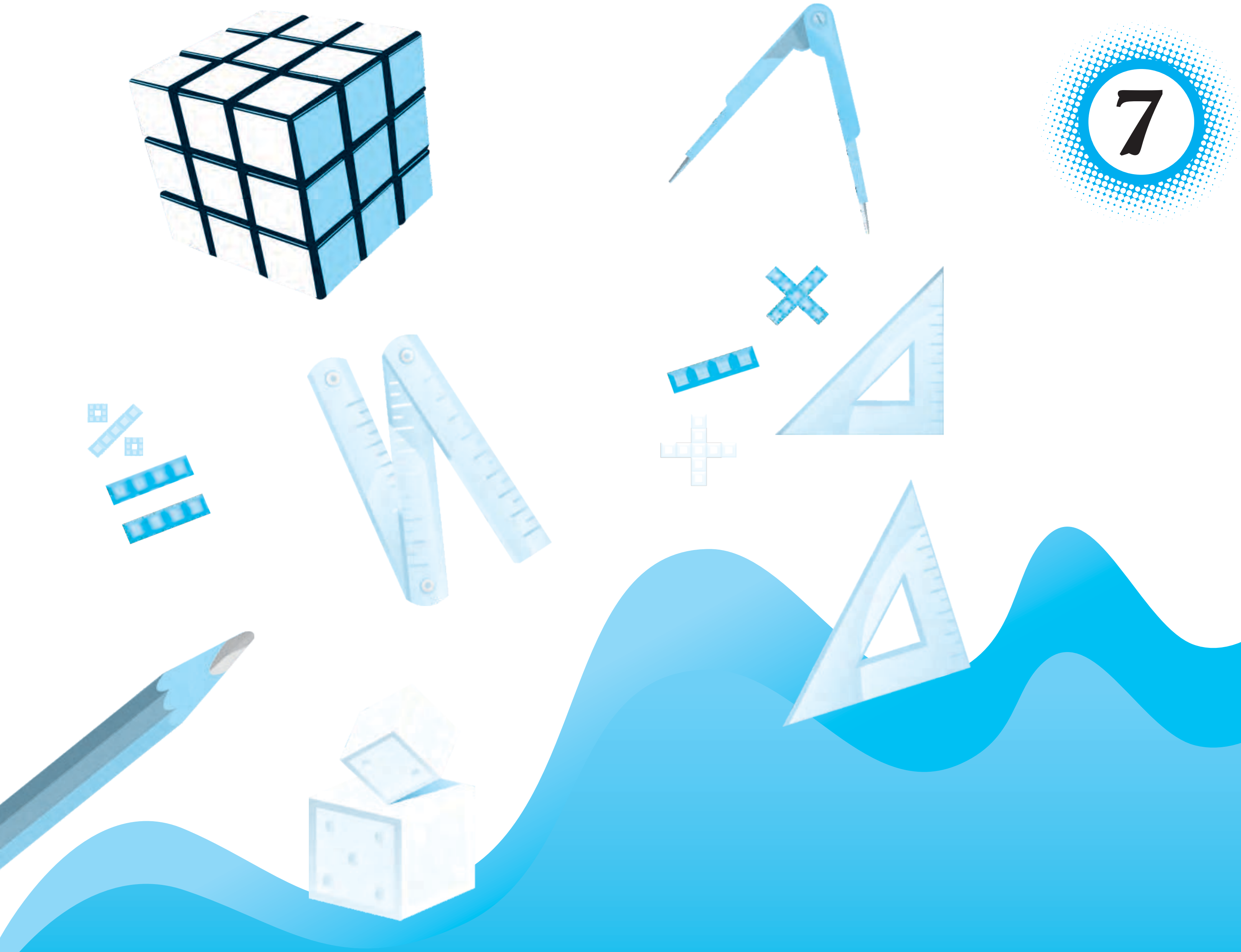
A Gateway to MATHEMATICS

7



A Gateway to

MATHEMATICS

PART-7

Published by:

Designed & Illustrated by :
EDIT ONE INTERNATIONAL

PREFACE

Mathematics is a demanding, challenging and dynamic subject which is deeply associated with the day to day life activities and experiences to different types of quantities. Every one uses mathematics in his/her daily life in various ways irrespective of their knowledge of mathematical concepts. Study of mathematics introduces to child the basic mathematical concepts and skills needed for the child to face real life problems.

A Gateway to Mathematics is a series of eight books from **Class I to VIII**, based on the latest reviews and guidelines of CCE (Continous and Comprehensive Evaluation) pattern issued by the NCERT and CBSE. Our objective is to empower the students with ideal and quality education. Each chapter is well-illustrated with relevant study material, stepwise solved examples and adequate practice questions are there on each topic. It helps the preceptor to increase the ability of a child to easy understand, analyze and solve the problems with accurate logical sequence.

All the books of this series have enough Diagrams, Clear Explanations, Maths Lab Activities to help children understand the several principles and patterns of mathematics intended for them.

Salient features of the series are:

* Interactive study approach.
* Easy to learn educational methodology.
* Simple and easy language has been used keeping in mind the comprehensive level of the students.
* Each topic has appropriate illustrations which help in visualization of abstract mathematical concepts.
* Examples and word problems to provide a variety of experience to children and to sharpen their observational skills.
* Facts to know to enable children to revisit the concepts previously learnt.
* Points to remember is given at the end of each chapter to highlight some important points of the topics.
* Hots Questions in every chapter to help children connect the topics with everyday life.
* Maths Lab Activities to explore and improve the child's memory potential and to utilize the rich and varied opportunities available outside a classroom situation.
* To develop creativity in the children, enough pattern exercises have been introduced.
* Revision exercises including MCQ's are given for self assessment of the learners.
* Any constructive suggestions for the improvement of this series are always appreciated.

— Publisher

CONTENTS

1 Knowing the Numbers

Recall what we have learnt in previous class about integers. We studied that integers comprise of whole numbers and negative numbers. We also learnt about the representation of integers on the number line, their comparative values, absolute value of an integer, addition and subtraction of integers. Let's briefly revise the main points again.

Integers

A combined set of negative numbers and whole numbers i.e, {....... –5, –4, –3, –2,–1, 0, 1, 2, 3, 4, 5, ...}.
[In this set, all the natural numbers are positive integers and others are negative integers except '0' . 0 (zero) is neither positive nor negative.]

Absolute Value of an Integer

The absolute value of an integer is the magnitude of the numerical value of an integer regardless of its sign (direction). It is denoted by the symbol '| |'. The absolute value of an integer is always positive or 0 (zero).

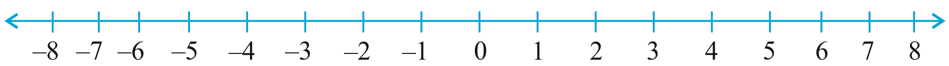
Example : $|-5| = 5$
$|-6 + 4| = |-2| = 2$
$|7 - 3 - 4| = |0| = 0$
Also, the corresponding positive and negative integers have the same absolute value.

Example : $|25| = 25$ and $|-25| = 25$

Ordering of Integers

Integers and whole numbers obey the same rule in their ordering when represented on a number line, i.e. an integer represented to the right of any integer is greater than that integer and vice-versa.

Examples : $8 > 6, 5 > 3, 3 > 1, -1 > -3, -3 > -5, -7 > -8$



1. For every positive integer placed to the right of zero, there is a negative integer to the left of zero placed at the same distance from zero.
2. Zero is greater than every negative integer and less than every positive integer.
3. Every positive integer is greater than every negative integer.

Facts to Know

- The beginning of integers dates back to Babylonia around 4000 years back.
- The first evidence of the use of negative integers had been found in China around 300 BC.

Addition and Subtraction of Integers

1. When two integers with the same sign (either positive or negative) are added, their absolute values are added and the common sign is assigned to their sum.

$(+75)+(+25) = 100$
$(-35)+(-15) = -50$

2. When two integers with different signs are added, first the difference of their absolute values is found and it carries the sign of the integer having greater absolute value.

$(+30)+(-12) = 18$ $\quad$ $(-30)+(12) = -18$
$(+12)+(-30) = -18$ $\quad$ $(-12)+(+30) = 18$

3. When we have to subtract two integers, we change the integer to be subtracted into its corresponding negative integer and then the two integer are simply added.

$(+18)-(+13)=(+18)+(-13)=5$ $\quad$ $(-18)-(-13)=(-18)+(+13)=-5$
$(+13)-(+18)=(+13)+(-18)=-5$ $\quad$ $(-13)-(-18)=(-13)+(+18)=5$
$(+18)-(-13)=(+18)+(+13)=31$ $\quad$ $(-18)-(+13)=(-18)+(-13)=-31$
$(+13)-(-18)=(+13)+(+18)=31$ $\quad$ $(-13)-(+18)=-13-18=-31$

Example 1 : **Add the following :**

(a) (–30) and (–15) (b) (–6) and 18 (c) 4 and (–12) (d) (–52) and 25

Solution :

(a) (-30) and (-15)
$= (-30)+(-15)$
$= -45$

(b) (-6) and 18
$= (-6)+18$
$= 12$

(c) 4 and (-12)
$= 4+(-12)$
$= -8$

(d) (-52) and 25
$= (-52)+25$
$= (-27)$

Example 2 : **Subtract the following :**

(a) (–8) – (–8) (b) 17 – (–33) (c) (–21) – (–8) (d) 20 – (–14)

Solution :

(a) $(-8)-(-8)$
$= -8+8=0$

(b) $17-(-33)$
$= 17+33=50$

(c) $(-21)-(-8)$
$= -21+8$
$= -13$

(d) $20-(-14)$
$= 20+14$
$= 34$

Exercise 1.1

1. **Arrange the following integers in ascending order :**
   (a) –28, 7, 1, 3, –7, –12, –1 (b) 11, –4, 8, 3, –5, 1, 6
2. **Arrange the following integers in descending order :**
   (a) –12, 15, 6, –14, –10, 3, –7, 9 (b) –16, 14, 2, 6, –8, –10, 0, 8, 13
3. **Write the absolute value of the following :**
   (a) $|-27+68|$ (b) $|21-18|$ (c) $|-83+0|$ (d) $|72-99|$
4. **Add the following :**
   (a) $33+(-17)$ (b) $-180+212$ (c) $0+(-919)$ (d) $(-815)+(-913)$
5. **Subtract the following :**
   (a) $-32-(-48)$ (b) $(-45)-(+92)$ (c) $-8-(-15)$ (d) $165-(-319)$
6. **Which value is higher ?**
   (a) –4°C or 7°C (b) 0°F or –4°F (c) –17°C or –14°C (d) 44°F or 0°F
7. **Complete the following sequences :**
   (a) –15, –12, –9, –6, –3, ____, ____, ____, ____ (b) 45, 51, 57, 63, ____, ____, ____, ____
8. **Write the opposites of the following statements :**
   (a) The frog jumped 4 steps forward.
   (b) Today's temperature is 4°c below normal.
   (c) Rahul reached at the platform 30 minutes before the arrival of train.
   (d) Start counting from 17 in ascending order.

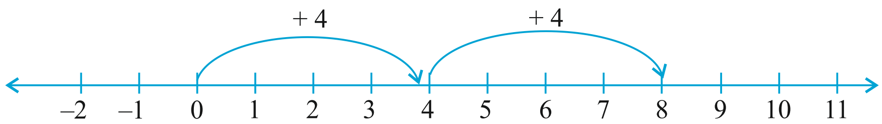
Multiplication of Ibtegers

In previous classes, we have learnt that multiplication is nothing but repeated addition. Therefore, we can find the product of any two integers using repeated addition method.

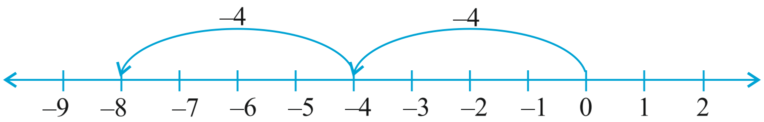
Example : $(+4)\times(+2) = (+4)\times 2 = (+4)+(+4) = 8$

$(-4)\times(+2) = (-4)\times 2 = (-4)+(-4) = -8$

On the number line, $(+4)\times(+2)$ means moving to the right of zero 2 times in steps of 4.



Similarly, $(-4)\times(+2)$ means moving to the left of zero 2 times in steps of 4.



Multiplication of Two Negative Numbers

Observe the following patterns :

$(-5)\times 4 = (-20)$

$(-5)\times 3 = (-15)$

$(-5)\times 2 = (-10)$

$(-5)\times 1 = (-5)$

$(-5)\times 0 = 0$

Here, we observe that when the multiplier is decreased by 1, the product increases by 5. Using this fact, we can proceed like this:

$(-5)\times(-1) = 5$

$(-5)\times(-2) = 10$

$(-5)\times(-3) = 15$

$(-5)\times(-4) = 20$

From the above given examples, we conclude that the product of two integers is the product of their absolute values and their signs are as follows :

(i) The product of two positive integers carries positive sign.

$(+9)\times(+8) = 72$

$(+12)\times(+7) = 84$

(ii) The product of two negative integers carries positive sign.

$(-15)\times(-6) = 90$

$(-9)\times(-8) = 72$

(iii) The product of two integers with opposite signs carries negative sign.

$(-17)\times(+4) = -68$

$(-7)\times(+13) = -91$

Facts to Know

- For multiplying integers we use the following rules :
  - (i) Product of two positive integers is positive (i.e. $+\times+=+$)
  - (ii) Product of two integers with opposite sign is negative (i.e. $+\times-=-$ or $-\times+=-$)
  - (iii) Product of two negative integers is positive (i.e. $-\times-=+$)

Example 3 : **Find the product of the following :**

(a) $(-13) \times 9$ (b) $(-15) \times (-15)$ (c) $(+17) \times 5$ (d) $(+6) \times (-19)$

Solution :

(a) $(-13) \times 9 = -117$ (minus × plus = minus)

(b) $(-15) \times (-15) = 225$ (minus × minus = plus)

(c) $(+17) \times 5 = 85$ (plus × plus = plus)

(d) $(+6) \times (-19) = -114$ (plus × minus = minus)

Example 4 : **Find the value of the following :**

(a) $(-12+7) \times 5$ (b) $(-2) \times (-4) \times 4$ (c) $(-4-5) \times (-5)$ (d) $(-6+11) \times (-2+3)$

Solution :

(a) $(-12+7) \times 5$

$= (-5) \times 5$

$= -25$ [$- \times + = -$]

(b) $(-2) \times (-4) \times 4$

first we multiply −2 and −4

$(-2) \times (-4) = 8$ [$- \times - = +$]

Now, multiply

$8 \times 4 = 32$

(c) $(-4-5) \times (-5)$

$= (-9) \times (-5)$

$= 45$ [$- \times - = +$]

(d) $(-6+11) \times (-2+3)$

$= 5 \times 1$

$= 5$

Division of Integers

It is well known that division is the inverse operation of multiplication. When we write $8 \times 4 = 32$, we can say $32 \div 4 = 8$ and $32 \div 8 = 4$. It means, for each multiplication statement there are two division statements.

So, when $(-7) \times 8 = -56$, then

$(-56) \div (-7) = 8$ and $(-56) \div 8 = -7$

Also ,when $(-4) \times (-7) = 28$, then

$28 \div (-7) = -4$ and $28 \div (-4) = -7$

Some more examples :

$(-5) \times 8 = (-40)$ So, $(-40) \div (-5) = 8$ and $(-40) \div 8 = (-5)$

$(-14) \times (-3) = 42$ So, $42 \div (-14) = -3$ and $42 \div (-3) = (-14)$

$(-6) \times 8 = (-48)$ So, $(-48) \div (-6) = 8$ and $(-48) \div 8 = (-6)$

$4 \times (-17) = (-68)$ So $(-68) \div 4 = -17$ and $(-68) \div (-17) = 4$

From the above examples, we can draw the following conclusions :

(i) When dividend and the divisor have the same sign (either both positive or both negative), the quotient carries a positive sign.

(ii) When dividend and divisor have different signs (one positive and other negative), the quotient carries a negative sign.

