pi r^2 h,$

$= \frac{1}{3} \times \frac{22}{7} \times 1.75 \times 1.75 \times 3 \text{ m}^3$

$= 9625 \text{ m}^3$

= 9.625 Kiloliters.

Example 5 : The diameter of a cone is 14 cm and its slant height is 9 cm. Find the area of its curved surface.

We know that the area S of the curved surface of a right circular cone of radius V and slant height l is given by,

Solution : $S = \pi rl$

Here, $r = \frac{14}{2}$ cm = 7 cm and l = 9 cm

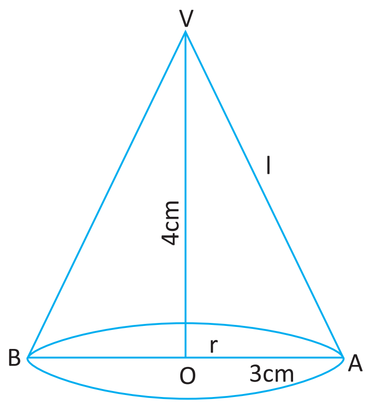
$\therefore \quad s = \frac{22}{7} \times 7 \times 9 \text{ cm}^2 = 198 \text{ cm}^2$

Example 6 : The radius of a cone of radius 3 cm and vertical height is 4 cm. Find the area of the curved surface.

Solution : We have r = 4 cm and h = 3 cm .

Let l cm be the slant height of the cone.

Then,



$l^2 = r^2 + h^2$

$l^2 = 3^2 + 4^2$

$l^2 = 25$

$l = \sqrt{25}$ cm = 5cm

$\therefore$ Area of the curved surface $= \pi rl$

$= \left[\frac{22}{7} \times 4 \times 5\right] \text{cm}^2$

$= 62.85 \text{ cm}^2$

Example 7 : How many lead shots each .03 cm in diameter can be made from a cuboid of dimensions 9 cm × 11cm × 12 cm ?

Solution The volume of the cuboid = $(9 \times 12 \times 11) \text{ cm}^3 = 1188 \text{ cm}^3)$

The radius of cone lead shot = $\left[\frac{0.3}{2}\right]$ cm = 0.15 cm^3

The volume of one lead shot

$= \left[\frac{4}{3} \times \frac{22}{7} \times 0.15 \times 0.15 \times 0.15\right] \text{cm}^3 = \left[\frac{99}{7000}\right] \text{cm}^3$

$\therefore$ The number of lead shots $= \frac{\text{Volume of cuboid}}{\text{Volume of 1 lead shot}}$

$= \left[1188 \times \frac{7000}{99}\right]$

= 84000

Example 8 : The lateral surface of a cylinder is equal to the curved surface of a cone. If the radius be the same, find the ratio of the height of the cylinder and slant height of the cone.

Solution : Let r be the radius of both the cylinder and the cone.

Let h and l respectively be the heights and slant height of the cylinder and the cone.

Then,

Lateral surface of the cylinder = curved surface of the cone.

$$2\pi rh = \pi rl$$

$$2h = l$$

$$\frac{h}{l} = \frac{1}{2}$$

$$h : l = 1 : 2$$

Exercise 16.2

1. **Find the volumes of the cones whose dimensions are -**

   (a) base radius is = 3.5 cm, height = 12 m

   (b) base radius is = 5 dm, height = 10.5 cm

   (c) base radius is = 21 cm height = 15 cm

2. Find the total surface area of a cone, if its slant height is 9 m and the radius of its base is 12 m.

3. The area of the base at a right circular cone is 314 cm^2 and its height is 15 cm. Find the volume of the cone.

4. The radius and height of a cone are in the ratio 4 : 3. The area of the base is 154 cm^3 . Find the area of the curved surface.

5. The volume of a right circular cone is 1232 cm^3. If the radius of its base is 14 cm, find its curved surface.

6. A cone and a cylinder are having the same base. Find the ratio of their heights if their volumes are equal.

7. A right triangle with its sides 5 cm, 12 cm and 13 cm is revolved around the side 12 cm. Find the volume of the solid so formed.

8. A conical vessel whose internal radius is 5 cm and height 24 cm is full of water. The water is emptied into a cylindrical vessel with internal radius 10 cm. Find the heights to which the water rises.

9. A right angled triangle in which the sides containing the right angle are 8 cm and 15 cm in length is curved around on the longer side. Find the volume of the solid thus generated. Also, find the total surface area of the solid so formed.

10. A tent is in the form of a right circular cylinder surmounted by a cone. The diameter of the cylinder is 24 meters. The height of the cylindrical portion is 14 metres, while the vertex of the cone is 19 metres above the ground. Find the area of the canvas required for the tent.

11. The cylinder is within the cube touching all the vertical faces. A cone is inside the cylinder. If their heights are same with the same base, find the ratio of their volumes.

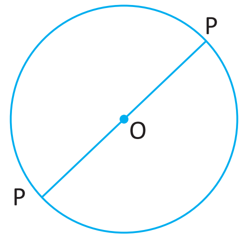
12. If the height of the cone is doubled and the radius of the base is kept the same as before, find the change in volume.

Volume and Surface Area of A Sphere

Introduction :

We shall learn in this section about volume and surface area of a sphere, hemisphere and spherical shell. The objects which are in the shape of a ball are known to have the shape of a sphere. We shall learn about these and their formula will be applied to solved some problems.

Definitions :



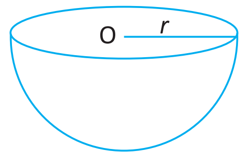
Sphere

Sphere : The set of all points in space which are equidistant from a fixed point, is called a sphere.

The fixed point is called its centre and the constant distance is called its radius.

In this fig. O is centre of sphere and POP is diameter and OP is radius of sphere. Intersects the sphere at a point P such that as,

OP = OP



Hemisphere

Hemisphere : A plane through the centre of a sphere divides the sphere into two equal parts, each of which is called a hemisphere.

Spherical shell : The difference of two solid concentric spheres is called a spherical shell.

A spherical shell has a finite thickness, which is the difference of the radii of the two solid spheres which determine it.

Volume of a sphere :

(i) The volume V of a sphere of radius r is given by,

$$V = \frac{4}{3}\pi r^3 \text{ cubic units.}$$

(ii) The volume V of a hemisphere of radius r is given by,

$$V = \frac{2}{3}\pi r^3 \text{ cubic units.}$$

(iii) Curved surface area of hemisphere

$$= (2\pi r^2) \text{ sq. units}$$

(iv) Total surface area of hemisphere

$$= (2\pi r^2 + \pi r^2)$$

$$= (3\pi r^2) \text{ sq. units}$$

(v) The volume v of a spherical shell whose outer and inner radii are R and r respectively is given by,

$$V = \frac{4}{3}\pi (R^3 - r^3) \text{ cubic units.}$$

Surface area of sphere :

We state the following formulas without proof:

(i) Surface areas of a sphere of radius r is given by,

$$s = 4\pi r^2 \text{ sq. units}$$

(ii) Curved surface area of a hemisphere of radius r is given by,

$$s = 2\pi r^2 \text{ sq. units}$$

(iii) Total surface area of a hemisphere of radius r is given by,

$$r = 2\pi r^2 + \pi r^2$$
$$= 3\pi r^2 \text{sq.units}$$

(iv) If R and r are outer and inner radii of a hemisphere shell then outer surface area = $4\pi r^2$ sq. units.

Illustrative Examples

Example 1 : Find the volume of sphere of radius 7 *cm*.

Solution : The volume of sphere = $V = \frac{4}{3}\pi r^3$ cubic units

Here, $r = 7cm$

$\therefore$ $V = \frac{4}{3} \times \frac{22}{7} \times 7 \times 7 \times 7 \text{ cm}^3$

$= 1437.33\, cm^3$

Example 2 : The curved surface area of a square is 1386 cm^2. Find its volume.

Solution : We know that the curved surface area of a sphere = $4\pi r^2 cm^2$

$\therefore$ $4\pi r^2 = 1386$

$$4 \times \frac{22}{7} \times r^2 = 1386$$

$$r^2 = \left[1386 \times \frac{7}{22} \times \frac{1}{4}\right] = \frac{441}{4}$$

$$r = \sqrt{\frac{441}{4}} = \sqrt{\frac{21 \times 21}{2 \times 2}} = \frac{21}{2}$$

$\therefore$ The volume of the sphere $= \frac{4}{2}\pi r^3$

$$= \left[\frac{4}{3} \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} \times \frac{21}{2}\right] \text{cm}^3$$

$$= 4851\, cm^3$$

Example 3 : A hemisphere bowl is made of steel sheet 0.5*cm* thick. The inside radius of the bowl is 4 cm. Find the volume of steel used in making the bowl.

Solution : We have $r = 4\,cm$ $\quad R = (4+0.5)\,cm = 4.5\,cm.$

Volume of the inner hemisphere $= \frac{2}{3}\pi r^3$

$= \left[\frac{2}{3}\times\frac{22}{7}\times 4\times 4\times 4\right]\text{cm}^3$

Volume of the outer hemisphere $= \frac{2}{3}\pi R^3$

$= \left[\frac{2}{3}\times\frac{22}{7}\times 4.5\times 4.5\times 4.5\right]\text{cm}^3$

$\therefore$ Volume of steel used $= \left[\frac{2}{3}\times\frac{22}{7}\times 4.5\times 4.5\times 4.5 - \frac{2}{3}\times\frac{22}{7}\times 4\times 4\times 4\right]\text{cm}^3$

$= \frac{2}{3}\times\frac{22}{7}\times\left[(4.5)^3 - (4)^3\right]\text{cm}^3$

$= \frac{214}{21}\times(91.125 - 64)\,\text{cm}^3$

$= \frac{44}{21}\times 27.125\,\text{cm}^3$

$= 56.83\,cm^3$

Example 4 : How many spherical bullets can be made out of a solid cube of lead whose edge measures 44 cm, each bullet being 4 cm in diameter.

Solution : Total bullets be x,

Radius of a spherical bullet $= \frac{4}{2}\,cm = 2\,cm$

Volume of a spherical bullet $= \frac{4}{3}\pi\times(2)^3\,cm^3$

$= \left[\frac{4}{3}\times\frac{22}{7}\times 8\right]\text{cm}^3$

Volume of x spherical bullets $= \left[\frac{4}{3}\times\frac{22}{7}\times 8\times x\right]\text{cm}^3$

Volume of the solid cube $= (44)^3 cm^3$

Volume of x spherical bullets = Volume of cube

$= \frac{4}{3}\times\frac{22}{7}\times 8\times x = (44)^3$

$= \frac{4}{3}\times\frac{22}{7}\times 8\times x = 44\times 44\times 44$

$x = \frac{44\times 44\times 44\times 3\times 7}{4\times 22\times 8}$

$= 2541\ cm^3$

Example 5 : Find the surface area of a sphere of radius 7 *cm.*

Solution : The surface area of a sphere

$$s = 4\pi r^2$$

$$\because \quad r = 7\,cm$$

$$\therefore \quad s = \left[4 \times \frac{22}{7} \times 7 \times 7\right] cm^2$$

$$= 616\,cm^2$$

Example 6 : Find the volume of a sphere whose surface area is 154 sq cm.

Solution : Surface area $= 154\,cm^2$

$$s = 4\pi r^2 = 154$$

$$4 \times \frac{22}{7} \times r^2 = 154$$

$$r^2 = \frac{154 \times 7}{4 \times 22} = \frac{49}{4} \qquad r = \sqrt{\frac{49}{4}} = \frac{7}{2}\,cm$$

Let V be the volume of the sphere. Then,

$$V = \frac{4}{3}\pi r^3$$

$$V = \left[\frac{4}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times \frac{7}{2}\right] cm^3$$

$$V = \left[\frac{1}{3} \times 11 \times 7 \times 7\right] cm^3$$

$$V = 179.66\,cm^3$$

Example 7 : A sphere, a cylinder and a cone are of the same radius and same height. Find the ratio of their curved surfaces.

Solution : r be the common radius of a sphere a cone and a cylinder.

Then height of the cone = height of the cylinder = height of the sphere = 2 r.

l be the slant height of the cone.

$$S_0 = l = \sqrt{r^2 + h^2} \qquad l = \sqrt{r^2 + 4r^2} = \sqrt{5r^2}$$

$$S_1 = 4\pi r^2$$

$$S_2 = 2\pi r \times 2r = 4\pi r^2$$

$$S_3 = \pi r l = \pi r \times \sqrt{5r} = \pi\sqrt{5r^2} = \pi r\sqrt{5}$$

$$\therefore \quad S_1 = S_2 = S_3 = 4\pi r^2 : 4\pi r^2 : \pi r\sqrt{5}$$

$$= 4 : 4 : \pi r\sqrt{5}$$

Example 8 : Show that the surface area of a sphere is the same as that of the lateral surface of a right circular cylinder that just enclosed the sphere.