Facts to Know

- For dividing integers we use the following rules:

(i) $- \div - = +$ (ii) $+ \div + = +$ (iii) $- \div + = -$ (iv) $+ \div - = -$

Example 5 : **Determine the quotient for the following :**

(a) $(-76) \div (-19)$ (b) $75 \div (-15)$ (c) $(-69) \div 23$ (d) $(-48) \div (-4)$

Solution : (a) $(-76) \div (-19)$

$= \dfrac{-76}{-19} = 4$ [$\therefore - \div - = +$]

(b) $75 \div (-15)$

$= \dfrac{75}{-15} = (-5)$ [$\therefore + \div - = -$]

(c) $(-69) \div 23$

$= \dfrac{-69}{23} = (-3)$ [$\therefore - \div + = -$]

(d) $(-48) \div (-4)$

$= \dfrac{-48}{-4} = 12$ [$\therefore - \div - = +$]

Example 6 : **Find the integer which gives (–54) when multiplied by (–6).**

Solution : Let the unknown integer be x,

According to the question,

$x \times (-6) = (-54)$

$\Rightarrow x = \dfrac{-54}{-6} = 9$ [$- \div - = +$]

Exercise 1.2

1. **Multiply the following :**
   (a) 21 and 0 (b) (–12) and (–12) (c) (–192) and 0
   (d) (–11) and (–12) (e) (–7) and 11 (f) $(-24) \times 5$
2. **Find the product of each of the following :**
   (a) $16 \times (-5)$ (b) $(-6) \times (-3) \times (-3)$ (c) $5 \times 4 \times 3 \times 2 \times 0$
   (d) $(-15) \times (-6)$ (e) $(-10) \times (-9) \times 5 \times 4$ (f) $(-10) \times (-10) \times 10$
   (g) $(-2) \times (-5) \times (-6) \times 0$ (h) $2 \times 3 \times 4 \times (-5) \times (-5)$
3. **Fill in the boxes for the following :**
   (a) $(-12) \times (-2) \times (-4) = \square$ (b) $(-2) \times (-2) \times 2 \times 2 = \square$ (c) $80 \div (-8) = \square$
   (d) $\square \div (-25) = (-3)$ (e) $(-24) \div \square = (-6)$ (f) $123 \div \square = (-41)$
   (g) $66 \div \square = (-11)$ (h) $84 \div (-12) = \square$ (i) $[(-2) \times \square] \div 6 = -1$
   (j) $(-5) \times (-2) \times 10 = \square$
4. **Determine the quotient for each of the following :**
   (a) $(-20) \div (-1)$ (b) $(-72) \div 6$ (c) $(-26) \div 13$
   (d) $36 \div (-3)$ (e) $(-126) \div 6$ (f) $(-111) \div (-3)$
   (g) $(-117) \div (-13)$ (h) $(-14) \div (-14)$ (i) $(-81) \div 9$
   (j) $(-100) \div (-10)$
5. **Simplify and find the absolute value of the following :**
   (a) $|(-5+6-5) \times (-3)|$ (b) $|(-18) \div 3|$ (c) $|\{(-8)-(-4)\} \div (-2)|$ (d) $|(-5) \times (-5) \times (-2) \times 2|$
6. Find the integer which gives (–51) when it is multiplied by (–17).
7. Find the quotient when divisor and dividend are (–4) and (–48) respectively.
8. An integer when divided by –7 gives 12. Find the integer.

Properties of Integers

Properties of Addition

Closure Property **:** If a and b are two integers, then a + b will always be an integer.

Examples : (–7) + (–8) = – 15

(–2) + 4 = 2

18 + (–12) = 6

(–16) + (–12) = –28

Commutative Property **:** If a and b are two integers, then their sum remains the same, irrespective of the order, i.e. a + b = b + a

Examples : (–12) + (–16) = (–16) + (–12) = (–28)

(–5) + 47 = 47 + (–5) = 42

10 + (–65) = (–65) + 10 = (–55)

(–17) + (–18) = (–18) +(–17) = –35

Associative Property **:** If a, b and c are three integers, then a + (b + c) = (a + b) + c. While adding these integers it is not necessary to add it in a particular order of their occurence. We can do it by grouping them as per our convenience.

Examples : [(–8) + 7] + (–15) = (–8) + [7 + (–15)]

⇒ (–1) + (–15) = (–8) + (–8)

⇒ (–16) = (–16)

Let's see another example :

[(–7) + (–8)] + (–11) = (–7) + [(–8) + (–11)]

⇒ (–15) + (–11) = (–7) + (–19)

⇒ (–26) = (–26)

Additive Identity **:** If 0 (zero) is added to any integer, its value remains same. Thus, for an integer 'a', a + 0 = 0 + a = a

Examples : (–29) + 0 = (–29)

0 + (–35) = (–35)

Additive Inverse **:** The sum of an integer and its opposite is always zero. If 'a' is an integer, then (–a) is its opposite and vice versa such that a + (–a) = 0 = (–a) + a

Examples : 29 + (–29) = 0 = (–29) + 29

(–18) + 18 = 0 = 18 + (–18)

(–95) + 95 = 0 = 95 + (–95)

In the above given examples, the integers of each pair, i.e. (29, –29), (18, –18) and (95, –95) are additive inverse of each other.

Property of 1 **:** If 1 is added to any integers, it gives its successor.

Examples : 11 + 1 = 12

So, 12 is the successor of 11.

Also ,

(–7) + 1 = (–6) So, – 6 is the successor of (–7).

Properties of Subtraction

Closure Property **:** If a and b are two integers, then a – b will always be an integer.

Examples : (–4) – (–8) = 4

(–8) – 12 = (–20)

4 – 11 = (–7)

(–17) – (–14) = (–3)

Commutative Property **:** If a and b are two integers, than, a – b ≠ b – a. It means commutative property is not applicable for the subtraction of integers.

Examples : (–7) – (–4) = (–3) but (–4) – (–7) = 3

11 – (–17) = 28 but (–17) – 11 = (–28)

(– 4) – 18 = (–22) but 18 – (–4) = 22

Associative Property **:** If a, b and c are three integers, then (a – b) – c ≠ a – (b – c). It means associative property is not applicable for the subtraction of integers.

Examples : [(–2) –4] – (–7) ≠ (–2) – [4 – (–7)]

⇒ (–6) – (–7) ≠ (–2) – 11

⇒ 1 ≠ –13

Property of Zero **:** If 0 (zero) is subtracted from any integer, its value remains same. Thus, for an integer 'a', a – 0 = a

Examples : (–32) – 0 = (–32)

(–75) – 0 = (–75)

99 – 0 = 99

Property of 1 **:** If 1 is subtracted from any integer, it gives its predecessor.

Examples : (–23) –1 = (–24)

83 – 1 = 82

(–87) – 1 = (–88)

In the above given examples, the integers (– 24), 82 and (–88) are predecessors of (–23), 83 and (–87) respectively.

Properties of Multiplication

Closure Property **:** If a and b are two integers, then a × b will always be an integer.

Examples : (–7) × (–8) = 56

(–8) × 4 = (–32)

7 × (–3) = (–21)

11×12 = 132

Commutative Property **:** If a and b are two integers, then their multiples remains the same, irrespective of the order, i.e. a × b = b × a

Examples : (–7) × (–12) = 84 = (–12) × (–7)

4 × (–9) = (–36) = (–9) × 4

(–13) × 6 = (–78) = 6 × (–13)

17 × 5 = 85 = 5 × 17

Associative Property **:** If a, b and c are three integers, then a × (b × c) = (a × b) × c. While multiplying these integers it is not necessary to multiply it as a particular order of their occurrence. We can do it by grouping them as per our convenience.

Examples : $[(-5)\times 6]\times(-7) = (-5)\times[6\times(-7)]$

$(-30)\times(-7) = (-5)\times(-42)$

$\Rightarrow\ 210 = 210$

Let's see another example :

$[(-12)\times(-5)]\times(-2) = (-12)\times[(-5)\times(-2)]$

$\Rightarrow\ 60\times(-2) = (-12)\times 10$

$\Rightarrow\ -120 = -120$

Multiplicative Identity : If 1 is multiplied to any integer, its value remains same. Thus, for an integer 'a'

$a\times 1 = 1\times a = a$

Examples : $(-87)\times 1 = 1\times(-87) = (-87)$

$43\times 1 = 1\times 43 = 43$

$(-27)\times 1 = 1\times(-27) = (-27)$

Multiplicative Inverse : The multiple of an integer and its multiplicative inverse is always 1. If 'a' is an integer, then $(\frac{1}{a})$ is its multiplicative inverse and vice-versa such that

$$a\times\frac{1}{a} = 1 = \frac{1}{a}\times a$$

Examples : $(-15)\times\frac{1}{(-15)} = 1 = \frac{1}{(-15)}\times(-15)$

$(-43)\times\frac{1}{(-43)} = 1 = \frac{1}{(-43)}\times(-43)$

Distributive Property

If a, b and c are three integers, then

$a\times(b+c) = (a\times b)+(a\times c)$

It means multiplication distributes over addition.

Examples : $(-7)\times[4+(-8)] = [(-7\times 4)]+[(-7)\times(-8)]$

$\Rightarrow\ (-7)\times(-4) = -28+56$

$\Rightarrow\ 28 = 28$

Facts to Know

- 1 and –1 are the only two integers whose respective multiplicative inverse remains same.

Properties of Division

Closure Property : If a and b are two integers, thus $a \div b$ will not always be an integer.

Examples : $(-2)\div 5 = -0.4$

$2\div 3 = 0.\overline{6}$

$(-4)\div 0 =$ not defined

$(-7)\div(-28) = 0.25$

Commutative Property : If a and b are two integers, then $a \div b \neq b \div a$. It means commutative property is not applicable for the division of integer.

Examples : $(-4)\div(8) \neq 8\div(-4)$

$\Rightarrow\ -0.5 \neq -2$

Let's see another example:

$(-12) \div (-4) \neq (-4) \div (-12)$

$\Rightarrow 3 \neq 0.\overline{3}$

Associative Property : If a, b and c are three integers, then $(a \div b) \div c \neq a \div (b \div c)$. It means associative property is not applicable for the division of integers.

Examples : $[(-16) \div 4] \div (-2) \neq (-16) \div [4 \div (-2)]$

$\Rightarrow (-4) \div (-2) \neq (-16) \div (-2)$

$\Rightarrow 2 \neq 8$

Let's see another example:

$[(-24) \div (-4)] \div 6 \neq (-24) \div [(-4) \div 6]$

$\Rightarrow 6 \div 6 \neq (-24) \div \left(\frac{-4}{6}\right)$

$\Rightarrow 1 \neq 36$

Property of 1 : If we divide any integer by 1, it gives the same integer as quotient. For any integer 'a', $a \div 1 = a$

Examples : $(-29) \div 1 = (-29)$

$(-75) \div 1 = (-75)$

Property of 0 (zero) : If we divide zero by any integer, the result is always zero. For any integer 'a',

$0 \div a = 0$

Example : $0 \div -92 = 0$

$0 \div 345 = 0$

$0 \div 645 = 0$

Facts to Know

- Zero divided by an integer is always zero.
- Any integer divided by zero is not defined.

Example 7 : Sum of two integers is (–73). If one of the integers is 46, find the other integer. Also verify the answer.

Solution : Let the required integer be 'x'

According to the question,

$x + 46 = (-73)$

$\Rightarrow x = (-73) - 46$

$\Rightarrow x = -119$

Hence, the required integer is (–119)

Verification :

$-119 + 46 = -73$

$\Rightarrow -73 = -73$

Example 8 : Subtract the sum of (–75) and (– 47) from the multiple of (–8) and (–12).

Solution : Sum of (–75) and (– 47)

$= (-75) + (-47)$

$= -122$

Multiple of (–8) and (–12)

= (–8) × (–12) = 96

∴ The required difference = 96 – (–122) = 218

Example 9 : Subtract the sum of 1 6, 14 and (–7) from the sum of 7, 19 and the additive inverse of 16.

Solution : Sum of 16, 14 and (–7)

= 16 + 14 + (–7) = 23

Additive inverse of 16 is (–16),

Sum of 7, 19 and (–16)

= 7 + 19 + (–16) = 10

∴ The required difference = 10 – 23 = (–13)

Example 10 : In a quiz competition, Rahul scored 70, (–20) and 40, whereas Saurav scored 50, 0 and 20 in three successive rounds. Who won the quiz competition and by what margin ?

Solution : Rahul's total score = 70 + (–20) + 40 = 90

Saurav's total score = 50 + 0 + 20 = 70

Rahul's score is 90 and Saurav's score is 70.

So, Rahul won by the margin of 90 – 70 = 20.

Example 11 : Evaluate the following using distributive property :

(a) (–6) × 97 (b) (–7) × 1002

Solution : (a) (–6) × 97 = (–6) × (100 – 3)

= [(–6) × 100] – [(–6) × 3] (by distributive property)

= (– 600) – (–18)

= (–600) + 18 = (–582)

(b) (–7) × 1002 = (–7) × (1000 + 2)

= [(–7) × 1000] + [(–7) × 2]

= (–7000) + (–14) = (–7014)

Example 12 : A submarine descends into sea water at the rate of 12 km per minute. What will be its position after 15 minutes ?

Solution : Since the submarine is going down, so the distance covered by it will be represented by negative integer.

Change in position of the submarine in one minutes = (–12) km

Position of the submarine after 15 minutes = (–12) × 15 = (–180) km

It means, after 15 minutes the submarine will be 180 km below the surface of sea water.

Example 13 : A room heater starts raising the temperature at the rate of 2°C per minute. Ankit feels comfortable at 34°C. In how much time will Ankit be happy ? [If current temperature is 0°C]

Solution : Times taken by room heater to reach at 34°C = $\frac{34}{2}$ = 17 min

Hence, after 17 minutes the temperature of the room will reach at 34°C that will make Ankit happy.

Exercise 1.3

1. Fill in the blanks :

(a) $20 \div \square = -1$ (b) $\square \div 25 = -1$ (c) $\square \div 6 = 0$

(d) $13 - (-13) = \square$ (e) $(-17) \times \square = 16 \times (-17)$ (f) $(-243) \times 0 =$

(g) $(-5) \times \frac{1}{\square} = 1$ (h) $(-10) \times [(-5) + 7] = [(-10) \times \square] + [(\square) \times 7]$

(i) $\square \times [3 + (-4)] = [(-8) \times 3] + [(-8) \times \square]$

(j) $[10 \times \square] \times (-6) = 10 \times [(-5) \times (-6)]$

2. Write T for True and F for False for the following statements :

(a) $(-5) \times 7 = -35$ $\square$ (b) $(-5) \times (-4) \times 2 = 5 \times 2 \times (-4)$ $\square$ (c) $(-4) - (-8) = -12$ $\square$

(d) $(-18) \times 0 = -18$ $\square$ (e) $(-8) \times (-14) \times 0 = 0$ $\square$ (f) $(-12) \div 4 = 4 \div (-12)$ $\square$

3. Calculate the following using suitable arrangements:

(a) $(127 \times 8) + (127 \times 12)$ (b) 105×7 (c) $(-125) + (-75) + 188 + (-38) + 25$

4. Prove the validity of the following using property of multiplication:

(a) $[57 \times (-127)] + [(-23) \times 57]$ (b) $(-8) \times [(-12) + (-16)]$

(c) $[(-21) \times [(-5) + (-7)]$ (d) $[(-105) \times 91] + [5 \times 91]$

5. Evaluate the following using distributive property:

(a) 18×102 (b) $(-5) \times 1003$ (c) 89×14 (d) $(-37) \times 97$

6. An integer divided by (–13) gives 9. Find the integer.

7. Find an integer which when multiplied with (–8) gives 72.

8. Tanya throws a stone 52 m vertically up in the air which later settled on the bottom of a lake 60 m deep. Find the total distance covered by stone.

Points to Remember

- Integers are the combined sets of negative numbers and whole numbers.
- The absolute value of an integer is the right magnitude of the value of an integer regardless of its sign.
- On a number line, every integer to the right of zero is greater than the integers to its left and vice versa.
- 0 is neither negative nor positive.
- Addition of integers follow closure, commutative and associative property i.e.,
  (a) The value of sum of two integers a and b is integer. (b) $a + b = b + a$
  (c) $(a + b) + c = a + (b + c)$
- Subtraction of integers follow only closure property. The value of subtraction of two integers a and b is integer.
- Multiplication of integers follow closure, communicative and associative property i.e.,
  (a) The product of two integers a and b is integer. (b) $a \times b = b \times a$ (c) $(a \times b) \times c = a \times (b \times c)$
- Integers follow distributive property, i.e. for any three integers a, b and c,
  $a \times (b + c) = (a \times b) + (a \times c)$
- For division of integers
  (a) The quotient carries a positive sign when dividend and divisor have same sign (either positive or negative).
  (b) The quotient carries a negative signs when dividend and divisor have different signs (one positive and other negative).
- For any integer 'a',
  (a) $a \div 0$ is not defined (b) $0 \div a = 0$ and (c) $a \div 1 = a$

EXERCISE

1. MULTIPLE CHOICE QUESTIONS (MCQs):

Tick (✓) the correct options :

(a) Which of the following is correct ?

(i) $a \div 0 = 0$ ☐ (ii) $0 \div a$ = not defined ☐

(iii) $0 \div a = 0$ ☐ (iv) $1 \div a = a$ ☐

(b) The smallest positive integer is :

(i) 0 ☐ (ii) 1 ☐ (iii) 99 ☐ (iv) 1000 ☐

(c) The additive inverse of (–7) is :

(i) $\frac{1}{7}$ ☐ (ii) $-\frac{1}{7}$ ☐ (iii) 7 ☐ (iv) None of these ☐

(d) The multiplicative inverse of 7x is :

(i) $\frac{1}{7x}$ ☐ (ii) $\frac{7}{x}$ ☐ (iii) $\frac{7x}{1}$ ☐ (iv) (–7x) ☐

(e) Which of the following relation is correct ?

(i) $(-24) \div (-4) = (-6)$ ☐ (ii) $(-72) \div 8 = (-9)$ ☐

(iii) $20 \div (-10) = 2$ ☐ (iv) $(-30) \div (-3) = -27$ ☐

(f) What is the value of expression:

$\{(-15) + 4 + (-7)\} \div \{(-3) - 9 + 18\}$

(i) 2 ☐ (ii) –3 ☐ (iii) 4 ☐ (iv) 6 ☐

(g) The expression $103 \times (-35)$ can be re-written as which of the following expression ?

(i) $10300 - 105$ ☐ (ii) $-10300 + 105$ ☐

(iii) $3500 + 105$ ☐ (iv) $-3500 - 105$ ☐

2. Find the product of the following :

(a) $(-13) \times (-11)$ (b) $(-4) \times 16$ (c) $(-6) \times (-15)$ (d) $(-125) \times 8$

3. Find the quotient :

(a) $(-64) \div (-16)$ (b) $(-84) \div 4$ (c) $0 \div (-7)$

(d) $168 \div (-8)$ (e) $(-216) \div (-6)$ (f) $120 \div (-15)$

4. Simplify :

(a) $\{(-80) \div 16\} \times (-2) + 6$ (b) $[\{(-3) - (-4)\} \times \{(-6) + (-8)\}]$

(c) $\{(-7) \times (-5) \times (-6)\} \div (-15)$ (d) $[24 \div \{(-6 + 18) - 6\}]$

5. Find the product using suitable properties as:

(a) $(-17) \times 29$ (b) $(-51) \times 13$

(c) $\{(-47) \times 21\} + (-47)$ (d) $[(-24) \times 37] + [(-24) + 13]$

(e) $(99 \times 17) + (17 \times 101)$ (f) $(-12) \times 25 \times 6 \times (-4)$

6. A monkey is trying to reach at the top of vertical slipperly pole 25 m high. In one jump he goes up by 3 m but slips by 1 m. In how many moves he will reach at the top?

7. Rinku gets ₹ 1500 as pocket money per month. He spends ₹ 750 eating outside and ₹ 300 as donation to a charity organisation. Find out Rinku's saving at the end of the year.

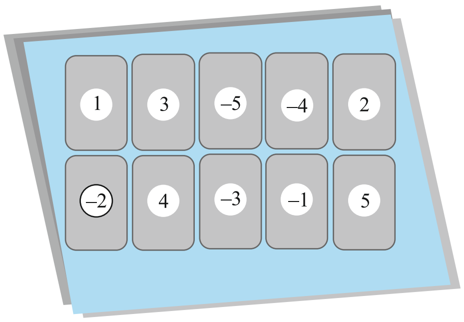
HOTS

Every floor of a 20 storey building is 5 m high. If a lift moves 2 metres every second, how long will it take to move from 3rd floor to 15th floor.

Lab Activity

Objective : To comprehend the operation of multiplication through a game.

Materials Required : 10 square-shaped one coloured cards with integers from –5 to 5 mentioned on it, square paper sketch pens and different coloured counters.



Procedure : The game is to be played ideally among 2 to 4 players.

Step 1 : Take a white colored chart paper. Cut it in squared shape.

Take a chart paper and draw a 9 × 9 grid on it using sketch pen.

32	33	34	35	36	37	38	39	40
31	30	29	28	27	26	25	24	23
14	15	16	17	18	19	20	21	22
13	12	11	10	9	8	7	6	5
–4	–3	–2	–1	0	1	2	3	4
–5	–6	–7	–8	–9	–10	–11	–12	–13
–22	–21	–20	–19	–18	–17	–16	–15	–14
–23	–24	–25	–26	–27	–28	–29	–30	–31
–40	–39	–38	–37	–36	–35	–34	–33	–32

Step 2 : Write the integers starting from – 40 to 40 as shown.

Step 3 : Keep the counters at 0. [Each player will play with only counter].

Step 4 : Rules to be followed:

Each player will choose two cards from the pack of 10 cards randomly. (Integers hidden on other side)

The player has to multiply the integers shown on the card. For example, the appearing numbers and (–4) and 5. So, (–4)×5=(–20). The player will put his counter at (–20) on the board.

Step 5 : Now its turn of the next player sitting next to the first player.

Step 6 : If the product is positive, the counter will move towards 40. If the product is negative it will move towards (–40).

Step 7 : Each player gets another chance to choose two cards after the completion of the first round.

Step 8 : The player whose counter crosses 40 first is the winner.

2 Rational Numbers

When one starts learning Mathematics, he first starts with counting numbers, i.e., 1, 2, 3, 4, 5, 6 ..... (Natural Numbers). Then he learns about 0 and numbers beyond 1 (Whole Numbers). Knowing about the negatives of natural numbers he learns about integers. Suppose one has to find out a part of a whole or a part of a set of objects, he may or may not get the desired result. Therefore, we need to extend the set of whole numbers so as to make the division of whole always possible. This new set of numbers is termed as the set of **rational numbers.** Here, rational number is any number that can be written as a **ratio of two integers.** It means a number is rational, if it can be written as a fraction where both numerator and denominator are integers.

The word 'rational' originates from the word 'ratio' because rational numbers are the ones that can be written in ratio form, $\frac{p}{q}$ where p and q are integers and $q \neq 0$.

Before knowing more about rational numbers, let's refresh the basics of number system.

Natural numbers : The counting numbers are called natural numbers. Thus, 1, 2, 3, 4, 5 ..... are natural numbers.

Whole Numbers : All natural numbers including 0 (zero) are called whole numbers. Thus, 0, 1, 2, 3...... are whole numbers.

We also learnt these properties related with whole numbers:

(a) The sum of two whole numbers is always a whole number.

Example : $0 + 8 = 8$
$17 + 25 = 42$
$29 + 97 = 126$

(b) The product of two whole numbers is always a whole number.

Example : $0 \times 17 = 0$
$8 \times 15 = 120$
$7 \times 19 = 133$

Division of integers by zero is not defined.
i.e $0 \div 7 = 0$
$0 \div (-6) = 0$

(c) The difference of two whole numbers is not always a whole number.

Example : $57 - 30 = 27$
$0 - 68 = (-68)$ (Not a whole number)
$75 - 87 = (-12)$ (Not a whole number)

(d) The division of two whole numbers does not always give a whole number as quotient.

Example: $12 \div 3 = 4$
$14 \div 6 = 2.\overline{3}$ (Not a whole number)
$49 \div 7 = 7$

Integers : The whole numbers with the negatives of natural numbers are called Integer.
Thus, ... –4, –3, –2, –1, 0, 1, 2, 3, 4, ... are all integers.
We learnt from above that subtraction of whole numbers may give us integers.

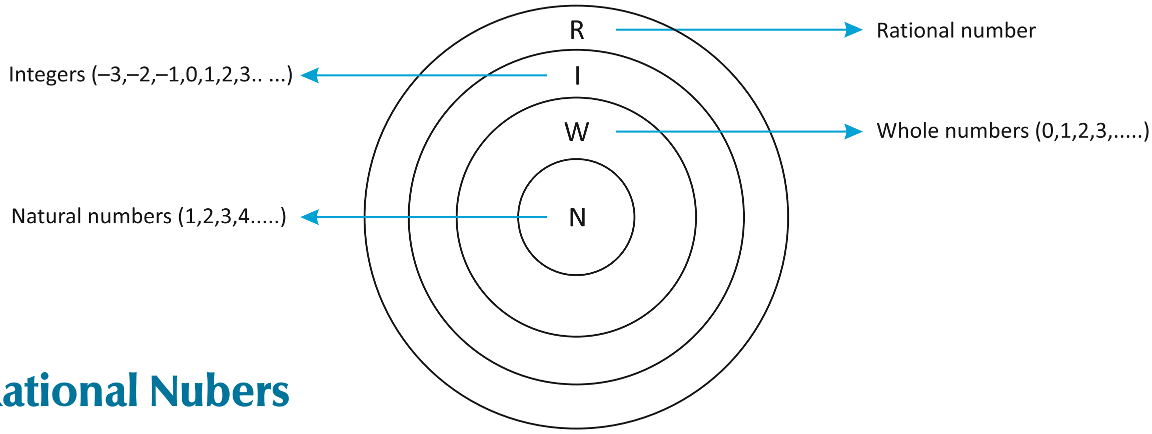
Also, if you recall the previous chapter, while dealing with properties of integers, we saw that the division of one integer by another integer may or may not be an integer.

Example : $24 \div 8 = 3$

$(-72) \div 18 = (-4)$

$(-5) \div 15 = \frac{-5}{15}$ [Not an integer]

$(-3) \div (-18) = \frac{1}{6}$ [Not an integer]



Rational Nubers

Therefore, we need to extend our number system in which negative and positive fractions must be included. This new system of numbers is known as rational number system.

A number that can be represented in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$, is called a rational number. The denominator of a fraction cannot be 0 (zero) because division of any number by zero is not defined.

Example : (a) $\frac{-6}{11}$ is a rational number as both (–6) and 11 are integers and denominator (11) is not equal to 0.

(b) $\frac{-3}{8}$ is a rational number as both (–3) and 8 are integers and denominator (8) is not equal to zero.

(c) 15 is a rational number as $15 = \frac{15}{1}$.

Both 15 and 1 are integers and the denominator (1) is not equal to 0 (zero).

From the above, we can conclude that rational numbers include both integers and fractions.

Facts to Know

- If both the numerator and denominator of a rational number are either both positive or both negative, it is a positive rational number.

Types of Rational Numbers

There are two types of rational numbers:

(a) Positive Rational Numbers **(b) Negative Rational Numbers**

(a) Positive Rational Numbers : A rational number is positive, if its numerator and denominator are either both positive or both negative.

Example : $\frac{2}{7}, \frac{8}{19}, \frac{23}{5}, \frac{0}{4}, \frac{7}{9}$ are positive rational integers

Also, $\frac{-7}{-18}, \frac{-2}{-9}, \frac{-13}{-77}, \frac{-1}{-6}$ are positive rational numbers, because $\frac{-7}{-18} = \frac{7}{18}, \frac{-2}{-9} = \frac{2}{9}, \frac{-13}{-77} = \frac{13}{77}$ and $\frac{-1}{-6} = \frac{1}{6}$

(b) Negative Rational Numbers : A rational number is negative, if either its numerator or denominator is negative.

Example : $\frac{-3}{7}, \frac{3}{-17}, \frac{18}{-29}, \frac{-37}{39}$ are negative rational numbers because

$$\frac{-3}{7} = \left(-\frac{3}{7}\right),\ \frac{3}{-17} = \left(-\frac{3}{17}\right),\ \frac{18}{-29} = \left(-\frac{18}{29}\right),\ \frac{-37}{39} = \left(-\frac{37}{39}\right)$$

Facts to Know

- A rational number $\frac{p}{q}$ is a fraction only when p and q are whole numbers and $q \neq 0$.
- All fractions are rational numbers but a rational number is not always a fraction.
- Negative fractions are also called negative rational numbers.
- All counting numbers (Natural numbers), whole numbers and integers are rational numbers.

Rational Numbers on a Number Line

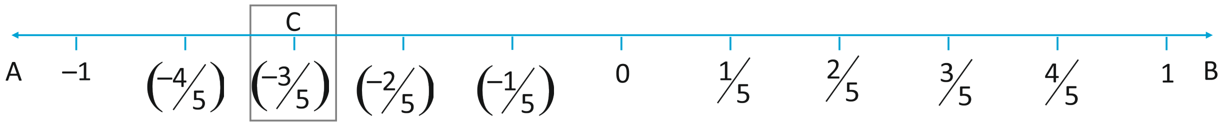
We already know how to represent fractions, integers, whole numbers and natural numbers on number line. Let's learn to denote rational numbers on a number line.

When we move from left to right on the number line, the number increases, whereas, if we move from right to left, the number decreases as shown in the given figure.

To represent a rational number on a number line, each unit length is divided into equal parts of the denominator of a rational number. Then one can easily mark the required rational number on line.

Example 1 : Represent $\frac{-3}{5}$ on a number line.

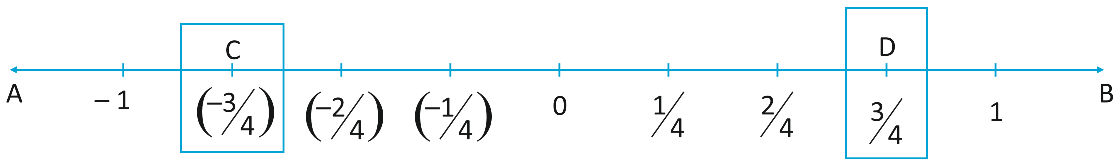
Solution : Here, the denominator of the rational number is 5. So we divide each unit length on the number line AB into 5 equal parts as shown is the figure.



The numerator to be denoted, is (–3), so counting 3 parts to the left of zero on the number line, mark it as point C. Point 'C' represents $\left(\frac{-3}{5}\right)$.

Example 2 : Represent $\left(\frac{-3}{4}\right)$ and $\frac{3}{4}$ on a number line.

Solution : Here, the denominator of the rational number is 4. So we divide each unit length on the number line AB into 4 equal parts as shown in the figure.

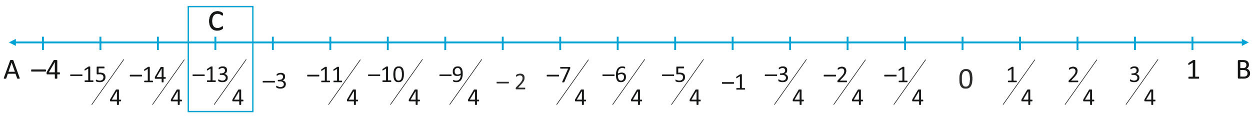


The numerators to be denoted are (–3) and 3. So counting 3 parts to the left of zero on the number line, mark it as point 'C' and counting 3 parts to the right of zero on the number line, mark it as point D. Point C and D represents $\left(-3/4\right)$ and $3/4$ respectively.

Example 3 : Represent $\frac{-13}{4}$ on a number line.

Solution : $\frac{-13}{4} = -3\frac{1}{4} = -3 + \left(\frac{-1}{4}\right)$

Here, the denominator of the rational number is 4. So we divide each unit length on the number line AB into 4 equal parts as shown in the figure.



Since the numerator to be denoted is –13, so count 13 parts to the left of zero on the number line and mark it as point 'C'. Point 'C' represents $\left(\frac{-13}{4}\right)$.

Standard Form of a Rational Number

A rational numbers is said to be in the standard form if its denominator is a positive integer and the numerator and the denominator are co-primes i.e., have no common factor other than 1.

Example : $-\frac{2}{7}, \frac{17}{47}, \frac{8}{19}$ and $\frac{-17}{6}$ are the rational numbers in standard form.