Solution : The radius of sphere be $r\ cm$. Surface area of the cylinder

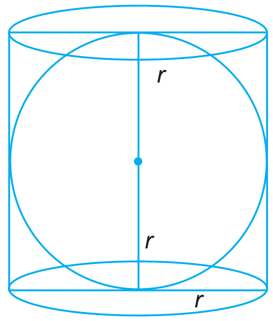
$$= 4\pi r^2 cm^2$$

The radius and height of a circular cylinder that just enclosed the sphere of radius r are r and $2r$ respectively.

$$\text{Surface area of the cylinder} = 2\pi r \times 2r$$

$$\text{We obtain that} = 4\pi r^2 cm^2$$

The surface area of the sphere is equal to the surface area of the cylinder that just encloses the sphere.



Exercise 16.3

1. **Find the volume of a sphere whose radius is:**

   (a) 3.5 *cm* (b) 10.5 *cm* (c) 4 *cm*

2. **Find the total surface area of hemisphere whose radius is:**

   (a) 21 *cm* (b) 2.8 *cm* (c) 6.3 *cm*

3. Find the volume of hemisphere of radius is 3.5 *cm.*
4. Find the surface area and total surface area of a hemisphere of radius 21*cm*.
5. A shopkeeper has one laddoo of radius 5 *cm*. With the same material, how many laddoos of radius 2.5 *cm* can be made.
6. A solid sphere of radius 3 *cm* is melted and then cast into small spherical balls each of diameter 0.6 *cm*. Find the number of balls thus obtained.
7. A cone and a hemisphere have equal bases and equal volumes. Find the ratio of their heights.
8. A spherical canon ball, 28 *cm* in diameter is melted and cast into a right circular conical shape, the base of which is 35 *cm* in diameter . Find the height of the cone, correct to one place of decimal.
9. A cylindrical jar of radius 6 *cm* contains oil. Iron spheres each of radius 1.5 *cm* are immersed in the oil. How many sphere are necessary to raise the level of the oil by two centimeters?
10. The volume of two spheres are in the ratio 64 : 27. Find the difference of their surface areas, if the sum of their radii is 7.
11. The surface area of a sphere is $452\frac{4}{7}$ cm^2. What is its volume?
12. The surface area of a sphere is 5544 cm^2. Find the diameter.
13. The radii of a spherical balloon in creases from 7 *cm* to 14 *cm*. Compare the surface areas of the balloon in the above two cases.

14. The internal and external diameter of a hollow hemispherical vessel are 24 *cm* and 25 *cm* respectively. The cost to point one sq. cm of the surface is 7 paise. Find the total cost to paint the vessel all over.

15. The diameter of a copper sphere is 6*cm*. It is beaten and drawn into the wire of diameter 0.2 *cm*. Find the length of the wire.

Points to Remember :

- The solid sphere with centre O and radius r is the region in space enclosed by the sphere.
- A plane through the centre of a sphere divides it into two equal parts each of which is called a hemisphere.
- The space occupied by a solid body is called its volume.
- (i) Volume of a cuboid $=(l \times b \times h)$ cubic units.
  (ii) Total surface area of a cuboid $= 2\,(lb + bh + lh)$ sq. units.
  (iii) Lateral surface area of a cuboid $= [2\,(l + b) \times h)]$ sq.units
  (iv) Diagonal of a cuboid $\sqrt{l^2 + b^2 + h^2}$ units.
- (i) Volume of a cube $= a^3$ cubic units.
  (ii) Total surface area of a cube $= (6a^2)$ sq. units.
  (iii) Lateral surface area of a cube $= (4a^2)$ sq. units.
  (iv) Diagonal of a cube $= \sqrt{3a}$ units
- For a cone of heigth, base radius *r* and slant height *I*, we have
  (i) $l^2 = (h^2 + r^2)$
  (ii) Volume of the cone $= (\frac{1}{3}\pi r^2 h)$ cubic units.
  (iii) Area of curved surface of the cone $= (\pi rl)$ sq. units.
  (iv) Total surface area of a cone $= \pi r\,(l + r)$ sq. units.
- For a sphere of radius *r*, we have
  (i) Volume of a sphere $= \left[\frac{4}{3}\pi r^3\right]$ cubic units.
  (ii) Surface area of the sphere $= (4\,\pi r^2)$ sq. units.
- For a hemisphere of radius *r*, we have
  (i) Volume of the hemisphere $= \left[\frac{2}{3}\pi r^3\right]$ cubic units.
  (ii) Curved surface area of the hemisphere $= (2\pi r^2)$ sq. units.
  (iii) Total surface area of the hemisphere $= (3\pi r^2)$ sq. units

EXERCISE

1. **MULTIPLE CHOICE QUESTIONS (MCQs) :**

   Tick (✓) the correct options.

   (a) Total surface area of the cube is ....

   (i) 6a sq. unit ☐ (ii) $6a^2$ sq. unit ☐ (iii) $6a^2$ unit ☐ (iv) 6a sq. unit ☐

   (b) Volume of right circular cylender is ....

   (i) πrh ☐ (ii) πrh^2 ☐ (iii) r^2h ☐ (iv) πr^2h ☐

   (c) Area of the curved surface of a right circular cone is ....

   (i) $\frac{1}{2}\pi rl$ ☐ (ii) πrl ☐ (iii) πr^2l ☐ (iv) $\frac{1}{3}\pi rl$ ☐

   (d) Volume of a sphere of radius r is given by ....

   (i) $\frac{4}{3}\pi r^3$ ☐ (ii) $\frac{2}{3}\pi r^3$ ☐ (iii) $\frac{4}{4}\pi r^2$ ☐ (iv) $2\pi r^2$ ☐

   (e) What is the surface area of a sphere of radius of 7 cm?

   (i) 618 cm^2 ☐ (ii) 516 cm^2 ☐ (iii) 720 cm^2 ☐ (iv) 616 cm^2 ☐

   (f) What is the slant height (l) of a cone if radius (r) = 3 cm, vertical height (h) = 4 cm?

   (i) 5 cm ☐ (ii) 25 cm ☐ (iii) 9 cm ☐ (iv) 16 cm ☐

2. Water is flowing at the rate of 3 km/hr through a circular pipe of 20 cm inter hall diameter into a circular cistern of diameter 10 *m* and depth 2 *m*. In how much time will the cistern be filled?
3. An iron pipe 20 *cm* long has exterior diameter equal to 25 *cm*. If the thickness of the pipe is 1 cm., find the whole surface area of the pipe.
4. A cylindrical tube, open at both ends is made of metal. The internal diameter of the tube is 10.4 cm and its length is 25 *cm*. The thickness of the metal is 8 *mm* every where. Calculate the volume of the metal.
5. **Find the volumes of the cones whose dimensions are -**

   (a) base radius is = 3.5 cm, height = 12 m

   (b) base radius is = 5 dm, height = 10.5 cm

   (c) base radius is = 21 cm height = 15 cm
6. A conical vessel whose internal radius is 5 cm and height 24 cm is full of water. The water is emptied into a cylindrical vessel with internal radius 10 cm. Find the heights to which the water rises.
7. A right triangle with its sides 5 cm, 12 cm and 13 cm is revolved around the side 12 cm. Find the volume of the solid so formed.

HOTS

1. What is the lateral surface area of a cuboid where length, breadth and height are 2a, 2b and 2c respectively?
2. What is the volume of a cylinder whose radius adn height is 1 unit each ?

Lab Activity

Objective : To obtain the formula of the surface area of a cylinder.

Materials Required : A closed right circular cylinder, drawing sheet, a pair of scissors, ruler, fevistic.

Procedure :

Step - 1 : Remove the top and the bottom of the circular cylinder [see fig. (a)]. These two circles are same radii.

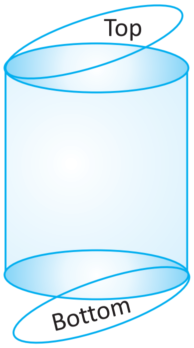


Fig. (a)

Step - 2 : Cut the curved portion of the cylinder vertically as shown in Fig. (b) and paste it on a drawing sheet. You obtain a rectangle.

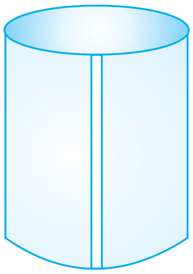
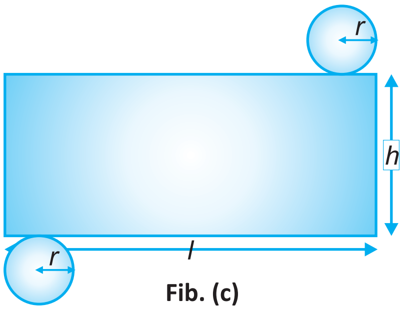


Fig. (b)

Step - 3 : Paste the two circles obtained in step (1) also as shown in Fig (c).



Fib. (c)

Step - 4 : Measure the radius of one of the circles. Let us write it as r units.

Step - 5 : Measure the length and breadth of the rectangle. Let us write these as l and h respectively.

Step - 6 : But l = circumference of the base of cylinder = $2\pi r$.

So curved surface area of cylinder = area of rectangle = $l \times h$

Area of each circular region = πr^2

Total surface area of cylinder = curved surface area + area of top + area of bottom

= $2\pi rh + \pi r^2 + \pi r^2$

= $2\pi rh + 2\pi r^2$

= $2\pi r\,(h+r)$

17 Statistics

Introduction

Statistics is a very broad subject, with applications in a vast number of different fields. In generally one can say that statistics is the methodology for collecting, analyzing, interpreting and drawing conclusions from information. It is a set of concepts, rules, and procedures that help us to:

- Organize numerical information in the form of tables, graphs and charts thus organizing and tabulating data.
- Understand statistical techniques underlying decisions that affect our lives and well-being.
- Make informed decisions.

We need information in the form of numerical figures in various fields. Each numerical figure is called observation and the collection of all observations is called the data.

Collection of observations is the first step in statistical investigations.

Raw data:

A collection of observations gathered initially is called raw data.

For example : Look at the following list of marks (out of 100) scored by 30 students of class VIII in a test:

55, 65, 15, 40, 35, 70, 90, 92, 84, 85
70, 75, 65, 72, 80, 78, 64, 88, 78, 76
55, 54, 52, 72, 70, 90, 85, 75, 65, 80

Data:

After collection of data, the investigator has to find ways to condense them in tabular form in order to study their salient features such an arrangement is called presentation of data.

Let the marks obtained by 30 students of class VIII in a class test, out of 50 marks according to their roll numbers be.

39, 25, 5, 33, 19, 21, 12, 41, 12, 21, 19, 1, 10, 8, 12, 17, 19, 17, 17, 41, 40, 12, 41, 33, 19, 21, 33, 5, 1, 21

The data in this form are called raw data or ungrouped data. The above raw data can be arranged in serial order as follow:

Roll no.	Marks	Roll no.	Marks	Roll no.	Marks
1.	39	11.	19	21.	40
2.	25	12.	1	22.	12
3.	5	13.	10	23.	41
4.	33	14.	8	24.	33
5.	19	15.	12	25.	19
6.	21	16.	17	26.	21
7.	12	17.	19	27.	33
8.	41	18.	17	28.	5
9.	12	19.	17	29.	1
10.	21	20.	41	30.	21

The raw data when arranged in ascending or descending order of magnitude is called an array or arrayed data.

Arithmetic mean:

Arithmetic mean or simply the mean of some given observations is defined as,

$$\text{Mean} = \left[\frac{\text{Sum of the given observations}}{\text{Number of these observations}}\right]$$

Illustrative examples:

Example1: Given below are marks (out of 100) obtained by 20 students of a class in mathematics in an annual examination:

23, 75, 56, 42, 70, 84, 92, 51, 40, 63

87, 58, 35, 80, 14, 63, 49, 72, 66, 61; find

(i) The lowest marks obtained

(ii) The highest marks obtained

(iii) The range of the given data

Solution: Arrange the above data in an ascending order, we get,

14, 23, 35, 40, 42, 49, 51, 56, 58, 61

63, 63, 66, 70, 72, 75, 80, 84, 87, 92

From the above data, we make the following observations.

(i) Lowest mark s obtained = 14

(ii) Highest marks obtained = 92

(iii) Range of the given data = (92 – 14) = 78

Example 2: The mean of 15 observations was found to be 34. Later on, it was detected that an observation 32 was misread as 23. Find the correct mean of the given observations.