Example 4 : Reduce the following in the standard form :

(a) $\frac{-4}{26}$ (b) $\frac{24}{-16}$

Solution : (a) The denominator of $\left(\frac{-4}{26}\right)$ is positive. To express it in standard form, first find the HCF of 4 and 26, which is 2. Now denominator is positive.

now $\frac{-4}{26} = \frac{(-4)\div 2}{26 \div 2}$

$= \frac{-2}{13}$

So, the standard form of $\left(\frac{-4}{26}\right)$ is $\left(\frac{-2}{13}\right)$.

(b) The denominator of $\left(\frac{24}{-16}\right)$ is negative. To make it positive, multiply both numerator and denominator by (–1), we get

$\frac{(24)\times(-1)}{(-16)\times(-1)} = \frac{-24}{16}$

Now find the HCF of 24 and 16, which is 8. Divide both numerator and denominator by 8, we get

$\frac{(-24)\div 8}{16 \div 8} = \frac{-3}{2}$

So, the standard form of $\left(\frac{24}{-16}\right)$ is $\left(\frac{-3}{2}\right)$.

Absolute Value of a Rational Number

The absolute value of a rational number is its positive numerical value, irrespective of the sign of numerator and denominator, i.e., $\left|\frac{p}{q}\right| = \frac{p}{q}$, $\left|\frac{-p}{q}\right| = \frac{p}{q}$, $\left|\frac{p}{-q}\right| = \frac{p}{q}$ and $\left|\frac{-p}{-q}\right| = \frac{p}{q}$

Example : (a) $\left|\frac{-7}{-6}\right| = \frac{7}{6}$ (b) $\left|\frac{-17}{18}\right| = \frac{17}{18}$

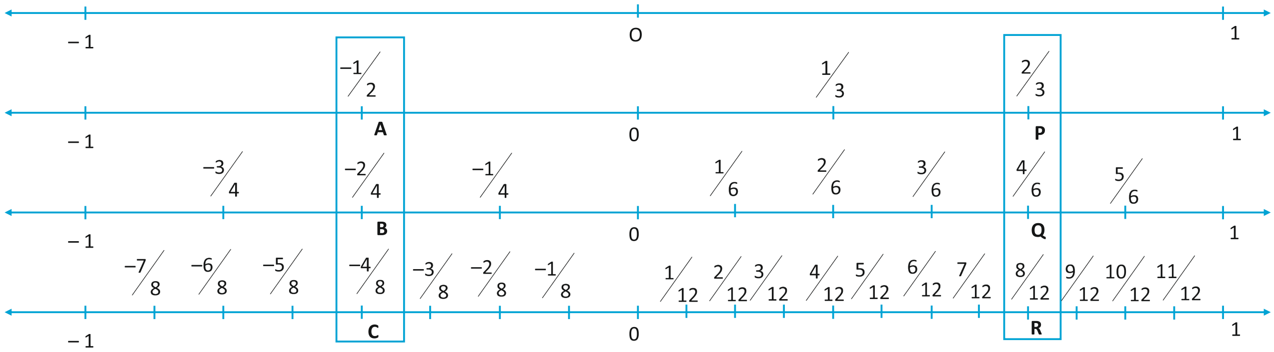
(c) $\left|\frac{27}{8}\right| = \frac{27}{8}$ (d) $\left|\frac{6}{-13}\right| = \frac{6}{13}$

Facts to Know

- The absolute value of rational number, when represented on a number line, is taken as its distance from zero (irrespective of the direction) and it is represented as $\frac{p}{q}$.
- A positive rational number is always greater than a negative rational number.

Equivalent Rational Numbers

Let's understand the concept of equivalent rational numbers through number lines. Draw the number lines as shown in the figure.



One can observe that points P, Q and R representing $\frac{2}{3}$, $\frac{4}{6}$ and $\frac{8}{12}$ respectively are equidistant from point O that represents (0) zero on the number line. In other words, the same point corresponds to these rational numbers. We can say rational numbers $\frac{2}{3}$, $\frac{4}{6}$ and $\frac{8}{12}$ are equivalent. Similarly points A, B and C, representing $\left(\frac{-1}{2}\right)$, $\left(\frac{-2}{4}\right)$, $\left(\frac{-4}{8}\right)$ and respectively are equidistant from point O that represents 0 on the number line. Hence, these are equivalent,

i.e., $\frac{-1}{2} = \frac{-2}{4} = \frac{-4}{8}$

Thus, rational numbers which can be represented by the same point on a number line are called equivalent rational numbers.

Let $\frac{a}{b}$ and $\frac{c}{d}$ be two rational numbers:

(a) $\frac{a}{b} = \frac{c}{d}$ or ad = bc, they are equivalent rational numbers.

(b) If $\frac{a}{b} > \frac{c}{d}$, then ad > cd (c) if $\frac{a}{b} < \frac{c}{d}$, then ad < bc

Equivalent rational numbers can be obtained by multiplying or dividing both the numerator and the denominator of the given rational number by the same non-zero integer.

Example : (a) $\frac{-7}{8} = \frac{-7\times2}{8\times2} = \frac{-14}{16}$ (b) $\frac{13}{18} = \frac{13\times3}{18\times3} = \frac{39}{54}$ (c) $\frac{3}{-13} = \frac{3\times(-3)}{-13\times(-3)} = \frac{-9}{39}$

$\frac{-7}{8} = \frac{-7\times(-5)}{8\times(-5)} = \frac{35}{-40}$ $\frac{13}{18} = \frac{13\times(-2)}{18\times(-2)} = \frac{-26}{-36} = \frac{26}{36}$ $\frac{3}{-13} = \frac{3\times6}{-13\times6} = \frac{18}{-78}$

Example 5 : Show that rational numbers $\frac{3}{5}$ and $\frac{18}{30}$ are equivalent.

Solution : We know that for two rational numbers $\frac{a}{b}$ and $\frac{c}{d}$ to be equivalent ad = bc.

Let $\frac{a}{b} = \frac{3}{5}$ and $\frac{c}{d} = \frac{18}{30}$

ad = 3 × 30 = 90 and bc = 5 × 18 = 90

Since ad = bc = 90, So $\frac{3}{5}$ and $\frac{18}{30}$ are equivalent.

Example 6 : Write four equivalent rational numbers of $\frac{6}{11}$.

Solution : $\frac{6}{11} = \frac{6\times2}{11\times2} = \frac{12}{22}$, $\frac{6}{11} = \frac{6\times3}{11\times3} = \frac{18}{33}$

$\frac{6}{11} = \frac{6\times4}{11\times4} = \frac{24}{44}$, $\frac{6}{11} = \frac{6\times5}{11\times5} = \frac{30}{55}$

So, four equivalent rational numbers of $\frac{6}{11}$ are $\frac{12}{22}, \frac{18}{33}, \frac{24}{44}$ and $\frac{30}{55}$.

Example 7 : Express $\frac{-17}{15}$ as a rational number with numerator 85.

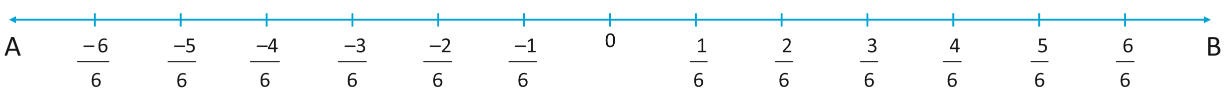
Solution : To express $\frac{-17}{15}$ as a rational number with numerator 85 which when multiplied by (–17), gives 85.

Since 85 ÷ 17 = 5, so multiply both the numerator and denominator by (–5) as (–17) × (–5) = 85.

$\frac{-17}{15} = \frac{(-17)\times(-5)}{15\times(-5)} = \frac{85}{-75}$ So, $\frac{-17}{15} = \frac{85}{-75}$

Comparison of Rational Numbers

Have a look at the number line representing rational numbers as shown in the figure.



From the figure, we learn that

$\frac{-6}{6} < \frac{-5}{6} < \frac{-4}{6} < \frac{-3}{6} < \frac{-2}{6} < \frac{-1}{6} < 0 < \frac{1}{6} < \frac{2}{6} < \frac{3}{6} < \frac{4}{6} < \frac{5}{6} < \frac{6}{6}$ (ascending order)

and $\frac{6}{6} > \frac{5}{6} > \frac{4}{6} > \frac{3}{6} > \frac{2}{6} > \frac{1}{6} > 0 > \frac{-1}{6} > \frac{-2}{6} > \frac{-3}{6} > \frac{-4}{6} > \frac{-5}{6} > \frac{-6}{6}$ (descending order)

The above mentioned rational numbers have same denominator. Therefore, by comparing the numerators we find out which is greater or smaller.

Example : $\frac{-4}{6} > \frac{-5}{6}$ (Since $-4 > -5$)

$\frac{-2}{6} < \frac{5}{6}$ (Since $-2 < 5$)

Other than this, all other rules for the comparison of integers and fractions are applicable to rational numbers also.

(i) All positive rational numbers are greater than 0.

(ii) All positive rational numbers are greater than all negative rational numbers.

(ii) All negative numbers are smaller than 0.

Example : $\frac{-3}{17} < \frac{-1}{17}$ [Since $(-3) < (-1)$]

and $\frac{3}{7} < \frac{18}{7}$ [Since $3 < 18$]

When the rational numbers have different denominators we change them as rational numbers with the same denominator and than compare. We will learn it through examples.

Example 8 : Arrange the following in descending order :

(a) $\frac{1}{12}, \frac{3}{12}, \frac{6}{12}, \frac{5}{12}, \frac{13}{12}$ (b) $\frac{-4}{13}, \frac{4}{13}, \frac{-1}{13}, \frac{12}{13}$

Solution : (a) Since the denominator of the given rational numbers is same i.e. 12, we write their numerators in descending order.

Here, $13 > 6 > 5 > 3 > 1$

So, $\frac{13}{12} > \frac{6}{12} > \frac{5}{12} > \frac{3}{12} > \frac{1}{12}$

(b) Since the denominator of the given rational numbers is same, i.e. 13, so, we write their numerators in descending order:

Here, $12 > 4 > -1 > -4$

So, $\frac{12}{13} > \frac{4}{13} > \frac{-1}{13} > \frac{-4}{13}$

Example 9 : Which is greater : $\frac{5}{6}$ or $\frac{7}{9}$?

Solution : Since the denominators of the given rational numbers are different, we change them as rational numbers with the same denominator and then compare.

$\frac{5}{6} = \frac{5 \times 3}{6 \times 3} = \frac{15}{18}, \frac{7}{9} = \frac{7 \times 2}{9 \times 2} = \frac{14}{18}$

$\frac{15}{18}$ [?] $\frac{14}{18}$

Since $15 > 14$ So, $\frac{15}{18} > \frac{14}{18}$ or, $\frac{5}{6} > \frac{7}{9}$

Facts to Know

- We can also use the following relations to compare two rational numbers :

(i) If $\frac{a}{b} > \frac{c}{d} = ad > bc$ (ii) If $\frac{a}{b} < \frac{c}{d} = ad < bc$

Exercise 2.1

1. **Which of the following is not a rational number ?**

$\frac{-2}{7}, \frac{4}{-13}, 1, \frac{2}{9}, 0, \frac{0}{7}, \frac{6}{0}, \frac{7}{5}$

2. **Convert the following rational numbers into integers:**

$\frac{-4}{-1}, \frac{-42}{14}, \frac{-36}{-18}, \frac{7}{-1}$

3. **Identify the positive rational numbers from the following :**

(a) $\frac{-3}{-4}$ (b) $\frac{6}{8}$ (c) $\frac{-8}{7}$ (d) $\frac{9}{-14}$ (e) $\frac{-8}{-14}$ (f) $\frac{-1}{7}$ (g) $\frac{2}{-6}$ (h) $\frac{-277}{643}$

4. Write any five positive and five negative rational numbers.

5. **Represent the following on a number line:**

(a) $\frac{-3}{4}$ (b) $\frac{-5}{3}$ (c) $\frac{3}{5}, \frac{1}{5}, \frac{7}{5}, \frac{4}{5}$ (d) $\frac{-7}{8}, \frac{-5}{8}, \frac{-3}{8}, \frac{1}{8}, \frac{3}{8}$

6. **Express the following in standard form :**

(a) $\frac{85}{280}$ (b) $\frac{27}{243}$ (c) $\frac{-12}{156}$ (d) $\frac{70}{357}$

7. **Write four equivalent rational numbers for the following:**

(a) $\frac{-3}{7}$ (b) $\frac{2}{3}$ (c) $\frac{-11}{7}$ (d) $\frac{6}{11}$

8. **Find the value of *x* in the following.**

(a) $\frac{x}{-9}=\frac{15}{12}$ (b) $\frac{-3}{4}=\frac{18}{x}$ (c) $\frac{4}{7}=\frac{-12}{x}$ (d) $\frac{-2}{5}=\frac{-8}{x}$ (e) $\frac{27}{15}=\frac{x}{24}$

9. **Observe the given patterns carefully and write the next two rational numbers for the following :**

(a) $\frac{7}{11}, \frac{5}{13}, \frac{3}{15}, \frac{1}{17}, \frac{-1}{19}$, ____, ____. (b) $\frac{5}{7}, \frac{7}{14}, \frac{9}{21}, \frac{11}{28}$, ____, ____.

(c) $\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}$, ____, ____. (d) $\frac{-3}{7}, \frac{-4}{7}, \frac{-5}{7}, \frac{-6}{7}$, ____, ____.

10. **Arrange the following rational numbers in ascending order :**

(a) $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{1}{5}, \frac{1}{6}$ (b) $\frac{-2}{7}, \frac{4}{4}, \frac{6}{7}, \frac{-3}{7}, \frac{-4}{7}$

11. **Arrange the following rational numbers in descending order:**

(a) $\frac{-2}{3}, \frac{4}{3}, \frac{2}{3}, \frac{-1}{3}, \frac{1}{3}, \frac{7}{3}, \frac{-7}{3}$ (b) $\frac{1}{3}, \frac{6}{3}, \frac{7}{3}, \frac{-13}{3}, \frac{-14}{3}, \frac{2}{3}$ (c) $\frac{-1}{2}, \frac{-3}{4}, \frac{4}{3}, \frac{2}{6}, \frac{2}{5}$ (d) $\frac{-7}{5}, \frac{-2}{5}, \frac{2}{5}, \frac{7}{10}, \frac{-6}{5}$

12. **Which of the two given rational numbers is smaller ?**

(a) $\frac{-3}{7}, \frac{-4}{9}$ (b) $\frac{14}{27}, \frac{7}{9}$ (c) $\frac{-6}{17}, \frac{-8}{15}$ (d) $\frac{-4}{7}, \frac{-3}{4}$

Addition of Rational Numbers

For addition of rational numbers, we should follow these steps:

(i) Express each rational number with positive denominator.

(ii) If the denominators are same, add their numerator and divide it by common denominator.

(iii) If the denominators are different, express them as equivalent rational numbers with same denominator. Now add the numerators and divide it by the common denominator of the equivalent rational numbers .

(iv) If required, change the result into standard form.

Example 10 : **Add the following:**

(a) $\frac{-8}{19}+\frac{-2}{57}$ (b) $\frac{-18}{29}+\left(\frac{-4}{29}\right)$

Solution : (a) $\frac{-8}{19}+\frac{-2}{57}=\frac{-8x3+(-2)x1}{57}$

$=\frac{-24+(-2)}{57}=\frac{-26}{57}$ [LCM of 19 and 57 in 57]

(b) $\frac{-18}{29}+\frac{-4}{29}=\frac{-18+(-4)}{29}$

$=\frac{-18-4}{29}=\frac{-22}{29}$

Example 11 : **Add the following:**

(a) $\frac{-17}{4}$ and $\frac{-37}{8}$ (b) $\frac{-4}{9}$ and $\frac{-7}{27}$

Solutions : (a) $\frac{-17}{4}+\left(\frac{-37}{8}\right)$ (8 is a multiple of 4)

$=\frac{-17\times2}{4\times2}+\left(\frac{-37}{8}\right)$

$=\frac{-34}{8}+\left(\frac{-37}{8}\right)$

$=\frac{-34-37}{8}=\frac{-71}{8}$

(b) $\frac{-4}{9}+\left(\frac{-7}{27}\right)$

$=\frac{-4\times3}{9\times3}+\left(\frac{-7}{27}\right)$

$=\frac{-12}{27}+\left(\frac{-7}{27}\right)$

$=\frac{-12-7}{27}=\frac{-19}{27}$

Example 12 : **Solve the following:**

(a) $\frac{2}{3}+\frac{4}{5}+\frac{3}{4}$ (b) $\frac{-3}{4}+\frac{7}{10}+\left(\frac{-5}{12}\right)$

Solution : (a) $\frac{2}{3}+\frac{4}{5}+\frac{3}{4}=\frac{2\times20}{3\times20}+\frac{4\times12}{5\times12}+\frac{3\times15}{4\times15}$

[LCM of 3, 5 and 4 = 60]

$=\frac{40}{60}+\frac{48}{60}+\frac{45}{60}$

$=\frac{40+48+45}{60}=\frac{133}{60}$

(b) $\frac{-3}{4}=\frac{-3\times15}{4\times15}=\frac{-45}{60}$

$\frac{7}{10}=\frac{7\times6}{10\times6}=\frac{42}{60}$

$\frac{-5}{12}=\frac{-5\times5}{12\times5}=\frac{-25}{60}$

[LCM of 4, 10 and 12=60]

Now, $\frac{-3}{4}+\frac{7}{10}+\left(\frac{-5}{12}\right)=\frac{-45}{60}+\frac{42}{60}+\left(\frac{-25}{60}\right)$

$=\frac{-45+42-25}{60}=\frac{-28}{60}=\frac{-7}{15}$

Subtraction of Rational Numbers

We know that subtraction is the inverse process of addition. We can subtract a rational numbers from another by adding its additive inverse to it. Let $\frac{p}{q}$ and $\frac{r}{s}$ be two rational numbers. To subtract $\frac{r}{s}$ from $\frac{p}{q}$ we add additive inverse of $\frac{r}{s}$ to $\frac{p}{q}$.

Thus, $\frac{p}{q} - \frac{r}{s} = \frac{p}{q} + \left(\frac{-r}{s}\right)$

Example 13 : Subtract $\frac{-7}{8}$ from $\frac{5}{7}$.

Solution : The additive inverse of $= \frac{-7}{8} = -\left(\frac{-7}{8}\right) = \frac{7}{8}$

Now, we have to add additive inverse of $\frac{-7}{8}$ to $\frac{5}{7}$

$$\frac{5}{7} - \left(\frac{-7}{8}\right) = \frac{5}{7} + \frac{7}{8}$$

$$= \frac{(5\times 8)+(7\times 7)}{56}$$

$$= \frac{40+49}{56}$$

$$= \frac{89}{56}$$

Example 14 : Subtract $\frac{5}{7}$ from $\frac{18}{7}$.

Solutions : $\frac{18}{7} - \frac{5}{7}$

$$\frac{18}{7} - \frac{5}{7} = \frac{18}{7} + \left(\frac{-5}{7}\right)$$

$$= \frac{18+(-5)}{7}$$

$$= \frac{13}{7}$$

Facts to Know

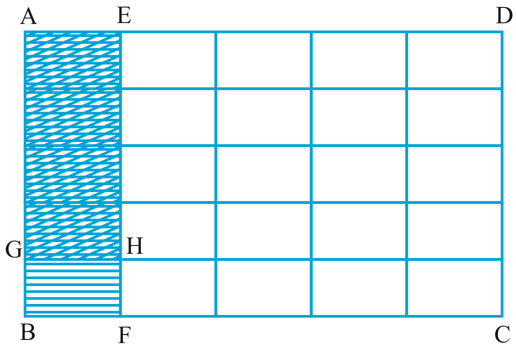
- There are infinite rational numbers between two consecutive rational numbers.

Multiplication of Rational Numbers

Let's learn multiplication of rational numbers $\left(\frac{4}{5}\times\frac{1}{5}\right)$ with the help of a diagram.

The shaded part ABFE shows $\frac{1}{5}$ of ABCD

The double-shaded part AGHE shows $\frac{4}{5}$ out of $\frac{1}{5}$.



From the given figure, it is clear that the double–shaded part shows $\frac{4}{25}$

So, $\frac{4}{5}$ of $\frac{1}{5}=\frac{4}{25}$

or $\frac{4}{5}\times\frac{1}{5}=\frac{4}{25}$

Also, we observe that $\frac{4}{5}\times\frac{1}{5}=\frac{4\times1}{5\times5}$

Hence, it can be concluded that the product of two rational numbers is a rational number whose numerator is the product of numerators of the denominators. If $\frac{p}{q},\frac{r}{s}$ and $\frac{t}{u}$ are three rational numbers, then

$$\frac{p}{q}\times\frac{r}{s}\times\frac{t}{u}=\frac{p\times q\times t}{q\times s\times u}=\frac{\text{Product of numerators}}{\text{Product of denominators}}$$

Example 15 : **Find the product of $\frac{-12}{17}$ and $\frac{8}{5}$**

Solution : $\frac{-12}{17}\times\frac{8}{5}=\frac{(-12)\times8}{17\times5}$

$=\frac{-96}{85}$

Example 16 : **Find the product of $\frac{2}{3},\frac{-4}{5},\frac{6}{7}$ and $\frac{-3}{4}$**

Solution : $\frac{2}{3}\times\left(\frac{-4}{5}\right)\times\frac{6}{7}\times\left(\frac{-3}{4}\right)=\frac{2\times(-4)\times6\times(-3)}{3\times5\times7\times4}=\frac{12}{35}$

Division of Rational Numbers

It is known to us that division is the inverse of multiplication i.e., if p and q two integers, then p ÷ q = $p\times\frac{1}{q}$, Here we multiply the dividend by the multiplicative inverse of the divisor. The same rule is applied for division of rational numbers. If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers, then $\frac{a}{b}\div\frac{c}{d}=\frac{a}{b}\times\frac{d}{c}$ $\left[\frac{c}{d}\neq0\right]$

Example 17 : **Divide $\frac{-8}{9}$ by $\frac{12}{13}$**

Solution : $\frac{-8}{9}\div\frac{12}{13}=\frac{-8}{9}\times\frac{13}{12}$ $\left[\textit{Multiplicative inverse of } \frac{12}{13} \textit{ is } \frac{13}{12}\right]$

$=\frac{-8\times13}{9\times12}=\frac{-26}{27}$

Facts to Know

- The sum of a rational number and its additive inverse is 0 (zero).
- The product of a rational number and its reciprocal is always 1.

Example 18 : Divide $\frac{-3}{7}$ by $\frac{3}{5}$

Solutions : $\frac{-3}{7} \div \frac{3}{5} = \frac{-3}{7} \times \frac{5}{3}$ [*Multiplicative inverse of* $3/5$ *is* $5/3$]

$= \frac{(-3) \times 5}{7 \times 3}$

$= \frac{-5}{7}$

Exercise 2.2

1. Solve the following:

(a) $\frac{3}{5} + \frac{6}{5} + \frac{8}{5}$ (b) $\frac{-5}{6} + \left(\frac{-3}{7}\right)$ (c) $\frac{4}{7} + \left(\frac{-2}{6}\right) + \left(\frac{-2}{3}\right)$

(d) $\frac{-3}{11} + \left(\frac{-4}{11}\right)$ (e) $\frac{5}{7} + \frac{4}{21} + \left(\frac{-3}{14}\right)$ (f) $\frac{-3}{4} + \left(\frac{-4}{5}\right) + \left(\frac{-6}{7}\right) + \left(\frac{-7}{8}\right)$

2. Add the following:

(a) $\frac{3}{4}, \frac{5}{6}$ and $\frac{-2}{3}$ (b) $\frac{-8}{7}$ and $\frac{-4}{7}$ (c) $\frac{13}{15}$ and $\frac{-7}{25}$

(d) $\frac{1}{3}$ and $\frac{-3}{4}$ (e) $\frac{5}{6}$ and $\frac{-6}{7}$ (f) $\frac{-7}{15}$ and $\frac{12}{5}$

3. Subtract the following :

(a) 14 from $\frac{-7}{8}$ (b) $\frac{-4}{5}$ from $\frac{-7}{12}$ (c) $\frac{4}{5}$ from $\frac{-4}{7}$

(d) $\frac{5}{7}$ from $\frac{7}{5}$ (e) $\frac{-3}{8}$ from $\frac{-7}{8}$ (f) $\frac{-5}{8}$ from $\frac{-11}{12}$

4. Find the product of the following:

(a) $\frac{4}{5} \times \left(\frac{-15}{21}\right) \times \frac{14}{25}$ (b) $\frac{-7}{8} \times \left(\frac{-5}{6}\right)$ (c) $\frac{-3}{4} \times \left(\frac{-5}{6}\right) \times \left(\frac{-7}{8}\right)$

(d) $\frac{-5}{7} \times \left(\frac{-8}{15}\right) \times \left(\frac{21}{16}\right)$ (e) $\frac{4}{7} \times \left(\frac{-3}{5}\right) \times \frac{14}{24}$ (f) $\frac{-11}{5} \times \left(\frac{-15}{22}\right) \times \frac{3}{5}$

5. Simplify the following:

(a) $\left\{\frac{4}{7} \times \frac{21}{16}\right\} \div \frac{7}{8}$ (b) $\frac{-5}{8} + \frac{6}{7} - \frac{2}{3}$ (c) $\frac{-3}{4} \times \left[\frac{3}{5} - \frac{2}{3}\right]$

(d) $\frac{-7}{4} \times \left[\frac{-7}{8} + \frac{9}{16}\right]$ (e) $\frac{-3}{5} - \frac{(-4)}{15} + \frac{-7}{10}$

6. The sum of two rational numbers is $\frac{-3}{7}$. If one of them is $\frac{2}{3}$, find the other number.

7. What should be added to $\left(\frac{-3}{4} + \frac{5}{7}\right)$ to get $\frac{-6}{14}$.

8. Find the product of $\frac{-3}{8}$ and the reciprocal of $\frac{15}{16}$.

Rational Numbers as Decimals

We can express a rational number into a decimal either by the long division method or by writing an equivalent rational number with the denominator as power of 10, and then write the equivalent rational number as a decimal.

Example 19 : **Express $\frac{7}{4}$ as a decimal number by writing an equivalent rational number with denominator as power of 10.**

Solution : $\frac{7}{4} = \frac{7 \times 25}{4 \times 25} = \frac{175}{100} = 1.75$

$\therefore \frac{7}{4} = 1.75$

Example 20 : **Express $\frac{3}{25}$ as a decimal number by using long division method.**

Solution : $\frac{3}{25} = 3 \div 25$

```
25) 3.0 (0.12
   -2 5
   ----
      5 0
     -5 0
     ----
        0
     ----
```

$\therefore \frac{3}{25} = 0.12$

Terminating and Non-Terminating Decimals

When a rational number is converted into decimals by division method, any one of the following two conditions will arise:

(a) The division process comes to an end after some steps, as there is no remainder left at certain point of time. Such decimals are called terminating decimals.

Example : $\frac{1}{2} = 0.5$, $\frac{-3}{8} = -0.375$ etc.

(b) The division process goes on indefinitely as there may be a remainder at each step. Such decimals are called non-terminating decimals.

Example : $\frac{1}{3} = 0.333....$

```
    0.333....
   ----------
3 ) 10
   -9
   ---
     10
    -9
    ---
      10
     -9
     ---
       1
     ---
```

The rational number in which the division process does not come to an end and keeps repeating, is called non-terminating repeating decimal. To represent such decimals, we put a bar sign (–) above the repeating part. So, $\frac{1}{3}$ = 0. 333... = $0.\overline{3}$.

Example 21 : **Express the following rational numbers as decimals using the division method. Do write whether these represent terminating or non-terminating decimals.**

(a) $\frac{5}{6}$ (b) $\frac{2}{7}$ (c) $\frac{5}{8}$

Solution : (a) $\frac{5}{6} = 5 \div 6$

```
6 ) 50000 ( 0.8333
   -48
     20
    -18
      20
     -18
       20
      -18
        2
```

$\therefore \frac{5}{6} = 0.8333 ...... = 0.8\overline{3}$

Thus, $\frac{5}{6} = 0.8\overline{3}$ represent non-terminating decimals.

(c) $\frac{5}{8} = 5 \div 8$

```
8 ) 5.000 ( 0.625
   -48
     20
    -16
      40
    - 40
       0
```

$\therefore \frac{5}{8} = 0.625$

Thus, $\frac{5}{8} = 0.625$ represents terminating decimals.

(b) $\frac{2}{7} = 2 \div 7$

```
7 ) 2.00000000 ( 0.28571428
   -14
     60
    -56
      40
     -35
       50
      - 49
         10
        - 7
          30
        - 28
           20
         - 14
            60
           -56
             4
```

$\therefore \frac{2}{7} = 0.28571428.... = 0.\overline{285714}$

Thus $\frac{2}{7} = 0.\overline{285714}$ represents non-terminating decimals.

It is to note that non-terminating non-repeating decimals can not be converted into rational number. Such type of numbers are called irrational numbers.

Facts to Know

- Every rational number can be converted into either a terminating decimal or non-terminating repeating decimal.
- Such decimals which are non-terminating and have no repeating parts are called irrational numbers.

Rule to Find Terminating or Non-terminating Repeating Decimals

Rule for Terminating Decimals : If a rational number is in its lowest term and its denominator has no multiple other than 2 or 5 or both.

Example : $\frac{1}{4}, \frac{3}{25}$ and $\frac{1}{250}$ represent terminating decimals.

Rule for Non-terminating Repeating Decimals : If a rational number is in its lowest term and its denominator has a prime factor other than 2 and 5.

Example : $\frac{3}{7}, \frac{5}{6}$ and $\frac{7}{15}$ are examples of non-terminating repeating decimals.

Example 22 : Without actual division, determine which of the following rational numbers have a terminating decimal representation:

(a) $\frac{7}{225}$ (b) $\frac{3}{16}$ (c) $\frac{9}{75}$

Solution : (a) In $\frac{7}{225}$ the denominator is 225.

We have, $225 = 5 \times 5 \times 3 \times 3$

Thus, 225 has 3 as a prime number that is other than 2 and 5.

Hence, $\frac{7}{225}$ must have a non-terminating decimal representation.

(b) In $\frac{3}{16}$ the denominator is 16.

We have, $16 = 2 \times 2 \times 2 \times 2$

Thus, 16 has 2 as the only prime factor.

Hence, $\frac{3}{16}$ must have a terminating decimal representation.

(c) In $\frac{9}{75}$ the denominator is 75.

We have, $75 = 5 \times 5 \times 3$

Thus, 75 has 3 as a prime number that is other than 2 and 5.

Hence, $\frac{9}{75}$ must have a non-terminating decimal representation.

Conversion of Non-terminating Repeating Decimals into Rational Numbers

There are two types of decimal representation of non-terminating repeating decimals:

(i) **Pure Repeating or Recurring Decimal :** A decimal presentation in which all the digits after the decimal point are repeated.

Example : 0.67, 0.7 and 0.123 are recurring decimals.

(ii) **Mixed Repeating or Recurring Decimal :** A decimal presentation in which at least one digit after the decimal point is non-repeating.

Example : $1.2\overline{345}$, $4.2\overline{35}$ and $1.01\overline{25}$ are mixed recurring decimals.

Let's learn conversion of non-terminating repeating decimals into rational numbers through examples.

Example 23 : Convert the following decimals in the form of $\frac{p}{q}$:

(a) $0.\overline{8}$ (b) $0.\overline{87}$ c) $7.\overline{23}$ (d) $0.\overline{723}$

Solution : (a) Let $x = 0.\overline{8}$ ....(i)

Here, only one digit in decimal part is repeated, we multiply it by 10, we get,

$10x = 8.\overline{8}$ .... (ii)

Subtracting (i) from (ii), we get,

$10x - x = 8.\overline{8} - 0.\overline{8}$

$\Rightarrow 9x = 8$

$\Rightarrow x = \frac{8}{9}$

(b) Let $x = 0.\overline{87}$ ......(i)

Here, only two digits in decimal part is repeated, we multiply it by 100, we get,

$100x = 87.\overline{87}$ ....(ii)

Subtracting (i) from (ii), we get,

$\Rightarrow \quad 99x = 87$

$\Rightarrow \quad x = \frac{87}{99}$

(c) Let $x = 7.\overline{23}$ ....(i)

Here, only two digits in decimal part is repeated, we multiply it by 100, we get,

$100x = 723.\overline{23} - 7.\overline{23}$ ....(ii)

Subtracting (i) from (ii) we get,

$100x - x = 723.\overline{23} - 7.\overline{23}$

$\Rightarrow \quad 99x = 716$

$\Rightarrow \quad x = \frac{716}{99}$

(d) Let $x = 0.723$

Here, we have 3 digits in the decimal part, out of which only one is repeating.