Solution: Calculated mean of 15 observations = 34

Sum of all these observations = (34 ´ 15) = 510

In these observations, 32 was misread as 23

Correct sum of these 15 observation = 510 – 23 + 32 = 519

Hence, correct mean = $\left[\frac{519}{15}\right]$ = 34.6

Example 3: There are 40 boys in a class. The mean height of 25 of them is 158 cm. if the mean height of the remaining boys is 154 cm. Find the mean height of the whole class.

Solution:

Mean height of 25 boys = 158 cm

Sum of the height of 25 boys = (158 ´ 25) cm

= 3950 cm

Remaining number of boys = (40 – 25) = 15

Mean height of 15 boys = 154 cm

Sum of the heights of these 15 boys = (154 ´ 15) cm

= 2310 cm

Sum of the heights of 40 boys = (3950 + 2310) cm

= 6260 cm

Mean height of the whole class = $\left[\frac{6260}{40}\right]$ cm = 156.5 cm.

Hence, the mean height of the whole class is 156.5 cm.

Exercise 17.1

1. Find the mean of the first ten natural numbers.
2. Find the mean of the first ten prime numbers.
3. Find the mean of all factors of 10.
4. Find the mean of first 10 even natural numbers.
5. **The marks obtained (out of 50) by 12 students in an examination are given below.**

   40, 32, 10, 44, 23, 35, 21, 36, 12, 15, 26, 24

   (a) Find the mean marks.

   (b) Find the range.
6. **The weights of new born babies (in kg) in a hospital on a particular day are as follows:**

   2.3, 2.2, 2.1, 2.7, 2.6, 3.0, 2.5, 2.9, 2.8, 3.1, 2.5, 2.8, 2.7, 2.9, 2.4

   (a) Rearrange the weights in descending order.

   (b) Determine the highest weight.

   (c) Determine the range.

   (d) How many babies weight more than 2.8 kg?
7. **Given below are the heights (in cm) of 11 boys of a class.**

   146, 143, 148, 132, 128, 139, 140, 152, 154, 142, 149

   Arrange the above data in an ascending order and find.

   (a) The height of the tallest boy.

   (b) The height of the shortest boy.

   (c) The range of the given data.

   (d) The mean height.
8. The mean of six numbers is 29. If one of the numbers is excluded, the mean of the remaining numbers becomes 31. Find the excluded number.
9. The mean of the five observations is 17. If the mean of the first three of these observations is 15 and that of the last three is 18. Find the third observation.
10. The mean of 75 numbers is 35. If each number is multiplied by 4, find the new mean.
11. The mean of five numbers is 27. If one number is excluded, their mean is 25. Find the excluded number.
12. The mean of 8 numbers is 15. If each number multiplied by 2. What will be the new mean?

Distribution

Frequency table is a method to present raw data in a form which one case easily understand the information contained in the raw data.

Frequency distributions are of two types:

(i) Discrete frequency distribution

(ii) Continous or grouped frequency distribution

Illustrative examples:

Example 1: Find the mean wages of 60 workers in a factory from the following frequency distribution table.

Wages in rupees	Frequency
800	25
850	10
900	12
950	8
1000	5
Total	60

Solution: We may calculate the mean as given below.

Wages (in rupees) x	Frequency f	fx		
800	25	25×800	=	20000
850	10	10×850	=	8500
900	12	12×900	=	10800
950	8	8×950	=	7600
1000	5	5×1000	=	5000
Total	60			51900

Example 2: The following data given marks out of 40, obtained by 30 students of a class in a test.

40, 12, 37, 17, 27, 30, 6, 2, 23, 19, 39, 25, 5, 33, 25, 5, 33, 19, 21, 12, 17, 19, 17, 12, 8, 10, 1, 9, 21, 13,

Arrange them in ascending order and present it as grouped data.

Solution: By arranging the marks in ascending order, we get:

1, 2, 5, 5, 6, 8, 9, 10, 12, 12, 12, 13, 17, 17, 17, 19, 19, 19, 21, 21, 23, 25, 25, 27, 30, 33, 33, 37, 39, 40

Marks	Number of students (Frequency)
1 - 10	8
11-20	10
21-30	7
31-40	5

Exercise 17.2

1. The number of members in 20 families are given below:

   4, 6, 5, 5, 4, 6, 3, 3, 5, 5, 3, 5, 4, 6, 7, 3, 5, 5, 7

   Prepare a frequency distribution data.

2. The following data gives the number of children in 40 families.

   1, 2, 6, 5, 1, 5, 1, 3, 2, 6, 2, 3, 4, 2, 0, 4, 4, 3, 2, 2, 0, 0, 1, 2, 2, 4, 3, 2, 1, 0, 5, 1, 2, 4, 3, 4, 1, 6, 2, 2

   Represent it in the form of a frequency distribution data.

3. Construct a frequency table for the following ages (in years) of 30 students using equal class intervals, one of them being 9-12, where 12 is not included.

   18, 12, 7, 6, 11, 15, 21, 9, 8, 13, 15, 17, 22, 19, 14, 21, 23, 8, 12, 17, 15, 6, 118, 23, 22, 16, 9, 21, 11, 16.

4. The marks scored by 40 students of class VIII in mathematics are given below:

   81, 55, 54, 73, 47, 35, 54, 38, 68, 52, 54, 45, 70, 83, 43, 54, 62, 64, 72, 92, 84, 76, 63, 43, 45, 26, 29, 68, 54, 73, 77, 50, 64, 35, 79, 64, 62, 72, 70, 54

   Prepare a frequency distribution table.

5. The monthly wages of 30 workers in a factory are given below.

   840, 851, 890, 885, 878, 840, 890, 833, 836, 848, 896, 804, 808, 890, 810, 869, 845, 820, 832, 812, 890, 868, 806, 840, 840, 810, 830, 835, 806.Prepare frequency distribution table.

6. The weights (in grams) of 40 oranges picked at random from a basket are as follows.

   50, 40, 65, 60, 55, 45, 30, 90, 85, 70, 75, 82, 85, 110, 70, 55, 35, 30, 35, 55, 75, 40, 100, 40, 110, 35, 45, , 84, 35.

   Construct a frequency table.

7. Find the mean weight of 50 boys from the following data:

Weight (in kg)	50	52	54	56	60
Number of boys	6	8	15	14	7

8. The marks scored by 20 students in a test are given below.

   54, 42, 68, 56, 62, 71, 78, 51, 72, 53, 44, 58, 47, 64, 41, 57, 89, 53, 84, 57.

   Complete the frequency table and find the greatest frequency.

9. **The heights of 25 girls were measured and recorded as given below find the mean:**

Height (in cm)	135	140	145	150	155	160
Number of girls	6	5	8	3	2	1

Graphical Representation of Data

Examples 1: The following table shows the expenditure in percentage incurred on the construction of a house in a city:

Item	Brick	Cement	Steel	Labour	Miscellaneous
Expenditure (in percentage)	15%	20%	10%	25%	30%

Represent the above data by a pie chart.

Solution: Total percentage = 100.

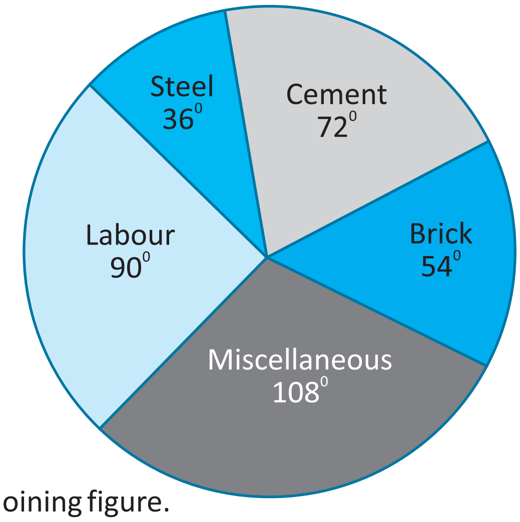
Center angle for a component = $\left(\frac{\text{value of the component}}{100} \times 360^{0}\right)$

Calculation of central angles

Item	Expenditure (in percentage)	Central angle
Brick	15%	$(15/100x360^{0} = 54^{0}$
Cement	20%	$(20/100x360^{0} = 72^{0}$
Steel	10%	$(10/100x360^{0} = 36^{0}$
Labour	25%	$(25/100x360^{0} = 90^{0}$
Miscellaneous	30%	$(30/100x360^{0} = 108^{0}$

Steps of construction:

1. Draw a circle of any convenient radius.
2. Draw a horizontal radius of the circle.
3. Draw sectors starting from the horizontal radius with central angles of 54^{0}, 72^{0}, 36^{0}, 90^{0} and 108^{0} respectively.
4. Shade the sectors differently using different colors and label them.



Thus, we obtain the required pie chart, shown in the adjoining figure.

Examples 2: The following data represent the favourite type of movie of a group of friends. Represent the data into a pie chart.

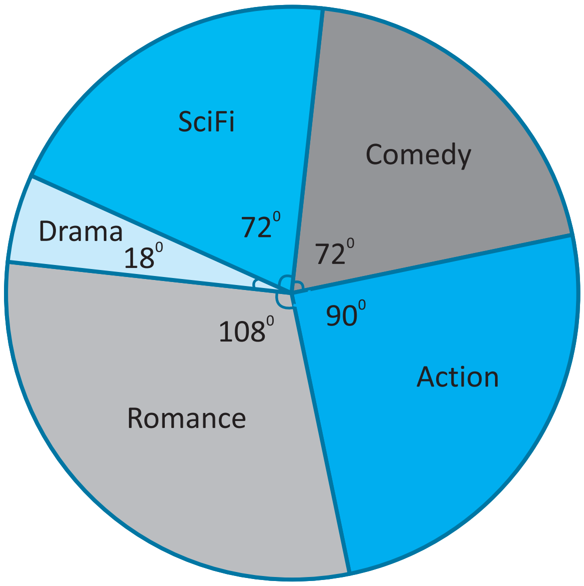
Favourite Type of Movie				
Comedy	**Action**	**Romance**	**Drama**	**SciFi**
4	5	6	1	4

Solution:

Comedy	Action	Romance	Drama	SciFi	TOTAL
4	5	6	1	4	20
4/20 = 20%	5/20 = 25%	6/20 = 30%	1/20 = 5%	4/20 = 20%	100%

Now you need to figure out how many degrees for each "pie slice" (correctly called a sector).
A Full Circle has 360 degrees, so we do this calculation:

Comedy	Action	Romance	Drama	SciFi	TOTAL
4	5	6	1	4	20
4/20 = 20%	5/20 = 25%	6/20 = 30%	1/20 = 5%	4/20 = 20%	100%
4/20 = 360^0 72^0	5/20 = 360^0 90^0	6/20 = 360^0 108^0	1/20 = 360^0 18^0	4/20 = 360^0 72^0	360^0



Exercise 17.3

1. Given below is the frequency distribution of the heights of 50 students of a class. Draw a histogram.

Class interval	140-145	145-150	150-155	155-160	160-165
Frequncy	7	13	19	10	6

2. Draw a histogram of the following data.

Class interval	10-15	15-20	20-25	25-30	30-35	35-45
Frequency	40	92	90	52	28	59

3. The following table shows the number of illiterate persons in the age group (10-15 years) in a town. Draw a histogram.

Age group (in years)	10-16	16-22	22-28	28-34	34-40	40-46
Number of persons	175	325	100	150	250	400

4. In a study of diabetic patients in a village, the following observations were noted. Draw a histogram.

Age in Years	10-20	20-30	30-40	40-50	50-60	60-70
Number of patients	90	40	60	20	120	30

5. Draw a histogram for the following data.

Class interval	20-25	25-30	30-35	35-40	40-45	45-50
Frequency	30	24	52	28	46	10

6. Draw a histogram for the following frequency distribution.

Class interval	101-150	151-200	201-250	251-300	301-350
Frequency	28	12	15	45	46

7. Number of work shops organized by a school in different areas during the last five years is as follows.

Years	95-96	96-97	97-98	98-99	99-2000
No. of workshops	26	35	43	54	62

Draw a histogram representing the above data.