First we multiply it by 100 so that only the repeating decimal is left on the right side of the decimal point.

$\therefore \quad 100x = 72.\overline{3}$ .... (i)

Now, only one digit is repeating, so we again multiply it by 10, we get

$1000x = 723.\overline{3}$ .... (ii)

Subtracting (i) from (ii), we get,

$1000 - 100x = 723.\overline{3} - 72.\overline{3}$

$\Rightarrow \quad 900x = 651$

$\Rightarrow \quad x = \frac{651}{900}$

Short-cut Method of Converting a Non-terminating Decimal into Rational Numbers:

To convert a recurring decimals into $\frac{p}{q}$ form, write repeated figure only once in the numerator and take as many nines in the denominator as the number of repeated digits

Example : $0.\overline{37} = \frac{37}{99}$ and $0.\overline{123} = \frac{123}{999}$

To convert a mixed recurring decimal into $\frac{p}{q}$ form, its numerator is obtained by removing the decimal point and bar and then subtract the non-repeating number. The denominator will carry as many nines as the number of digits in the repeating part followed by as many zero as the numbers of digits in the non-repeating part after decimal point.

Example : $0.2\overline{37} = \frac{237-2}{990} = \frac{235}{990}$

Example 24 : **Evaluate the following using short-cut method:**

(a) $3.\overline{67} + 4.\overline{58}$ (b) $2.3\overline{53} - 1.1\overline{25}$

Solution : (a) $3.\overline{67} + 4.\overline{58} = (3+4) + (0.\overline{67} + 0.\overline{58})$

$= 7 + \left(\frac{67}{99} + \frac{58}{99}\right)$

$$= 7 + \frac{125}{99}$$

$$= 7 + 1\frac{26}{99}$$

$$= 8\frac{26}{99}$$

(b) $2.3\overline{53} - 1.1\overline{25}$

$$= (2 + 0.3\overline{53}) - (1 + 0.1\overline{25})$$

$$= \left(2 + \frac{353-3}{990}\right) - \left(1 + \frac{125-1}{990}\right)$$

$$= 2 + \frac{350}{990} - 1 - \frac{124}{990}$$

$$= (2-1) + \left(\frac{350}{990} - \frac{124}{990}\right)$$

$$= 1 + \frac{350-124}{990}$$

$$= 1 + \frac{226}{990}$$

$$= 1\frac{113}{495}$$

Exercise 2.3

1. Convert the following into decimals by writing an equivalent rational number with denominator as power of 10:

(a) $\frac{13}{5}$ (b) $\frac{-7}{25}$ (c) $\frac{3}{125}$ (d) $\frac{-7}{40}$

2. Without actual division state whether the following rational numbers represent terminating or non-terminating decimals :

(a) $\frac{5}{18}$ (b) $\frac{17}{8}$ (c) $\frac{-23}{75}$ (d) $\frac{27}{64}$

(e) $\frac{-7}{25}$ (f) $\frac{12}{29}$ (g) $\frac{8}{125}$ (h) $\frac{-19}{20}$

3. Convert the following rational numbers into decimal numbers:

(a) $\frac{2}{15}$ (b) $\frac{3}{7}$ (c) $\frac{7}{8}$ (d) $\frac{17}{25}$

(e) $\frac{12}{24}$ (f) $\frac{13}{4}$ (g) $\frac{125}{8}$ (h) $\frac{12}{25}$

4. Convert the following decimals into rational numbers:

(a) 1.25 (b) 7.025 (c) $3.\overline{7}$ (d) $2.3\overline{45}$

(e) $0.2\overline{3}$ (f) $3.\overline{356}$ (g) $0.7\overline{8}$ (h) $1.02\overline{3}$

5. **Find the value of the following :**

(a) $2.\overline{3}+3.\overline{4}$ (b) $3.\overline{34}+6.\overline{78}$ (c) $1.2\overline{35}-0.\overline{785}$ (d) $18.\overline{63}-7.\overline{683}$

6. If $\frac{x}{y} = 2.3\overline{6} + 1.7\overline{3}$, find $\frac{x}{y}$.

7. If $\frac{x}{y} = 1.\overline{356} - 1.\overline{067}$, find the least value of x and y.

8. **Which of the following decimals can be expressed as rational numbers ?**

(a) 0.3333 .... (b) 0.127272727............. (c) 3.4010010001.............

(d) 13.35735735 .... (e) 2.0010020003............. (f) 7.125125.............

Points to Remember

- A rational number can be expressed in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$.
- All fractions are rational numbers but all rational numbers are not fractions.
- All counting numbers, whole numbers and integers are rational numbers.
- All rational numbers can be represented on a numbers line.
- The absolute value of a rational number is equal to its positive numeric value.
- For two rational numbers $\frac{a}{b}$ and $\frac{c}{d}$:

  (i) $\frac{a}{b} = \frac{c}{d} \Rightarrow ad = bc$ (Equivalent) (ii) $\frac{a}{b} > \frac{c}{d} \Rightarrow ad > bc$ (iii) $\frac{a}{b} < \frac{c}{d} \Rightarrow ad < bc$
- A rational number is in standard form if the HCF of its numerator and denominator is 1.
- For two rational numbers $\frac{a}{b}$ and $\frac{c}{d}$, $\frac{a}{b} + \frac{c}{d} = 0$ then $\frac{c}{d}$ is called the additive inverse of $\frac{a}{b}$ and vice versa.
- The product of a rational number and its reciprocal is always 1.
- A rational number can be converted into decimal either by long division method or by writing an equivalent rational number with the denominator as power of 10.
- Rational numbers can be expressed either as terminating or non-terminating repeating decimals.
- Decimals which are non-terminating and non-repeating are called irrational numbers.

EXERCISE

1. **MULTIPLE CHOICE QUESTIONS (MCQs):**

Tick (✓) the correct options :

(a) Only rational number whose absolute value is 0, is

(i) 0 ☐ (ii) –1 ☐ (iii) +1 ☐ (iv) –9 ☐

(b) Additive inverse of $\frac{-17}{14}$ is :

(i) $\frac{14}{17}$ ☐ (ii) $\frac{17}{14}$ ☐ (iii) $\frac{17}{-14}$ ☐ (iv) $\frac{-14}{-17}$ ☐

(c) Only rational number which is neither positive nor negative is :

(i) 0 ☐ (ii) 1 ☐ (iii) 100 ☐ (iv) None of these ☐

(d) The largest rational numbers is :

(i) 2^n ☐ (ii) 10^n ☐ (iii) $\left(\frac{p}{q}\right)^n$ ☐ (iv) non determinable ☐

(e) Multiplicative inverse of $\frac{-7}{18}$ is :

(i) $\frac{18}{7}$ ☐ (ii) $\frac{18}{-7}$ ☐ (iii) $\frac{7}{18}$ ☐ (iv) $\frac{-7}{-18}$ ☐

(f) $\frac{-312}{169}$ in the standard form is :

(i) $\frac{24}{14}$ ☐ (ii) $\frac{27}{13}$ ☐ (iii) $\frac{24}{13}$ ☐ (iv) $\frac{29}{17}$ ☐

(g) If $\frac{-x}{6}=\frac{42}{-36}$, then the value of x is:

(i) 7 ☐ (ii) −7 ☐ (iii) 14 ☐ (iv) −14 ☐

(h) The example of irrational number is :

(i) $\frac{2}{5}$ ☐ (ii) $\frac{3}{7}$ ☐ (iii) $\frac{7}{100}$ ☐ (iv) 0.1234564325624.... ☐

2. Represent the following on the numbers line:

(a) $\frac{-5}{3}$ (b) $\frac{-3}{4}$ (c) $\frac{-2}{5}$ and $\frac{2}{5}$ (d) $\frac{-3}{5}$ and $\frac{3}{5}$

3. Convert the following rational numbers in standard form:

(a) $\frac{-65}{225}$ (b) $\frac{161}{343}$ (c) $\frac{-125}{1000}$ (d) $\frac{55}{3250}$

4. Find the absolute value of the following rational numbers:

(a) $\frac{-3}{4}+\frac{3}{7}-\frac{6}{8}$ (b) $\frac{5}{9}+\left(\frac{-6}{24}\right)$ (c) $\frac{-2}{3}-\frac{3}{5}$ (d) $3-\frac{4}{5}$

5. In each of the following pairs of rational numbers, which is greater ?

(a) $\frac{3}{4},\frac{5}{7}$ (b) $\frac{-7}{3},-\frac{5}{3}$ (c) $\frac{5}{11}$ and $\frac{-3}{-7}$ (d) $\frac{-2}{3},\frac{3}{4}$

6. Arrange the following rational numbers in ascending order:

(a) $\frac{2}{3},\frac{3}{4},\frac{5}{8},\frac{-7}{8},\frac{-5}{6}$ (b) $\frac{-5}{14},\frac{3}{10},\frac{-3}{7},\frac{-6}{35}$ (c) $\frac{-2}{3},\frac{-5}{6},\frac{2}{3},\frac{5}{6},\frac{-1}{3},\frac{1}{3}$ (d) $-2,\frac{-1}{2},\frac{1}{2},0,2,\frac{-3}{5},\frac{3}{5}$

7. Simplify :

(a) $\frac{-3}{7}+\left(\frac{-4}{7}\right)+\left(\frac{-6}{7}\right)$ (b) $\frac{3}{5}+\left(\frac{-4}{14}\right)+\left(\frac{-2}{5}\right)+\frac{6}{15}$ (c) $\frac{-6}{17}+\frac{5}{51}$ (d) $\frac{-1}{2}-\frac{2}{3}-\frac{3}{4}-\frac{5}{6}$

8. Solve :

(a) $\frac{-7}{17}-0$ (b) $\frac{-12}{13}-\left(\frac{-15}{26}\right)$ (c) $\frac{-8}{26}-\left(\frac{24}{26}\right)$ (d) $\frac{9}{13}-\left(\frac{-4}{13}\right)$

9. Simplify: (a) $\frac{-3}{8}\times\frac{16}{15}\times\left(\frac{-24}{75}\right)\times\frac{20}{12}$ (b) $\frac{-6}{5}\times\left(\frac{-4}{7}\right)\times\left(\frac{21}{-12}\right)\times\left(\frac{-3}{-4}\right)$

10. Divide the sum of $\frac{7}{8}$ and $\frac{-3}{4}$ by the product of $\frac{-2}{3}$ and $\frac{-3}{4}$.

11. What rational number should we multiply to $\frac{-5}{6}$ to get the product 24 ?

12. If $x=\frac{-3}{4}$ and $y=\frac{-2}{3}$, find the value of $3x-4y$.

13. **Find the value of *x* for the following :**

(a) $\frac{-6}{7} + x = \frac{-7}{8}$ (b) $\frac{2}{5} \div x = \frac{-3}{7}$ (c) $x - \left(\frac{-5}{7}\right) = \frac{7}{5}$

14. **Convert the following rational numbers into decimals:**

(a) $\frac{2}{5}$ (b) $\frac{6}{7}$ (c) $\frac{3}{50}$ (d) $\frac{5}{12}$

15. **Convert the following decimals into rational numbers:**

(a) 0.025 (b) 3.15 (c) $25.\overline{5}$ (d) $0.\overline{379}$

16. **Simplify of the following:**

(a) $4.2\overline{3} + 3.\overline{79}$ (b) $2.\overline{37} - 3.\overline{25} + 1.\overline{23}$ (c) $0.\overline{78} + 0.6\overline{7}$ (d) $3.7\overline{86} + 37.\overline{86}$

17. If $\frac{p}{q} = 3.\overline{7} + 7.\overline{3}$, find $\frac{p}{q}$.

18. **Evaluate the following:**

(a) $2.\overline{3} \times 1.\overline{2}$ (b) $1.2\overline{5} \div 0.\overline{5}$

HOTS

There are 100 students in a school. Each student is required to participate in an extracurricular activity. The choices are art, cricket, basketball and swimming. $\frac{3}{10}$ of students are in art, $\frac{1}{10}$ are in cricket and 17 play basketball. How many students are going to participate in swimming?

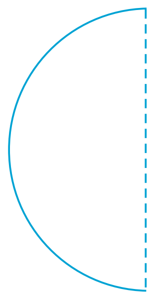
Lab Activity

Objective : To multiply two rational numbers by folding circular paper.

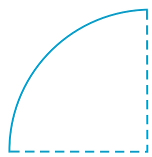
Materials Required : Circular paper strip and sketch pen.

Procedure : Let us find the product of two rational numbers, say $\frac{2}{4}$ and $\frac{1}{2}$. Take circular paper strip and follow these steps :

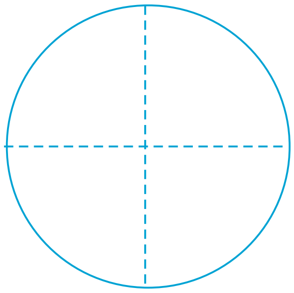
Step 1 : Fold the circular strip into two halves



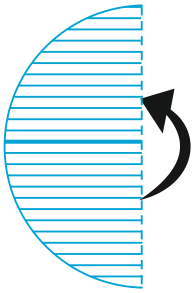
Step 2 : Fold the strip again.



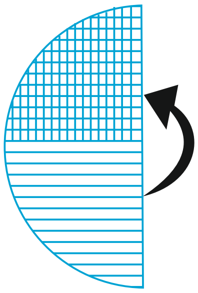
Step 3 : Unfold the strip and your will get the shape like this.



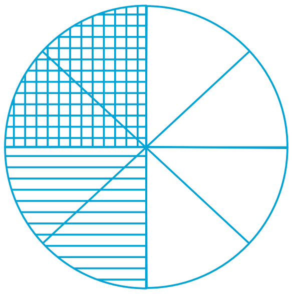
Step 4 : Shade two parts with horizontal lines and fold the rest unshaded part.



Step 5 : Now, fold the strip horizontally to divide the strip into two parts and shade it with vertical lines in one out of two parts.



Step 6 : Unfold the strip. You will find that 2 out of 8 parts are double shaded. This illustrates that the double shaded region represents $\frac{2}{8}$ of the whole strip.



Conclusion : $\frac{2}{4} \times \frac{1}{2} = \frac{2}{8}$

3 Fractions

A fraction is a part of whole. We have studied already about fractions, their addition and subtraction in previous class. Fractions are one of the most important concepts in Mathematics. We come across their application in our daily life. For example, we say half cup of milk i.e. one part out of two equal parts. Similarly, $\frac{2}{3}$ of a kilometre, $1\frac{1}{2}$ metres of cloth, etc. We are also familiar about two parts of a fraction i.e., **numerator** and **denominator**.

For Example : $\frac{4}{7}$ 4 ← **Numerator**, 7 ← **Denominator**

Facts to Know

- An improper fraction is a combination of whole number and proper fraction.

Type of Fractions

Proper fraction: This fractions has numerator less than denominator.

Examples : $\frac{2}{5}, \frac{7}{19}, \frac{4}{13}, \ldots$

A proper fraction is always less than 1.

Improper fraction : This fraction has numerator bigger than denominator.

Examples : $\frac{6}{5}, \frac{15}{11}, \frac{17}{13}, \ldots$

Mixed fraction : This fraction has both whole number part and fractional part.

Examples : $2\frac{1}{4}, 5\frac{3}{7}, 4\frac{2}{7} \ldots$

While dealing with improper fraction and mixed fraction, we should keep their denominators same.

Unit fraction This fraction has numerator 1.

Examples : $\frac{1}{17}, \frac{1}{5}, \frac{1}{11}, \ldots$

Like fractions : These fractions have same denominators.

Examples : $\frac{7}{6}, \frac{5}{6}, \frac{13}{6}, \ldots$

Unlike fractions : These fractions has different denominators.

Examples : $\frac{1}{5}, \frac{7}{8}, \frac{1}{2}, \ldots$

We have learnt that only like fractions can be added or subtracted. For example : $\frac{2}{7} + \frac{3}{7} = \frac{5}{7}$; $\frac{7}{9} - \frac{5}{9} = \frac{2}{9}$

If we have to add or subtract unlike fractions, then we must convert unlike fractions into like or equivalent fractions with same denominators.