8. Construct a histogram for the following data.

Monthly School fee (in Rupees)	30-60	60-90	90-120	120-150	150-180	180-210
No. of Schools	6	15	13	19	9	3

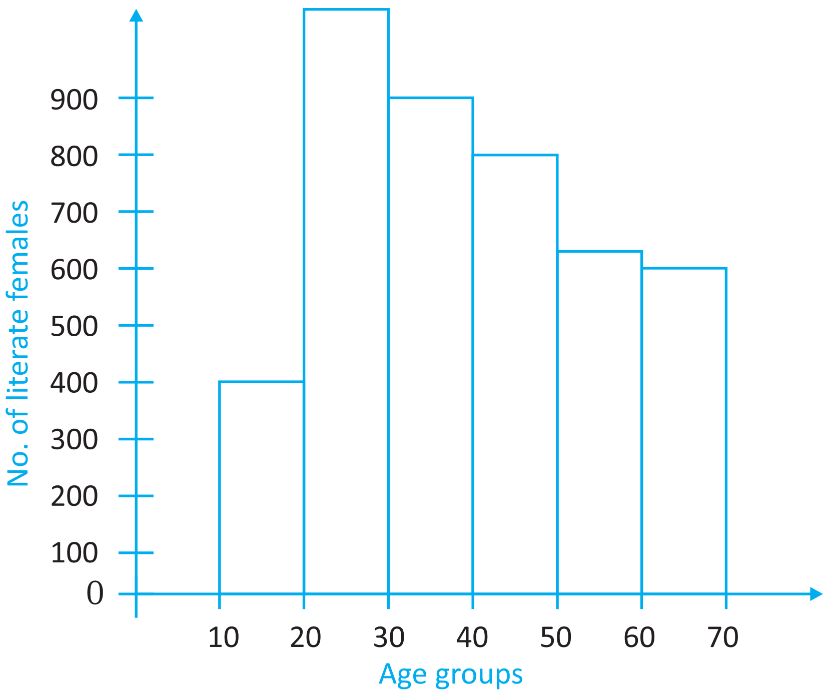
9. **The following histogram shows the number of literate females in the age group 10 to 70 years in the town.**

(a) Write the age group in which the number of literate females is lowest?

(b) In which age group literate females are highest?

(c) What are the class marks of the classes?

(d) What is the width of the class?

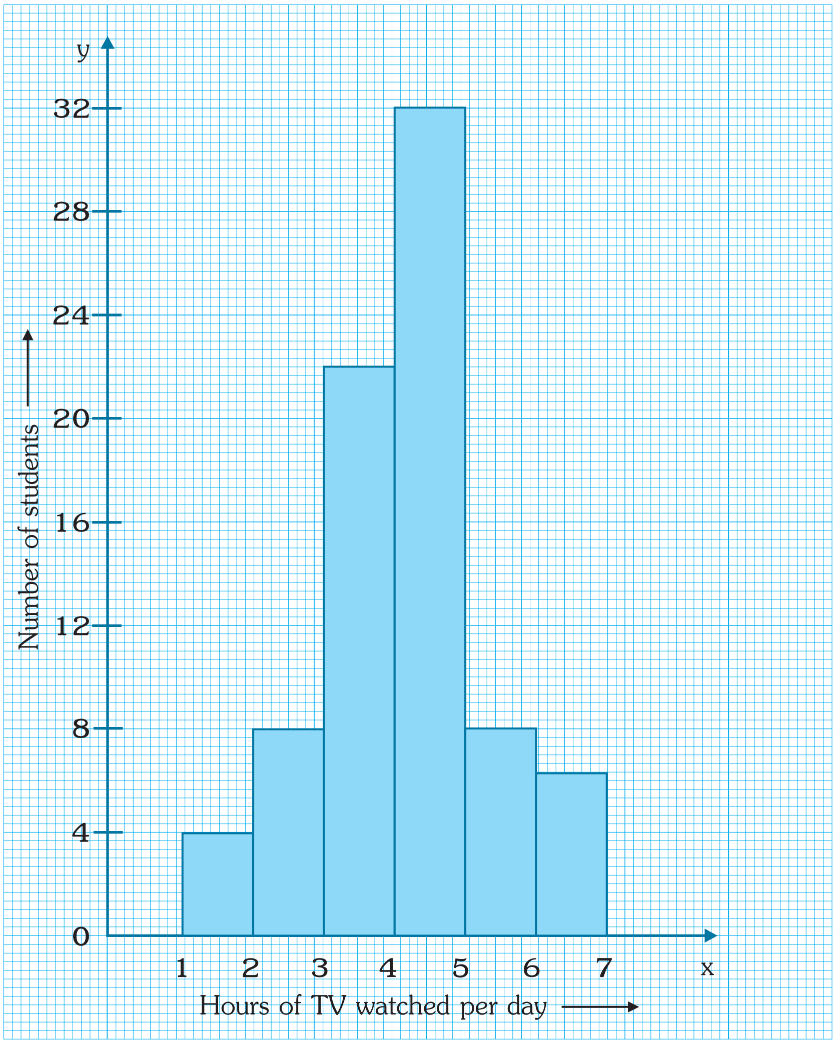


10. **The number of hours for which students of a particular class watched television during holidays, is shown though the given graph :**

(a) For how many hours did the maximum number of students watch TV?

(b) How many students watched TV for less than 4 hours?

(c) How many students spent more than 5 hours in watching TV?



Points to Remember :

- Each numerical figure is called an observation and the collection of all observations is called the data.
- The raw data can be arranged is any one of the following ways:
  (i) Serial order
  (ii) Ascending order
  (iii) Descending order
- The raw data when put in ascending or descending order of magnitude is called an array or arrayed data.
- The number of lines an observation occurs in the given data, is called the frequency of the observation.
- Frequency distributions are of two types:
  (i) Discrete frequency distribution
  (ii) Continuous or grouped frequency distribution.
- Frequency table is a method to present raw data in the form of a table showing the frequency of various observations.
- The difference between upper limit and lower limit of a class interval is called the class size.
- The middle value of a class interval is called its class mark.
- A frequency histogram is a graphical representation of a frequency distribution in the form of rectangles with class intervals as basses and heights propotional to corresponding frequencies there is no gap between any two successive rectangle.

EXERCISE

1. MULTIPLE CHOICE QUESTIONS (MCQs) :

Tick (✓) the correct options.

(a) Collection of numerical values of unorganised from is called—

(i) raw data ☐ (ii) frequency ☐ (iii) statistics ☐ (iv) grouped data ☐

(b) Data can be represented pictorially graphically by—

(i) histogram ☐ (ii) Pie chart ☐ (iii) bar graph ☐ (iv) all the these ☐

(c) The range of the data 64, 67, 57, 60, 59, 71 is—

(i) 71 ☐ (ii) 14 ☐ (iii) 57 ☐ (iv) 128 ☐

(d) There are ...... limits in each class.

(i) 4 ☐ (ii) 3 ☐ (iii) 2 ☐ (iv) 16 ☐

(e) The middle value of a class interval is called its—

(i) limit ☐ (ii) frequency ☐ (iii) class mark ☐ (iv) none of these ☐

(f) Histogram is a pictorial representation of the granted data in the form of—

(i) circles ☐ (ii) rectangles ☐ (iii) tangents ☐ (iv) diameters ☐

(g) In a pie chart, the data are represented in a circle by

(i) sectors ☐ (ii) chords ☐ (iii) tangents ☐ (iv) diameters ☐

2. Find the lower class limit and upper class limit for the following.

(a) 65 – 70 ☐ (b) 110–125 ☐ (c) 0–10 ☐ (d) 15–25 ☐

3. Find the size of the class in the following.

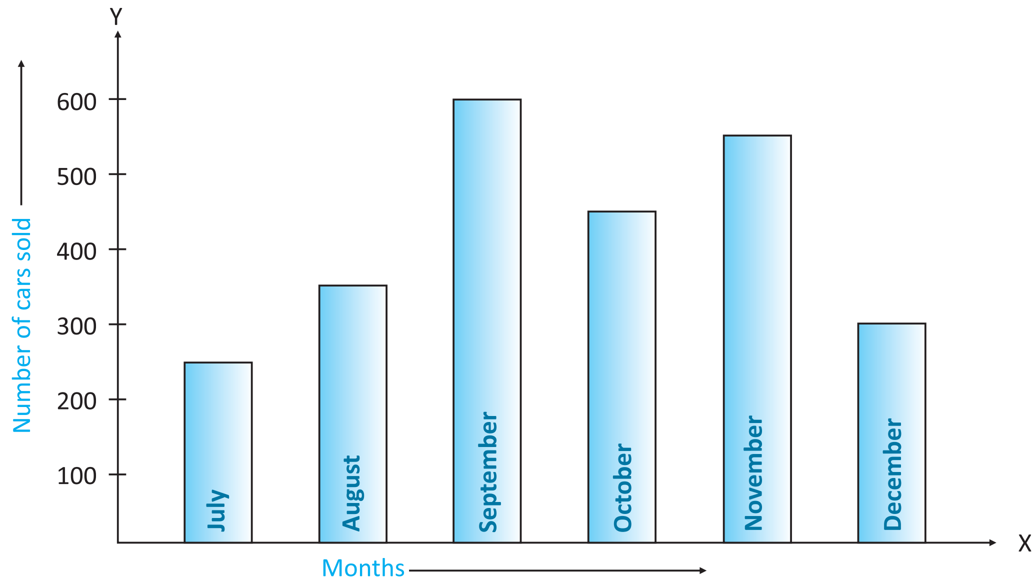
(a)	(b)
0 – 8	0 – 12
8 – 16	12 – 24
16 – 24	24 – 36
24 – 32	36 – 48
32 – 40	48 – 60

4. A dice was thrown 40 times and the following scores were obtained.

3, 2, 4, 6, 3, 5, 1, 2, 4, 4, 6, 5, 4, 1, 2, 5, 6, 4, 2, 2, 2, 6, 5, 3, 4, 1, 2, 6, 4, 2, 1, 3, 5, 5, 4, 2, 3, 1, 1, 6

Prepare a frequency table using tally makes for the scores and draw the bar graph.

5. Read the graph carefully and answer the following questions.



(a) What information is given by the bar graph ?

(b) How many cars were sold during the months under survey ?

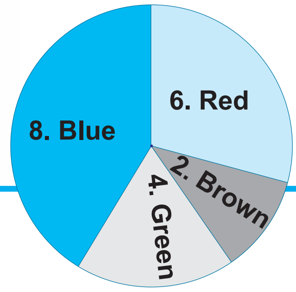
(c) What is the average number of cars sold by during the months under survey ?

(d) Name the months in which 450 or more cars were sold.

(e) Name the months in which 300 or less cars were sold.

HOTS

You ask 20 of your friends what their favourite colour is. The pie chart below show how many picked each colour. What percent of your friends picked red?



Lab Activity

Objective : To represent the time spent by a student in a day through a pic chart.

Materials Required : Chart paper, geometry box, sketch pens.

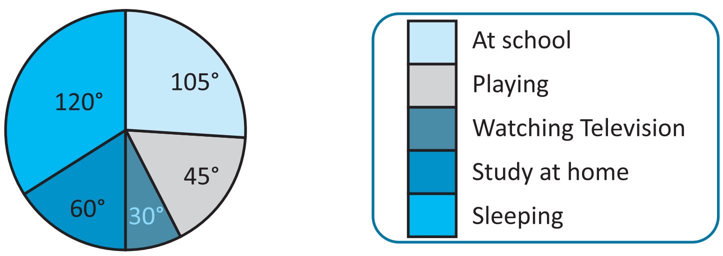
Procedure :

1. Represent the information in a tabular form as shown below.

Activity	At school	Playing	Watching television	Study at home	Sleeping
Time spent (in hr)	7	3	2	4	8

2. Draw a circle of any reduces on a chart paper divide the circle into 5 sectors because the students is indulged in five activity subdivide the total angle at the centre of the circle 360° in the ratio of the hours spent in different activities.

Activity	Time spent (in hr)	Central angle for the respective sector
At school	7	$\frac{7}{24} \times 360° = 105°$
Playing	3	$\frac{3}{24} \times 360° = 45°$
Watching television	2	$\frac{2}{24} \times 360° = 30°$
Study at home	4	$\frac{4}{24} \times 360° = 60°$
Sleeping	8	$\frac{8}{24} \times 360° = 120°$



3. Make a pattern or shade these sectors so that we can differentiate between activities.

18 Introduction to Graph

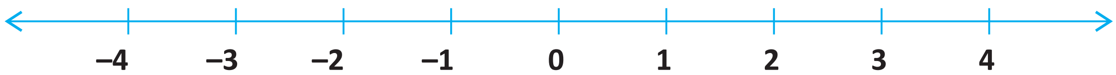
Introduction

Graphs are used to represent the numerical data in the visual form so that it can easily be understand. Thus, graphs are also defined as "the visual representation of data".

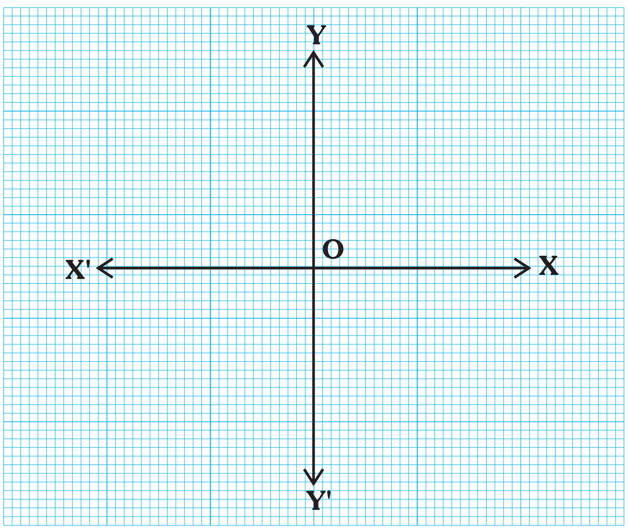
Linear Graphs

Sometimes, a graph is in the form of a broken line. A line graph consists of bits of line segments joined consecutively.

We have studied about the co-ordinate system, where we have locate the points in a plane as shown below :



On a graph paper, draw two mutually perpendicular lines X^1 OX and YO Y^1, intersecting each other at point O.

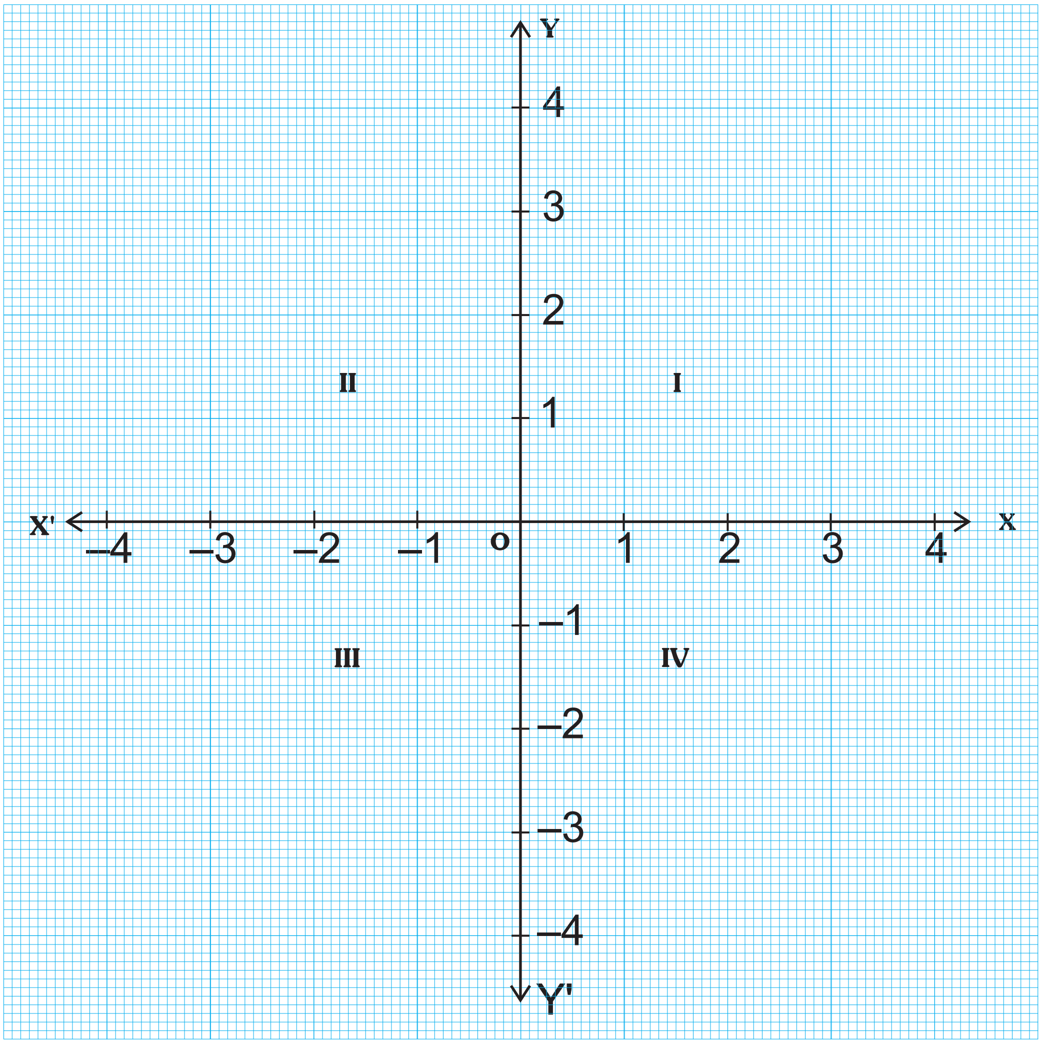


These lines are known as **co-ordinate axes**.

Line X' OX is called **x-axis** and line YOY' is called **y-axis**.

Point O is called **point of origin**.

The plane of the paper in which these co-ordinate axes are drawn is called **cartesian plane.**



The two axes divide the cartesian plane into **four parts**. Each part is referred as **quadrant**. Each quadrant is numbered as I, II, III and IV in the anti-clockwise direction. We read them as **quadrant I**, **quadrant II**, **quadrant III** and **quadrant IV**.

Sign : Starting from O, on the right-hand side of the y-axis, every end-point of a square on the x-axis, represents a positive integer.

On the left hand side of *y*-axis, every end-point of a square on *x*-axis, represents a negative integer.

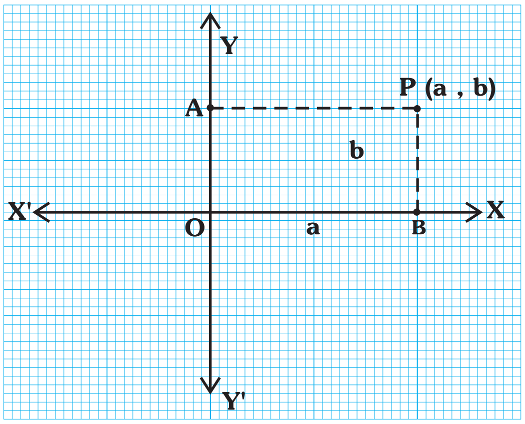
Above the x-axis, every end-point of a square on y-axis represents a positive integer.

Below the axis, every end-point of a square on y-axis, represents a negative integer.

Ordered Pair

Writing a pair of numbers in a specified order is called an **ordered pair**.
For example : (*a* , *b*) is an ordered pair with *a* at the first place and *b* at the second place.

Co-ordinate of A Point



Let P be any point on a graph paper, at a distance of *a* units from x-axis and *b* units from y-axis.

Then, the co-ordinates of P are P (a , b).

Here, *a* is referred as *x - co-ordinate* or *abscissa* of P

and *b* is referred as *y-co-ordinate* or *ordinate* of P.

In the above figure, co-ordinates of A are (a , 0),

co-ordinates of B are (0 , b)

Example 1 : Plot the following points on the graph paper :

(i) A(3,4)
(ii) B(4,–4)
(iii) C(–3,4)
(iv) D(–4,–3)
(v) E(5,0)
(vi) F(0,4)
(vii) G(–3,0)
(viii) H(0,–5)

Solution : Let X'OX and YOY' be the co-ordinate axes.

(i) For A(3,4) : On the x-axis, take 3 units to the right of y-axis and then on the y-axis take 4 units above the x-axis.

(ii) For B(4,–4) : On the x-axis, take 4 units to the right of y-axis and then on the y-axis take 4 units below the x-axis.

(iii) For C(–3,4) : On the x-axis, take 3 units to the left of y-axis and then on the y-axis take 4 units above the x-axis.

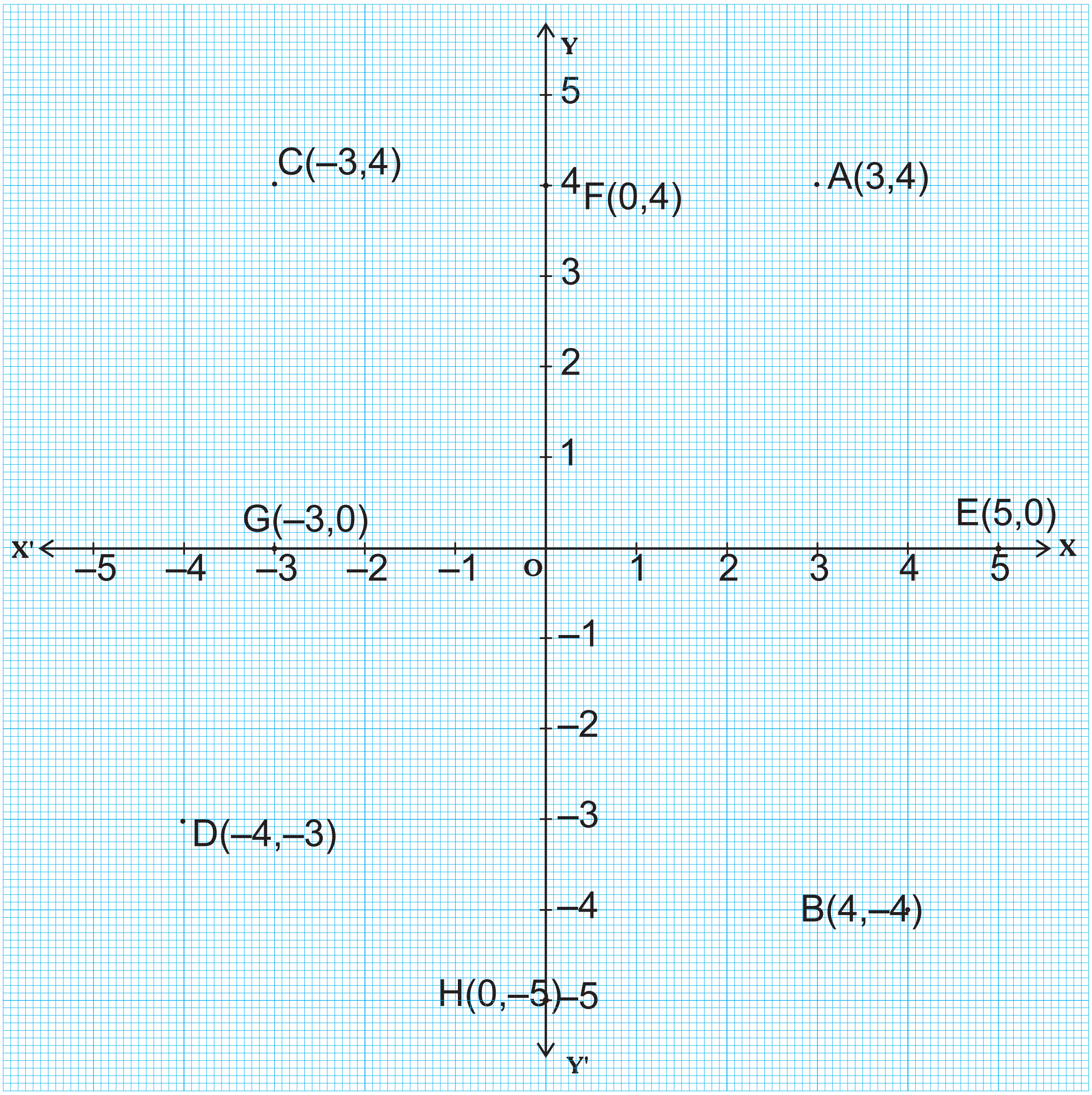
(iv) For D(–4,–3) : On the x-axis, take 4 units to the left of y-axis and then on the y-axis take 3 units below the x-axis.

(v) For E(5,0) : On the right of y-axis, take 5 units on x-axis and we get the point E(5,0).

(vi) For F(0,4) : Take 4 units on y-axis above the x-axis and we get F(0,4).

(vii) For G(–3,0) : Take 3 units on the x-axis on the left of y-axis and we get G(–3,0).

(viii) For H(0,–5) : Take 5 units on y-axis below the x-axis and we get H(0,–5).