Example 1 : **Add $\frac{3}{7}, \frac{5}{3}, \frac{11}{9}$**

Solution : We first take out LCM of 7, 3, 9 which comes out as 63.

Now, we convert each denominator into 63.

So, $\frac{3}{7}=\frac{3\times 9}{7\times 9}=\frac{27}{63}$, $\frac{5}{3}=\frac{5\times 21}{3\times 21}=\frac{105}{63}$, $\frac{11}{9}=\frac{11\times 7}{9\times 7}=\frac{77}{63}$

$\frac{27}{63}+\frac{105}{63}+\frac{77}{63}=\frac{209}{63}=3\frac{20}{63}$

Here, $\frac{27}{63}$ is equivalent to $\frac{3}{7}$, $\frac{105}{63}$ is equivalent to $\frac{5}{3}$, $\frac{77}{63}$ is equivalent to $\frac{11}{9}$

Facts to Know

- Equivalent fractions are fractions that have same values as that of a fraction or represent the same part of an object.

Example 2 : **Convert an improper fraction $\frac{23}{6}$ into a mixed fraction.**

Solution : We can write $\frac{23}{6}$ as $3\frac{5}{6}$.

Therefore, $3\frac{5}{6}$ is a mixed fraction in which denominator is same as that of improper fractions.

Example 3 : **Identify and write unit fractions?**

$\frac{5}{7}, \frac{7}{3}, \frac{1}{3}, \frac{10}{17}, \frac{1}{20}, \frac{1}{2}, \frac{7}{37}, \frac{1}{11}$

Solution : Unit fractions must have numerator as 1.

Therefore, $\frac{1}{3}, \frac{1}{20}, \frac{1}{2}, \frac{1}{11}$ are unit fractions.

Example 4 : **Raina purchased $6\frac{1}{2}$ kg guava and $5\frac{1}{3}$ kg peach. Calculate the total weight of fruits purchased by her.**

Solution : The total weight of fruits $=\left(6\frac{1}{2}+5\frac{1}{3}\right)$ kg

$=\left(\frac{13}{2}+\frac{16}{3}\right)\text{kg}=\left(\frac{13\times 3+16\times 2}{6}\right)\text{kg}=\left(\frac{39+32}{6}\right)\text{kg}$

$=\frac{71}{6}\text{ kg}=11\frac{5}{6}\text{ kg}$

Exercise 3.1

1. **Add the following proper fractions:**

(a) $\frac{5}{7}+\frac{3}{7}+\frac{1}{7}$ (b) $\frac{6}{13}+\frac{7}{13}+\frac{5}{13}$ (c) $\frac{7}{15}+\frac{3}{15}+\frac{2}{15}$

2. **Subtract the following improper fractions by converting them into equivalent fractions:**

(a) $\frac{3}{2}-\frac{9}{7}$ (b) $\frac{45}{17}-\frac{5}{2}$ (c) $\frac{14}{3}-\frac{5}{7}$

3. **Add the following:**

(a) $\frac{17}{5}+3\frac{3}{5}+\frac{25}{10}$ (b) $2\frac{3}{6}+3\frac{1}{2}+4\frac{5}{9}$ (c) $1\frac{1}{5}+3\frac{1}{2}+6\frac{4}{10}$

4. **Write 4 equivalent fractions for each of the following:**

(a) $\frac{4}{5}$ (b) $\frac{8}{11}$ (c) $\frac{13}{15}$

5. The sum of two numbers is $12\frac{3}{4}$. If one number is $5\frac{1}{4}$, then find the other.

Multiplication of Fractions

Multiplying a Fraction by Whole Number

To multiply a fraction by whole number, we should remember:

$$\text{Fraction} \times \text{Whole Number} = \frac{\text{Numerator of the fraction} \times \text{Whole number}}{\text{Denominator of the fraction}}$$

We know that multiplication is a repeated addition.

For example, $2 \times \frac{3}{7} = \frac{3}{7} + \frac{3}{7} = \frac{6}{7}$

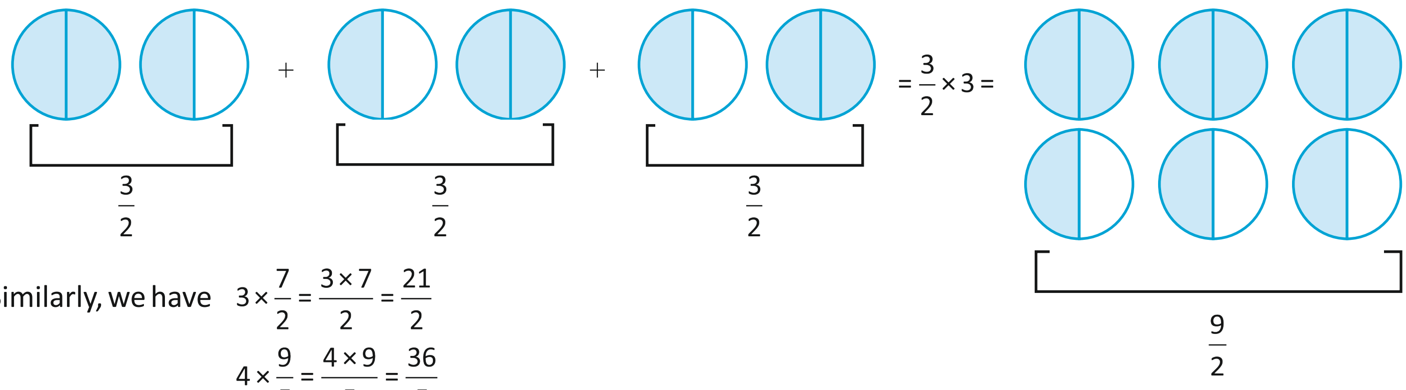
Here, $\frac{3}{7}$ is added two times, so that we get $\frac{6}{7}$.

In this case, we have multiplied proper fraction $\frac{3}{7}$ with whole number 2.

Suppose, we multiply improper fraction $\frac{3}{2}$ with whole number 3. We get

$$\frac{3}{2} \times 3 = \frac{3}{2} + \frac{3}{2} + \frac{3}{2} = \frac{3+3+3}{2} = \frac{9}{2}$$

Let us denote it with the help of pictures given below.



Similarly, we have $3 \times \frac{7}{2} = \frac{3 \times 7}{2} = \frac{21}{2}$

$4 \times \frac{9}{5} = \frac{4 \times 9}{5} = \frac{36}{5}$

$7 \times \frac{8}{3} = \frac{7 \times 8}{3} = \frac{56}{3}$

If we want to multiply a mixed fraction with a whole number we first convert mixed fraction into an improper fraction, then multiply.

For example: $3\frac{3}{4} \times 5 = \frac{15}{4} \times 5 = \frac{75}{4} = 18\frac{3}{4}$

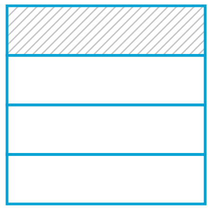
$2\frac{7}{5} \times 3 = \frac{17}{5} \times 3 = \frac{51}{5} = 10\frac{1}{5}$

$3\frac{1}{4} \times 5 = \frac{13}{4} \times 5 = \frac{65}{4} = 16\frac{1}{4}$

Multiplication of a Fraction by a Fraction

What do you understand by $\frac{1}{2}$ of $\frac{1}{4}$? It means $\frac{1}{2} \times \frac{1}{4}$.

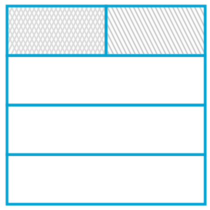
Let us use square as 1 unit. Now, we divide it into 4 equal parts with the help of horizontal lines as shown here. One part is shaded to show $\frac{1}{4}$



Square

Again, we divide into two equal halves (parts) and shade the one half as shown here.

It shows $\frac{1}{2}$ of $\frac{1}{4} = \frac{1 \times 1}{2 \times 4} = \frac{1}{8}$



Square

It means that dark shaded portion is $\frac{1}{8}$ part of square.

Therefore, multiplication of two fractions can be written as:

$$\text{Product of two fractions} = \frac{\text{Product of numerators}}{\text{Product of denominators}}$$

Multiplication of more than two Fractions

While multiplying three or more fractions, we convert each and every mixed fraction, if they are, into improper fraction. After that, we multiply numerators of all the improper fractions with each others. We also multiply the denominators of all the fractions with each other.

Finally, we divide the product of numerators by the product of denominators.

We must reduce the fraction so obtained into the lowest form and express the answer as a mixed fraction, if the answer in an improper fraction.

Example 5 : **Simplify** $2\frac{3}{3} \times \frac{2}{7} \times 11\frac{1}{6} \times 48\frac{2}{2}$

Solution :

$$2\frac{3}{3} \times \frac{2}{7} \times 11\frac{1}{6} \times 48\frac{2}{2}$$

$$= \frac{9}{3} \times \frac{2}{7} \times \frac{67}{6} \times \frac{98}{2}$$

$$= 67x7 = 469$$

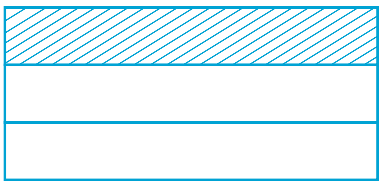
Facts to Know

- Product of a fraction does not change if the order is changed.
- Product of a fraction with 1 is the fraction itself.
- Product of a fraction with zero (0) is always zero (0).

Fraction as an Operator "of"

Suppose, we express 1 as a rectangle. Now, we divide it into three equal parts as shown in the figure.

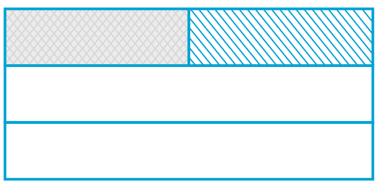
Shade one part and represent it as $\frac{1}{3}$.



$\frac{1}{3}$ part

Now, we divide $\frac{1}{3}$ into two equal halves. Thus, $\frac{1}{2}$ of $\frac{1}{3}$ is represented by the darker shade as shown in the figure.

$\frac{1}{2}$ of $\frac{1}{3}$ part



It means $\frac{1}{2}$ of $\frac{1}{3}$ $= \frac{1}{2} \times \frac{1}{3} = \frac{1 \times 1}{2 \times 3} = \frac{1}{6}$

Hence, we observe that "of" represents multiplication only.

Reciprocal of a Fraction

To get the reciprocal of a fraction, just turn fraction upside down. In other words, swap over the Numerator and Denominator.

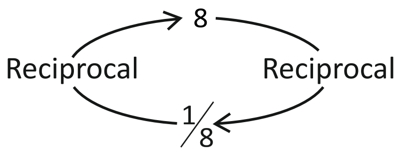
Examples :

Fraction	Reciprocal
$3/8$	$8/3$
$5/6$	$6/5$
$1/3$	$3/1 = 3$
$19/17$	$17/19$

Every non-zero fraction when multiplied by its reciprocal (or multiplicative inverse), gives 1.

Examples : $\frac{99}{78} \times \frac{78}{99} = 1$, $\frac{2}{3} \times \frac{3}{2} = 1$, and so on.

If you take the reciprocal of a reciprocal you end up back where you started.



Facts to Know

- The word "Reciprocal" comes from the Latin word reciprocus meaning returning.

If a number is greater than 1, then its reciprocal is less than 1.

For example : 2 is greater than 1 but its reciprocal $\frac{1}{2}$ is less than 1.

If a number is less than 1 (i.e., between 0 and 1), then its reciprocal is greater than 1.

For example : $\frac{1}{2}$ is less than 1 but its reciprocal is greater than 1.

Example 6 : Find the reciprocal of each of the following :

(a) $\frac{5}{7}$ (b) $2\frac{1}{3}$ (c) 3 (d) $\frac{6}{11}$

Solution : (a) Reciprocal of $\frac{5}{7}$ is $\frac{7}{5}$.

(b) Reciprocal of $2\frac{1}{3}$ or $\frac{7}{3}$ is $\frac{3}{7}$.

(c) Reciprocal of 3 is $\frac{1}{3}$.

(d) Reciprocal of $\frac{6}{11}$ is $\frac{11}{6}$.

Exercise 3.2

1. Find the product of the following :

(a) $5 \times \frac{7}{11}$ (b) $2 \times \frac{19}{4}$ (c) $6 \times \frac{7}{5}$ (d) $15 \times \frac{2}{5}$ (e) $\frac{8}{9} \times 7$ (f) $\frac{64}{9} \times 3$

2. Find the product of the following :

(a) $\frac{27}{9} \times \frac{7}{2}$ (b) $\frac{6}{11} \times \frac{21}{7}$ (c) $\frac{5}{3} \times \frac{7}{2}$ (d) $5\frac{2}{3} \times \frac{3}{2}$ (e) $7\frac{1}{9} \times 3\frac{2}{16}$ (f) $\frac{8}{45} \times \frac{7}{9}$

3. Find the value of the following :

(a) $\frac{5}{9} \times 1\frac{2}{3} \times 3\frac{3}{5}$ (b) $\frac{2}{3} \times \frac{3}{4} \times \frac{4}{5}$ (c) $\frac{1}{4} \times \frac{8}{9} \times \frac{3}{4}$ (d) $\frac{1}{7} \times \frac{3}{5} \times \frac{4}{19} \times \frac{17}{3}$

4. The cost of 1 m of ribbon is ₹ $67\frac{1}{2}$. Find the cost of $1\frac{1}{3}$ m of ribbon.

5. Avinash walks $7\frac{1}{2}$ km in 4 hours. How much distance will be covered by him in 12 hours?

6. A cow grazes for 2 hours in a field and gives $2\frac{1}{2}$ litres of milk. Find the quantity of milk given by cow if it grazes for 6 hours.

Division of Fractions

Division of a Fractions by Whole Number

To divide a fraction by whole number, we multiply the bottom number of fraction by whole number.

or

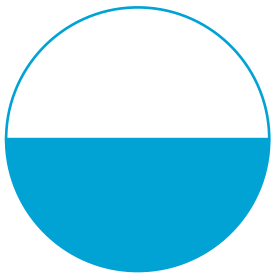
Fraction ÷ Whole number = Fraction × Reciprocal of the whole number

Example 7 : Does $\frac{1}{2} \div 3$ equal $\frac{1}{6}$?

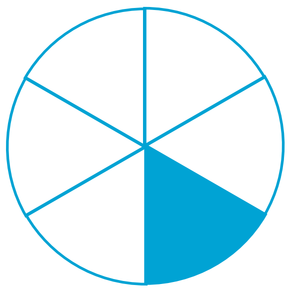
Solution : We look at the pizzas below. . .

When half of pizza is divided into 3 equal parts, each person gets one sixth of a whole pizza, i.e., $\frac{1}{2 \times 3} = \frac{1}{6}$

A Half :



Divided by 3 :



Answer : $\frac{1}{6}$

Similarly, $\frac{2}{5} \div 4$ gives $\frac{2}{5 \times 4} = \frac{2}{20}$ or $\frac{1}{10}$

Division of a Fractions by another Fraction

We multiply first fraction by reciprocal of second fraction.

So, First fraction ÷ Second fraction = First fraction × Reciprocal of second fraction.

Hence, $\frac{3}{4} \div \frac{1}{4} = \frac{3}{4} \times \frac{4}{1} = \frac{3 \times 4}{4 \times 1} = \frac{12}{4} = 3$

and $\frac{24}{3} \div \frac{6}{2} = \frac{24}{3} \times \frac{2}{6} = \frac{24 \times 2}{3 \times 6} = \frac{48}{18} = \frac{8}{3} = 2\frac{2}{3}$

In divisions having mixed numbers, convert the mixed numbers into improper fractions and then solve.

For example : $3\frac{7}{5} \div \frac{3}{2} = \frac{22}{5} \div \frac{3}{2} = \frac{22}{5} \times \frac{2}{3} = \frac{22 \times 2}{5 \times 3} = \frac{44}{15}$

$1\frac{1}{2} \div \frac{6}{4} = \frac{3}{2} \times \frac{4}{6} = \frac{3 \times 4}{2 \times 6} = \frac{12}{12} = 1$

Division of a Whole Number by a Fraction

To perform this, we multiply whole number by the reciprocal of fraction.