Exercise 18.1

1. **Plot the following points on the graph paper :**

(i) A(2,3)	(ii) B(3,–2)	(iii) C(–4,2)	(iv) D(–2,–1)
(v) E(3,0)	(vi) F(0,6)	(vii) G(–5,0)	(viii) H(0,–4)

2. **Identify the x-coordinate of each of the following points:**

(i) A(–2,5)	(ii) B(3,2)	(iii) C(–5,0)	(iv) D(–8,0)
(v) E(0,9)	(vi) F(–3,–4)	(vii) G(2,–3)	(viii) H(0,–7)

3. **Identify the y-coordinate of each of the following points:**

(i) A(–3,7)	(ii) B(7,8)	(iii) C(–7,–9)	(iv) D(6,–5)
(v) E(9,0)	(vi) F(0,8)	(vii) G(–3,0)	(viii) H(0,–8)

4. What are the co-ordinates of origin ?

5. **Without actually plotting the points, state in which quadrant each of the following points lie:**

(i) A(5,9) (ii) B(2,–4) (iii) C(–7,–2) (iv) D(–5,4)

6. Plot the points A(0,0), B(4,0), C(4,4) and D(0,4) on the graph paper and show that they form a square.

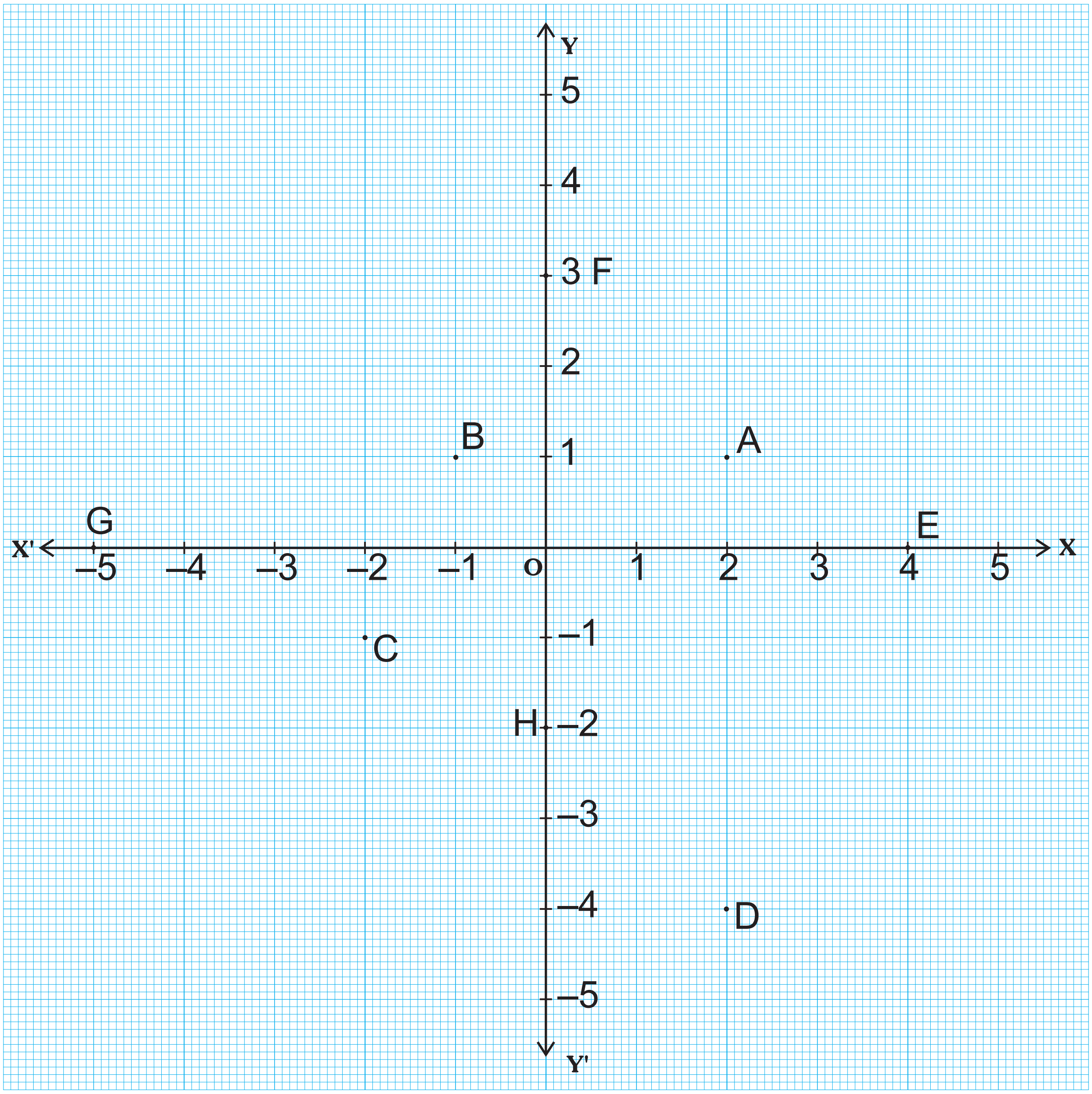
7. Plot the points A(0,0), B(7,0), C(7,3) and D(0,3) on the graph paper and show that ABCD forms a rectangle.

8. **Which of the following points:**

A(5,0), B(0,0), C(–5,8), D(8–5), E(–3,0), F(0,–3)

(i) lie on x-axis ? (ii) lie on y-axis ?

9. Write down the coordinates of points A,B,C,D,E,F,G and H; located on the graph paper given below :



PLOTTING POINTS FOR DIFFERENT KINDS OF SITUATIONS

Graph of Perimeter Vs Length of Squares

We know that, **Perimeter of Square = 4 × Side of Square**

1. Let, **P = 4a**

   where , P is the perimeter of square and *a* is the side of square.

   Draw a graph for the above relation.

2. From the above graph, find the value of P, when :

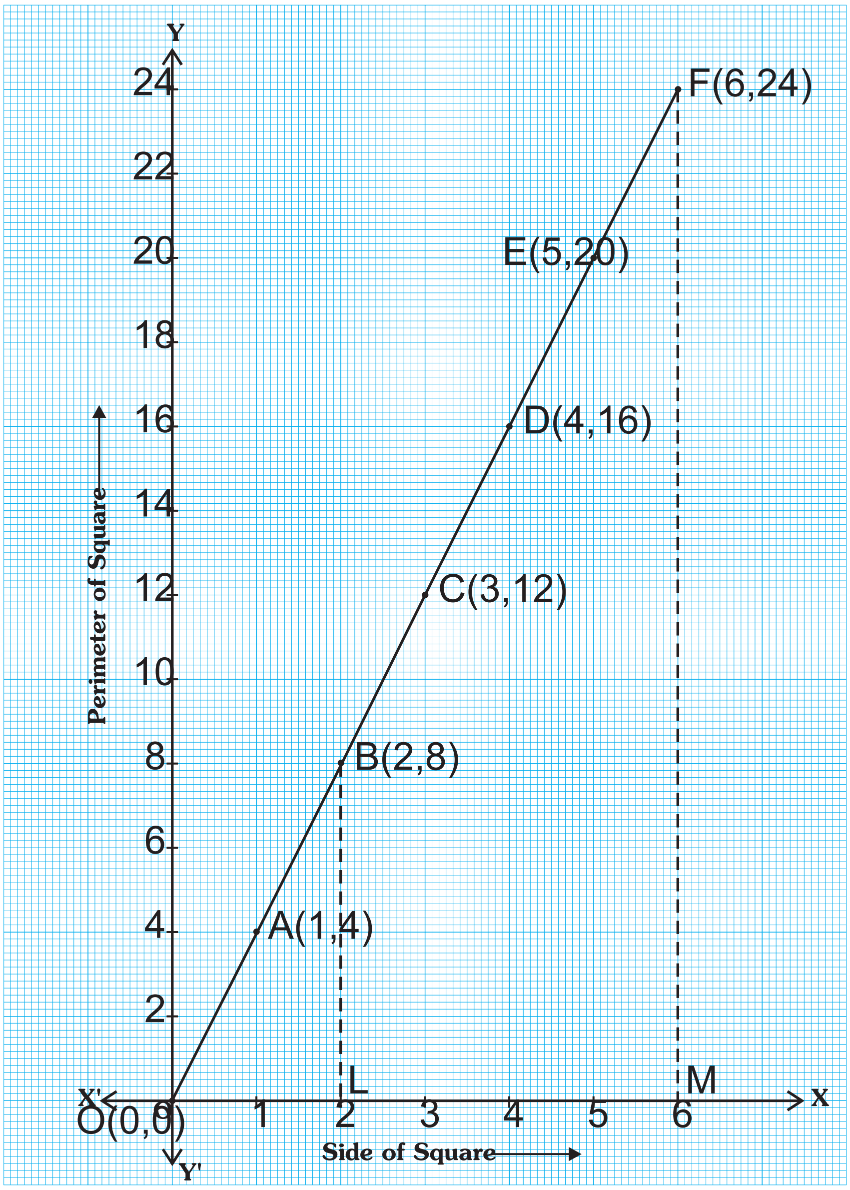
   (i) a = 2 (ii) a = 6

Solution : Given function, P = 4a

For different values of *a*, the corresponding values of P are given below:

a	0	1	2	3	4	5
P = 4a	0	4	8	12	16	20

Now, plot the points O(0,0), A(1,4), B(2,8), C(3,12), D(4,16) and E(5,20) on the graph paper. Join them successively to obtain the required graph.



Scale :
Along x-axis, take 1 cm = 1 unit
Along y-axis, take 1 cm = 2 units

Reading off from the graph

(i) On the x-axis, take the point L at a = 2. Draw LB ^ x-axis, meeting the graph at B.

Clearly, BL = 8 units

∴ P = 20

(ii) On the x-axis, take the point M at a = 6. Draw MF ^ x-axis, meeting the graph at F.

Clearly, MF = 24 units

∴ P = 24

Graph of Area As A Function of Side of A Square

We know that,

Area of Square = (Side)^2

1. Let, $A = x^2$

   where, A is the area of square

   and x is the side of square.

   Draw a graph for the above function.

2. From the above graph, find the value of A, when :

   (i) $x = 5$

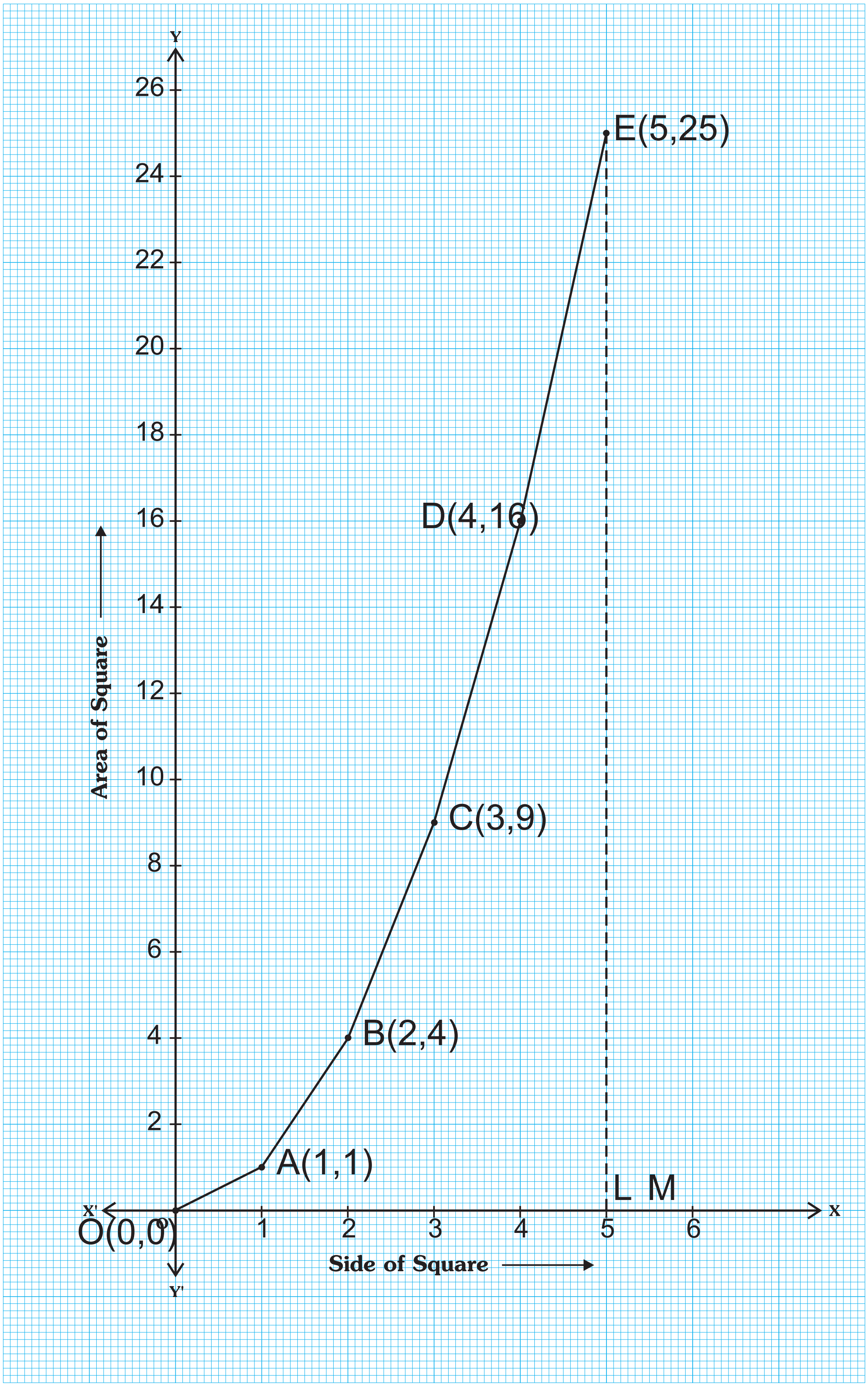
Solution : Given function, $A = x^2$

For different values of x, the corresponding values of A are given below :

x	0	1	2	3	4
$A = x^2$	0	1	4	9	16

Now, plot the points O(0,0), A(1,1), B(2,4), C(3,9) and D(4,16) on the graph paper. Join them successively to obtain the required graph.

Scale : Along x - axis, take 1cm = 1 unit
y - axis, take 1 cm = 2 units



Reading off from the graph

(i) On the x-axis, take point L at x = 5. Draw LE ^ x-axis, meeting the given graph at E.

Clearly, EL = 25 units

∴ A = 25 units

In the above graph a linear graph ? We observe that this graph is not a straight line, but it is curved line. So, it is not a linear graph.

Graph of Multiples of Different Numbers

1. Draw a graph of the function, y = 3x

2. From the graph, find the value of y, when

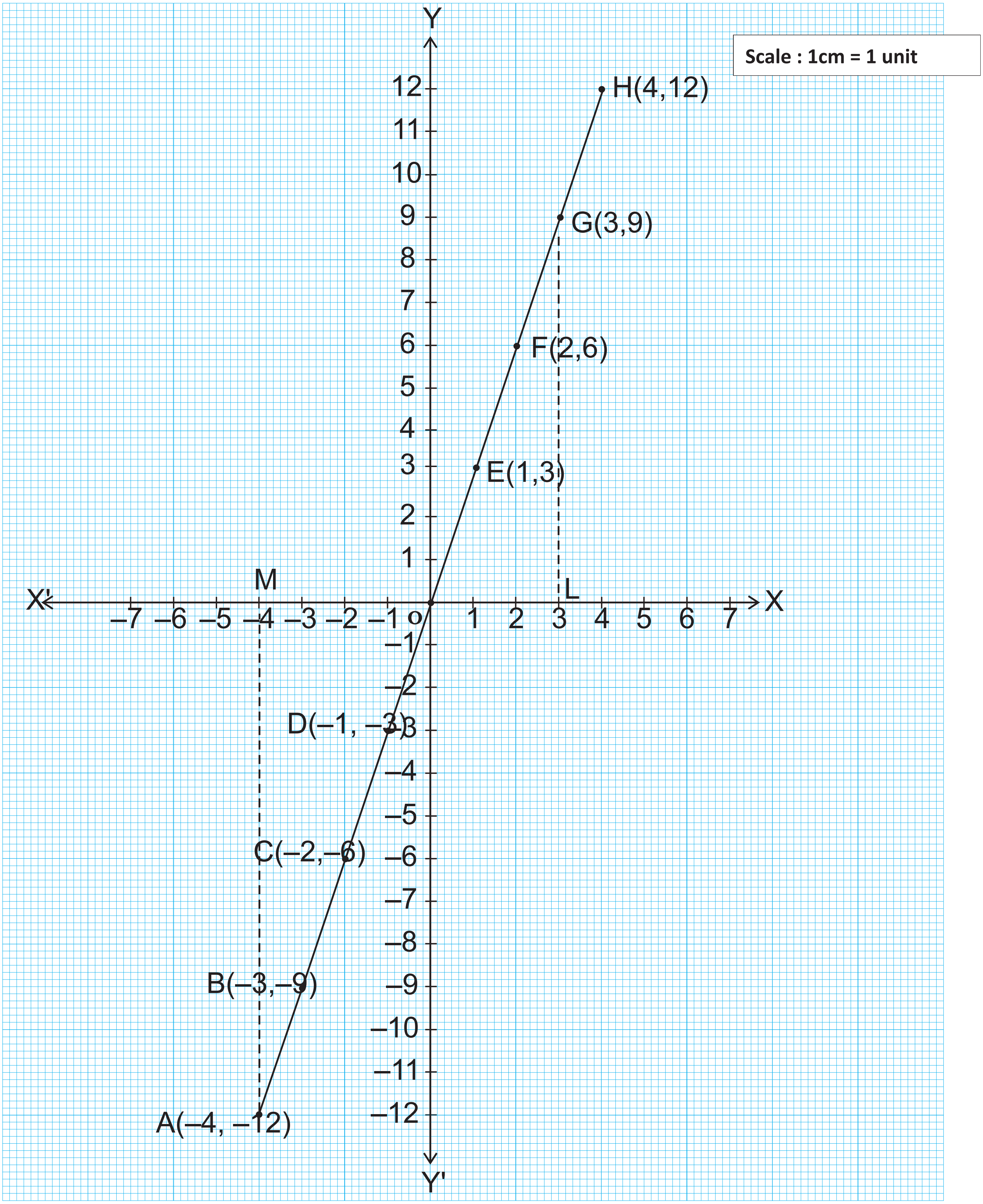
(i) x = 3 (ii) x = –4

Solution : Given function, y = 3x

For different values of x, the corresponding values of y are given below:

x	–4	–3	–2	–1	0	1	2	3	4
y = 3*x*	–12	–9	–6	–3	0	3	6	9	12

Plot the points A(–4,–12), B(–3,–9), C(–2,–6), D(–1,–3), 0(0,0). E(1,3), F(2,6), G(3,9) and H(4,12) on the graph paper. Join them successively to obtain the required graph.


Scale : 1cm = 1 unit
Y
X'
X
Y'
H(4,12)
G(3,9)
F(2,6)
E(1,3)
D(–1, –3)
C(–2,–6)
B(–3,–9)
A(–4, –12)
M
L
o
12
11
10
9
8
7
6
5
4
3
2
1
–1
–2
–3
–4
–5
–6
–7
–8
–9
–10
–11
–12
–7 –6 –5 –4 –3 –2 –1
1 2 3 4 5 6 7

Reading off from the graph :

(i) On the x-axis, take the point L at x = 3. Draw LG ^ x-axis , meeting the graph at G.

Clearly, GL = 9

∴ y = –9

(ii) On the x - axis, take the point M at x = –4. Draw MA ^ x-axis, meeting the graph at A.

Clearly, MA = –12

∴ y = –12

Graph of Simple Interest Vs Number of Years

Simple interest on a certain sum is ` 20 per year.

Then, $S = 20 \times x$

where, S is the simple interest and x is the number of years.

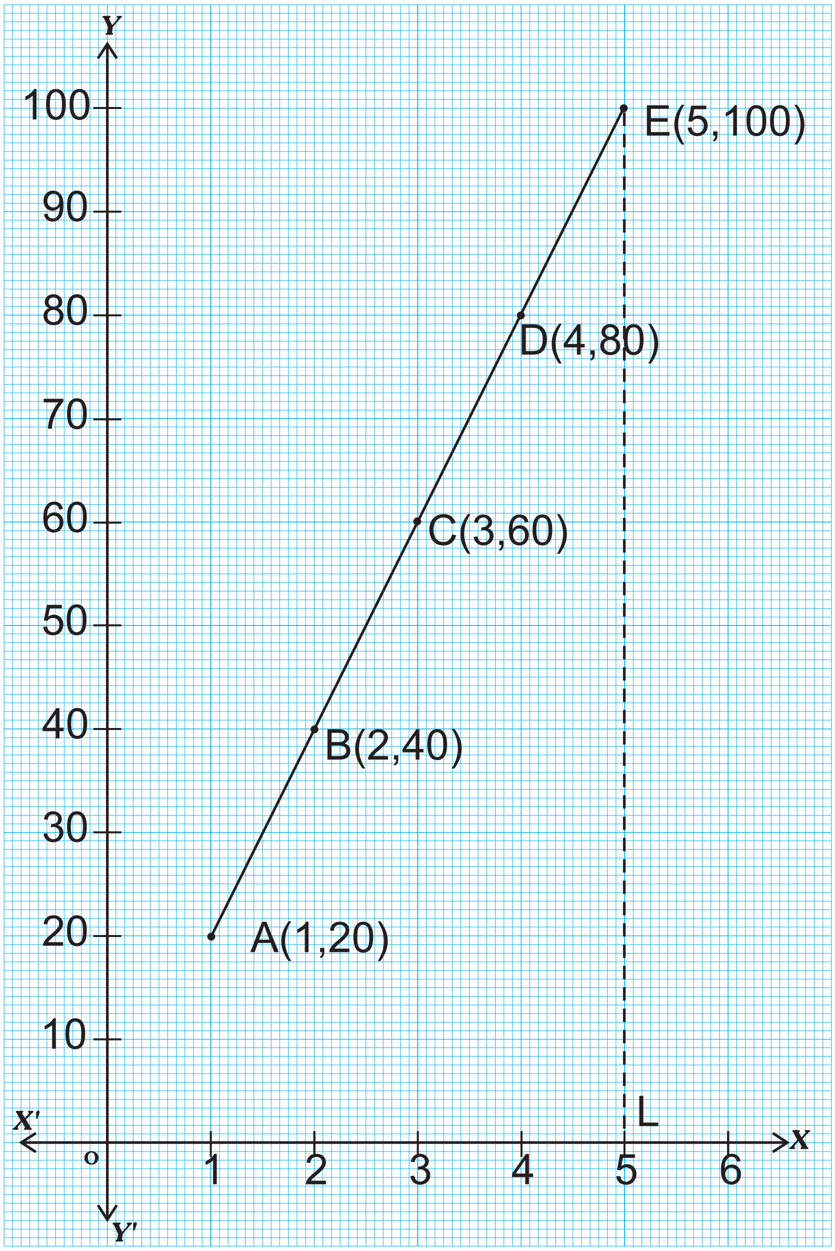
1. Draw a graph for the above function.
2. From the above graph, find the value of S, when x = 5

Solution : Given function, $S = 20x$

For different values x, the corresponding values of S are given below :

x	1	2	3	4
$S = 20x$	20	40	60	80

Plotting the points A(1,20), B(2,40), C(3,60), D(4,80) on the graph paper. Join them successively to obtain the required graph.



Scale :

Along x-axis, take 1cm = 1unit

Along y-axis, take 1cm = 10 units

Reading off from the graph :

On the x-axis, take the point L at x = 5

Draw LE ^ x-axis, meeting the graph at E.

Clearly, EL = 100 units

$\therefore$ S = 100

Reading of Distance Vs Time Graph

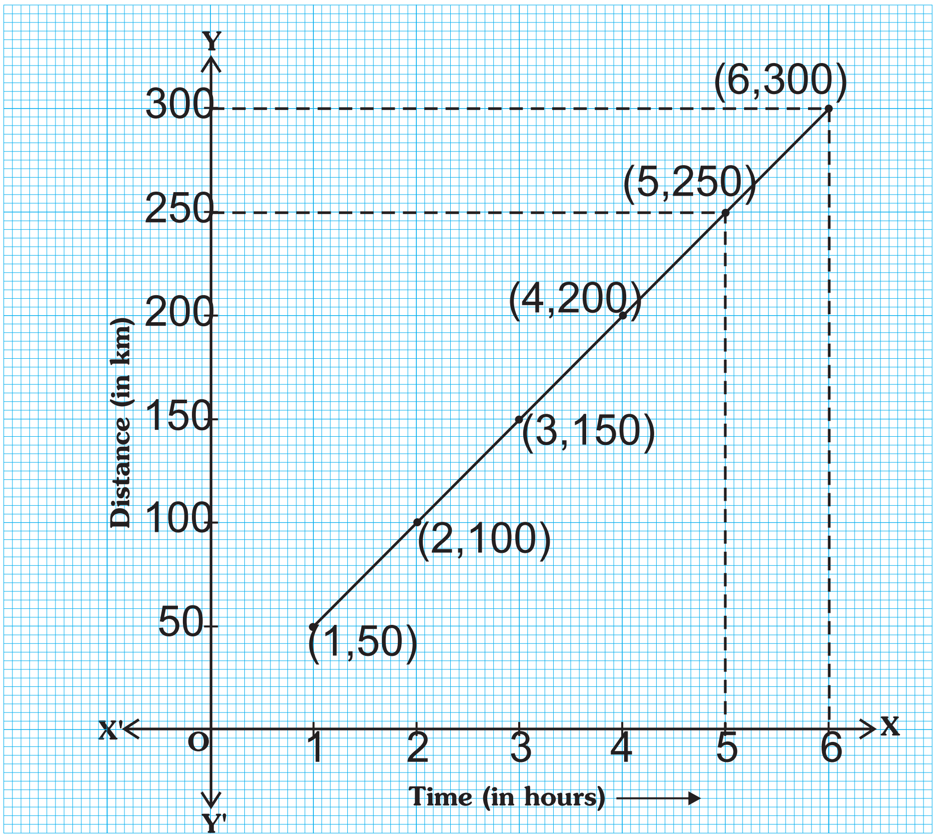
As we know, distance is directly proportional to time. So, as the distance increases, time also increases, or vice-versa.

Let us suppose a car travels at a speed of 50 km/hr. Then the distance travelled be D in x hours be :

$$D = 50 \times x$$

Taking *D* and *x* be two variables, we can draw graph representing the above relation between distance and time.

x	1	2	3	4
D = 50*x*	**50**	**100**	**150**	**200**



Scale :

Along x-axis, take 1cm = 1unit

Along y-axis, take 1cm = 50units

Read the above graph carefully, answer the following questions:

(i) Find the distance covered in 5 hours.

(ii) Find the time taken for the distance of 300 km.

Solution : It is clear from the above graph :

(i) Distance covered in 5 hours = 250 km

(ii) Time taken for the distance of 300 km = 6 hours

Exercise 18.2

1. (i) Draw the graph for the function, P = 4x.

(ii) From the graph, find the value of P, when

(a) x = 4 (b) x = 5 (c) x = 6

2. (i) Draw the graph for the function, $A = x^2$.

(ii) From the graph, find the value of A, when

(a) x = 3 (b) x = 5 (c) x = 6

3. (i) Draw the graph for the function, y = 2x.

(ii) From the graph, find the value of y, when

(a) x = 4 (b) x = 5 (c) x = 6

4. **Draw the graph for the following :** **[NCERT]**

(i)

Side of Square (in cm)	**2**	**3**	**3·5**	**5**	**6**
Perimeter (in cm)	**8**	**12**	**14**	**20**	**24**

Is it a linear graph ?

(ii)

Side of Square (in cm)	**2**	**3**	**4**	**5**	**6**
Area (in cm^2)	**4**	**9**	**16**	**25**	**36**

Is it a linear graph ?

5. **Draw the graph for the interest on deposits for a year :** **[NCERT]**

Deposits (in ₹)	**1000**	**2000**	**3000**	**4000**	**5000**
Simple Interest (in ₹)	**80**	**160**	**240**	**320**	**400**

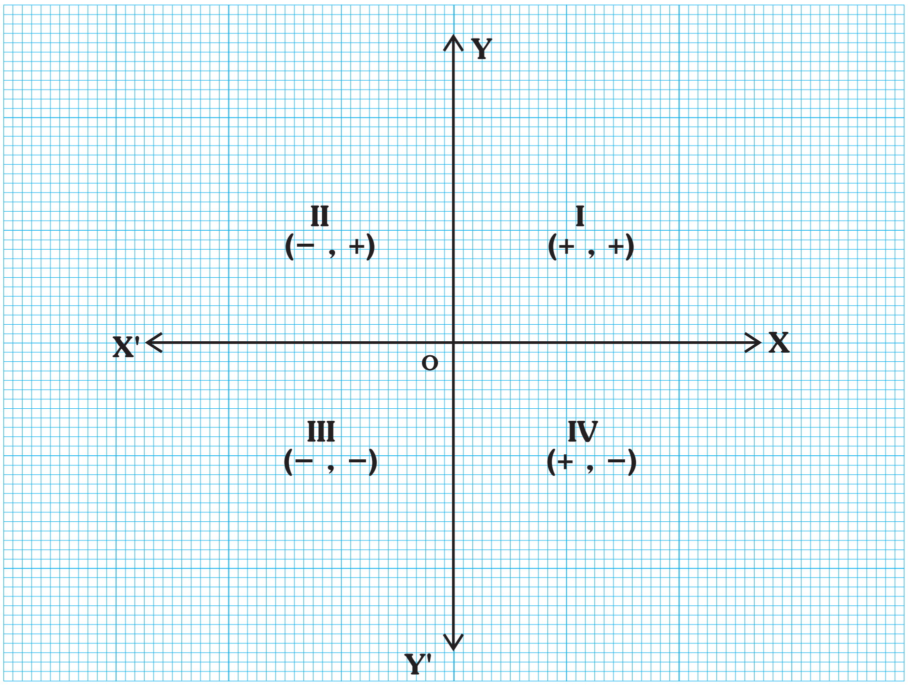
(i) Does the graph pass through the origin ?

(ii) Use the graph to find the interest on ₹ 2500 for a year.

(iii) To get an interest of ₹ 280 per year, how much money should be deposited ?

Points to Remember :

- In rectangular coordinate system, the hofizontal line X'OX is called x-axis while the vertical line Y'OY is called y-axis. The point of intersection of the lines XOX' and YOY' is called the origin.
- The plane is divided by the axes into four regions each being called quadrant.
- In first quadrant, x is ositive and y is positive.
  In second quadrant, x is negative and y is positive.
  In third quadrant, x is negative and y is negative.
  In fourth quadrant, x is positive and y is negative.
- Graphical representation of data, being visual input, is easy to understand.
- The coordinate (x, y) means, we move x units along x-axis and then move y units along y-axis.
- The Graphs of different data can be drawn with the help of the coordinates.
- Graphs help us in predicting different important results.
- The relation between dependent variable and independent variable can be represented by a graph.



➢ **Signs :**

Region	Quadrant	Signs of Co-ordinate
XOY	I	(+ , +)
YOX'	II	(– , +)
X'OY'	III	(– , –)
Y'OX	IV	(+ , –)

➢ Abscissa is the perpendicular distance of a point from y-axis.
➢ Ordinate is the perpendicular distance of a point from x-axis.
➢ Abscissa is positive on the right of y-axis.
➢ Abscissa is negative on the left of y-axis.
➢ Abscissa of any point on the y-axis is always zero.
➢ Ordinate is positive above x-axis.
➢ Ordinate is negative below x-axis.
➢ Ordinate of any point on the x-axis is always zero.
➢ Co-ordinates of origin are always (0 , 0)

EXERCISE

1. MULTIPLE CHOICE QUESTIONS (MCQs) :

Tick (✓) the correct options.

a. In which of the following quadrants does the point P(4,5) lie?

(i) I ☐ (ii) II ☐ (iii) III ☐ (iv) IV ☐

b. In which of the following quadrants does the point P(–7,–8) lie?

(i) I ☐ (ii) II ☐ (iii) III ☐ (iv) IV ☐

c. In which of the following quadrants does the point Q(–7,3) lie?

(i) I ☐ (ii) II ☐ (iii) III ☐ (iv) IV ☐

d. In which of the following quadrants does the point R(2,–5) lie?

(i) I ☐ (ii) II ☐ (iii) III ☐ (iv) IV ☐

e. Co-ordinates of origin are :

(i) (0,0) ☐ (ii) (0,1) ☐ (iii) (1,0) ☐ (iv) None of these ☐

f. x-co ordinate is also called :

(i) Origin ☐ (ii) Abscissa ☐ (iii) Ordinate ☐ (iv) None of these ☐

g. y-co ordinate is also called :

(i) Origin ☐ (ii) Abscissa ☐ (iii) Ordinate ☐ (iv) None of these ☐

h. The cartesian plane has ______ axes.

(i) One ☐ (ii) Two ☐ (iii) Three ☐ (iv) Four ☐

i. The x-co ordinate of every point on the y-axis is :

(i) Zero ☐ (ii) One ☐ (iii) Two ☐ (iv) Three ☐

j. The y-co ordinate of every point on the x-axis is :

(i) Zero ☐ (ii) One ☐ (iii) Two ☐ (iv) Three ☐

2. State the quadrants in which the points with the following coordinate lie :

(i) P (3, 2) (ii) Q (5, –4) (iii) R (–5, –4) (iv) S (–5, 4)

3. Plot the points (5, 6) on a graph sheet. Is it the same as the point (6, 5)?

4. Write the coordinates of a point:

(i) lying on x-axis to the left of origin at a distance of 6 units,

(ii) lying on y-axis at a distance of 5 units below origin.

5. Plot the points A(2, 1), B(–1, 3), C(5, –1) on a graph paper using the same coordinate axes. Join the points A, B, C. What do you observe?

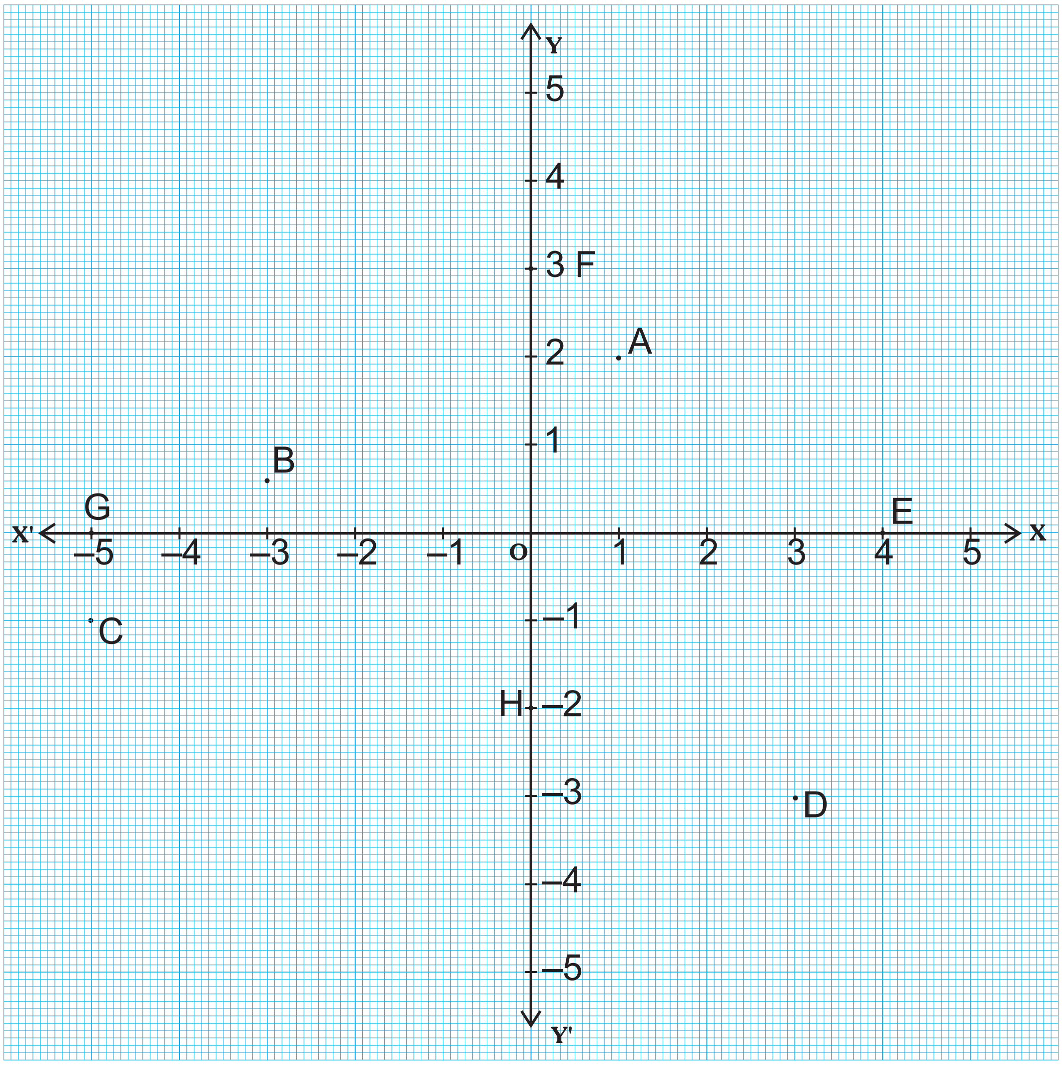
6. Plot the following points on graph paper:

(i) A (3, 4) (ii) B (–3, –4) (iii) C (–3, –4) (iv) D (3, –4)

7. Find the distance of following points from x-axis:

(i) (3, 4) (ii) (–4, 5) (iii) (–5, 5) (iv) (0, 5)

8. Write down the coordinates of the points A, B, C and D in the given figure:



HOTS

A sum of ₹ 5000 is deposited in bank at the rate of 10% simple interest. Plot a graph with interest and number of years as variable.

Lab Activity

Take some sheets of graph paper and try to draw different types of shapes. Also try to find out the co-ordinates of vertex of these shapes.

OR

Make a powerpoint presentation on graphs.