Example 8 : Divide the following :

(a) $15 \div \frac{3}{5}$ (b) $8 \div \frac{7}{4}$ (c) $5 \div 3\frac{4}{7}$

Solution : (a) $15 \div \frac{3}{5} = \frac{15}{1} \div \frac{3}{5} = \frac{15}{1} \times \frac{5}{3} = \frac{15 \times 5}{1 \times 3} = \frac{75}{3} = 25$

(b) $8 \div \frac{7}{4} = \frac{8}{1} \div \frac{7}{4} = \frac{8}{1} \times \frac{4}{7} = \frac{8 \times 4}{1 \times 7} = \frac{32}{7}$

(c) $5 \div 3\frac{4}{7} = \frac{5}{1} \div \frac{25}{7} = \frac{5}{1} \times \frac{7}{25} = \frac{35}{25} = \frac{7}{5}$

Example 9 : Mohanty cuts 21 m long cloth into pieces of $3\frac{1}{2}$ m length each. How many pieces of the cloth did he get ?

Solution : Length of the cloth = 21 m

Length of each piece = $3\frac{1}{2}$ m

So, number of pieces = $21 \div 3\frac{1}{2}$

$= 21 \div \frac{7}{2}$

$= \frac{21 \times 2}{7} = \frac{42}{7} = 6$ Hence, Mohanty got 6 pieces of cloth.

Exercise 3.3

1. Find the value of the following:

(a) $\frac{3}{5} \div \frac{1}{2}$ (b) $2\frac{1}{2} \div \frac{3}{5}$ (c) $5\frac{1}{6} \div \frac{7}{2}$ (d) $\frac{13}{6} \div \frac{11}{3}$

2. Divide the following:

(a) $\frac{2}{9} \div 3$ (b) $\frac{1}{2} \div 5$ (c) $\frac{3}{11} \div 4$ (d) $4\frac{3}{7} \div 7$

3. Divide the following:

(a) $11 \div \frac{3}{4}$ (b) $100 \div \frac{3}{10}$ (c) $5 \div 3\frac{4}{7}$ (d) $14 \div \frac{5}{6}$

4. The product of two numbers is $14\frac{1}{2}$. If one number is $4\frac{5}{7}$, find the other number.

5. The cost of a pen is ₹ $12\frac{3}{4}$. How many pens can be purchased for ₹ 204?

6. One box can hold $7\frac{1}{4}$ kg of Rajma dal. How many kilogram of Rajam dal can 18 such boxes hold?

7. The price of 11 footballs is ₹ $536\frac{1}{4}$. Find the price of 1 football.

8. $\frac{1}{4}$ liter of mustard oil costs is ₹ $27\frac{1}{2}$. How much would $8\frac{3}{5}$ liters of mustard oil cost?

Points to Remember

- A fraction is a part of whole.
- A fraction has two part - Numerator and Denominator.
- It can be written as :

$$\text{Fraction} = \frac{\text{Numerator}}{\text{Denominator}}$$

- Fraction can be classified into Proper fraction, Improper fraction, Mixed fraction, Unit fraction, Like fractions, Unlike fraction etc.
- Equivalent fractions are fractions that have same values as that of fraction.
- To multiply a fraction by a whole number, we first multiply numerator of fraction with whole number, then divide it by the denominator of the fraction.
- To multiply a mixed fraction with a whole number, we first convert it into an improper fraction, then multiply.
- Multiplication of two fractions can be written as :

$$\text{Product of two fractions} = \frac{\text{Product of Numerator}}{\text{Product of Denominator}}$$

- Product of a fraction with zero (0) is always zero (0).
- Reciprocal of a fraction means swap over the Numerator and Denominator.
- To divide a fraction by whole number, we multiply the bottom number of fraction by the whole number.
- To perform division by a fraction, we multiply fraction by the reciprocal of second fraction.
- To divide whole number by a fraction, we multiply whole number by the reciprocal of fraction.

EXERCISE

1. **MULTIPLE CHOICE QUESTIONS (MCQs):**

Tick (✓) the correct options :

(a) Which one of the following is a proper fractions ?

(i) $\frac{15}{11}$ ☐ (ii) $\frac{17}{14}$ ☐ (iii) $\frac{6}{10}$ ☐ (iv) $\frac{15}{7}$ ☐

(b) Which one of the following is a unit fraction?

(i) $\frac{17}{7}$ ☐ (ii) $\frac{50}{17}$ ☐ (iii) $\frac{1}{18}$ ☐ (iv) $\frac{5}{14}$ ☐

(c) What is the final output of $\frac{2}{7}+\frac{6}{11}+\frac{5}{7}$?

(i) $\frac{117}{66}$ ☐ (ii) $\frac{119}{15}$ ☐ (iii) $\frac{109}{7}$ ☐ (iv) $\frac{119}{77}$ ☐

(d) What is the product of $2\frac{2}{3}$ and 6 ?

(i) 17 ☐ (ii) 22 ☐

(iii) 16 ☐ (iv) 15 ☐

(e) Shikha reads $8\frac{1}{2}$ page of a book in one hour. How many pages will she read in $2\frac{1}{2}$ hours?

(i) $17\frac{13}{4}$ ☐ (ii) $18\frac{13}{4}$ ☐ (iii) $12\frac{13}{4}$ ☐ (iv) $17\frac{15}{7}$ ☐

(f) On simplifying $\frac{7}{5}\div\left(\frac{3}{2}\times\frac{1}{8}\right)$, we get

(i) $\frac{115}{12}$ ☐ (ii) $\frac{112}{15}$ ☐ (iii) $\frac{66}{15}$ ☐ (iv) $\frac{109}{11}$ ☐

(g) If we add $2\frac{1}{4}$ and $3\frac{1}{4}$, we get

(i) $\frac{31}{2}$ ☐ (ii) $\frac{11}{2}$ ☐

(iii) $\frac{15}{4}$ ☐ (iv) $\frac{19}{7}$ ☐

(h) $\frac{78}{14}$ is same as

(i) $5\frac{8}{14}$ ☐ (ii) $6\frac{8}{14}$ ☐

(iii) $4\frac{8}{13}$ ☐ (iv) None of these ☐

2. By what number should we multiply $8\frac{3}{4}$ to get $26\frac{2}{8}$?

3. The cost of $7\frac{3}{4}$ kg of Jaggery is ₹ $201\frac{1}{2}$. At what price per kg Jaggery will be sold?

4. Kanchan covers $2\frac{1}{2}$ km in 1 hour. How much distance she covers in 6 hours?

5. The cost of $2\frac{1}{3}$ kg of tomatoes is ₹70. How much will $4\frac{2}{3}$ kg tomatoes cost?

6. $\frac{32}{3}$ litres cold drink is kept for distribution among 8 children. How much cold drink would one child get?

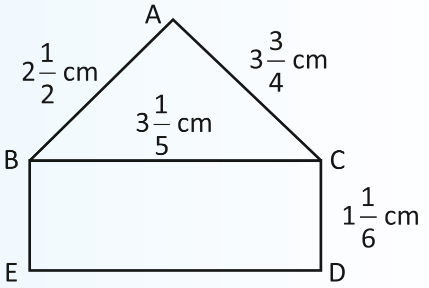
7. There are 56 students in a class. Suppose $\frac{7}{8}$ of them are boys. Find out the number of girls in the class.

8. To publish a monthly school magazine, ₹ $2\frac{1}{5}$ are collected from each student. If the amount collected is ₹ $512\frac{3}{5}$, find the number of students who contributed for this cause.

9. Kiran travelled $26\frac{4}{5}$ km by train, $18\frac{3}{5}$ km by bus and $3\frac{3}{5}$ km by cycle. Find the total distance travelled by her.

HOTS

Find the perimeters of ΔABC and rectangle BCDE of the given figure. Whose perimeter is greater and by how much?



Lab Activity

Objective : To multiply two fractions.

Materials Required : 20 small sized broad ice-cream sticks, glue, A4 size paper, green sketch pen.

Procedure :

Step 1 : Take an **A4 size paper** horizontally.

Step 2 : Paste small sized broad ice-cream sticks on A4 sized paper with the help of **glue.** Look at the structure.

Step 3 : The first column is made up of horizontally pasted sticks shows $\frac{1}{4}$ of total structure.

Step 4 : Shade the top four sticks with **green sketch pen.**

These shaded sticks **show** $\frac{4}{5}$ **of** $\frac{1}{4}$ structure.

Conclusion : So, you can observe that $\frac{4}{5}$ of $\frac{1}{4}$ shows $\frac{4}{20}$ th part of the complete structure i.e., $\frac{4}{5} \times \frac{1}{4} = \frac{4}{20}$.

4 Decimals

The word "Decimal" comes from Latin word **decima** which means a tenth part. Decimal of numbers is based on 10 and contains a decimal point. Decimals are also called as decimal fractions with denominators 10, 100, 1000 and so on. These can be written as :

$$\frac{4}{10} = 0.4, \quad \frac{4}{100} = 0.04, \quad \frac{9}{1000} = 0.009 \text{ and so on.}$$

To understand decimal numbers, you must know about place value chart. Let us take a number, say 7352.678, in place value system.

Thousands	Hundreds	Tens	Ones	Decimal	Tenths	Hundredths	Thousandths
(1000)	(100)	(10)	(1)		$\left(\frac{1}{10}\right)$	$\left(\frac{1}{100}\right)$	$\left(\frac{1}{1000}\right)$
7	3	5	2	.	6	7	8

As we move left from decimal, each position (value) is 10 times bigger.
As we move right from decimal, each position (value) is 10 times smaller.
In the above place value chart (table)

The digit 2 in Ones place is written as 2 × 1, i.e., 2
The digit 5 in Tens place is written as 5 × 10, i.e., 50
The digit 3 in Hundreds place is written as 3 × 100, i.e., 300
The digit 7 in Thousands place is written as 7 × 1000, i.e., 7000
The digit 6 in Tenths place is written as $6 \times \frac{1}{10} = \frac{6}{10}$, i.e., 0.6
The digit 7 in Hundredths place is written as $7 \times \frac{1}{100} = \frac{7}{100}$, i.e., 0.07
The digit 8 in Thousandths place is written as $8 \times \frac{1}{1000} = \frac{8}{1000}$, i.e., 0.008
So, the number in place value chart becomes 7352.678

A decimals number always represents Whole number part and Decimals part, separate by a decimal point.

7352.678	
Whole number part	Decimal part
Decimal point	

This number has three places of decimals.

Whole number part and decimal part are also called as integral part and fractional part respectively.

Facts to Know

- According to Scientist / Historian Joseph Needham, decimal fractions were first developed and used in China in 1st century BC, and then spread to Middle East and from there to Europe.

Addition and Subtraction of Decimals

We can add like decimals which have same number of places of decimal. For example, 5.39, 6.02, 92.73 can be added because they have same number of decimal places. But we can not add unlike decimals (which have different number of places of decimal). For example 3.29, 7.7, 9.832 can not be added. To add unlike decimals, we convert them into like decimals by just adding zeroes on the right of the decimal point so that the value of numbers do not change.

For example : 7.7 = 7.700, 3.29=3.290 and so on.

We add

$$\begin{array}{r} 7.700 \\ 3.290 \\ +9.832 \\ \hline 20.822 \\ \hline \end{array}$$

Decimal numbers can also be written in expanded form. For example; $35.732 = 30 + 5 + \frac{7}{10} + \frac{3}{100} + \frac{2}{1000}$

Exercise 4.1

1. **Add the following :**
   (a) 224.0 + 3.794 (b) 3.24 + 7.905 (c) 604.02 + 39.734 (d) 593.288 + 52.697
2. **Subtract the following :**
   (a) 3.797 – 2.07 (b) 137.2 – 25.738 (c) 17.30 – 7.198 (d) 287.35 – 65.95
3. **Write in expanded form :**
   (a) 67.48 (b) 925.379 (c) 249.007 (d) 287.239
4. **Convert the following unlike decimals into like decimals :**
   (a) 7.432, 84.002, 72.5 (b) 5.39, 28.024, 987.7 (c) 25.73, 19.553, 6.759, 9.3
5. Deepali walked 7.65 km on first day, 17.04 km on second day and 25.02 km on third day. How much distance did she walk in all ?

Multiplication of Decimal Fractions

Multiplication of a Decimals by 10, 100, 1000, ...

When we multiply a decimal by 10, the decimal point moves one place to the right and when we multiply a decimal by 100, the decimal point moves two places to the right. Similarly, when we multiply a decimal by 100, the decimal point moves three places to the right.

Example 1 : Multiply the following :

(a) 9.47 × 1000 (b) 94.234 × 100 (c) 78.25 × 10

Solution :
(a) 9.47 × 1000 = 9470 (The decimal point shifts three places to the right)
(b) 94.234 × 100 = 9423.4 (The decimal point shifts two places to the right)
(c) 78.25 × 10 = 782.5 (The decimal point shifts one place to the right)

Example 2 : The cost of 1 kg packet of detergent cake is ₹ 52.75. What will be the cost of 100 packets, if each packet weighs 1 kg ?

Solutions : It is very clear that we have to multiply 52.75 by 100 to find out the cost of 100 packets.
Therefore, ₹ 52.75 × 100 = ₹ 5275

Multiplication of a Decimal be a Whole Numbers

- To multiply a decimal number by a whole numbers :
- Ignore the decimal point and multiply the digits.
- Place the decimal point in the answer so that it has the same number of decimal places as that of number (decimal numbers)

Example 3 : **Calculate 39.98 × 7**

Solution :

```
   39.98      ←— 2 decimal places in the number
 ×     7
  279.86      ←— 2 decimal places in the answer
```

Do not forget to place the decimal point two places from the right-hand end of the answer.

Example 4 : **Calculate 0.0005 × 8**

Solution :

```
  0.0005
 ×     8
  0.0040     or  0.004
```

Example 5 : **Calculate 47.4 × 45**

Solution :

```
   47.4
 × 45
   2370
  1896×
  2133.0
```

Facts to Know

- Any trailing zeros in the decimal places can be omitted.

Multiplication of a Decimal Number by Another Decimal Number

To multiply the two decimals :

- Ignore the decimal points and multiply the digits.
- Place a decimal point in the answer so that it has the same number of decimal places as the sum of decimal places in the given numbers.

Example 6 : **Calculate 0.8 × 0.9**

Solution :

```
   0.8      ←— 1 decimal place
 × 0.9      ←— 1 decimal place
  0.72      ←— 2 decimal places in the answer
```

Thus, 0.8 × 0.9 = 0.72. In this example, we have ignored the decimal points first and multiplied 8 by 9. Then, you have placed the decimal point at 2 places of decimal from right.

Example 7 : **Multiply 114.2 by 2.14**

Solution :

```
    114.2
     2.14
     4568
    1142×
   2284××
  244.388
```

The decimal places in multiplicand (114.2) = 1

and decimal places in multiplier (2.14) = 2

Sum of decimal places = 2 + 1 = 3

Thus, we place a decimal point after 3 places from right-hand end.

Example 8 : **Multiply 18.81 by 1.11**

Solution :

```
      18.81   ← 2 decimal places
     x1.11    ← 2 decimal places
      1881
     1881
    1881
   20.8791    ← 4 decimal places
```

Exercise 4.2

1. Multiply the following :

(a) 7.95×10 (b) 1.45×100 (c) 1.53×1000 (d) 6.75×100

(e) 0.55×10 (f) 3.567×1000 (g) 279.01×1000 (h) 287.33×10

2. Find the product of the following :

(a) 0.874×401 (b) 3.5×40 (c) 0.96×91 (d) 3.3×9

(e) 9.28×17 (f) 745.20×37 (g) 532.2×49 (h) 8.9×2.75

3. Find the product of the following :

(a) 0.429×33.5 (b) 15.78×1.1 (c) 3.05×1.29 (d) 279.7×2.14

(e) 45.7×3.36 (f) 19.13×7.3 (g) 0.75×1.25 (h) 1.35×0.123

4. Nayan can travel at 54.67 km per hour by his car. How far can he travel in 3 hours ?

5. Find the area of a square of sides 17.90 m each. [Hint : Area of square = side × side]

Division of Decimals

When we have learnt the technics to multiply two decimals. Now, we shall learn about division of two decimals.

Division of a Decimal by 10, 100, 1000, ...

We have divide a decimal by 10, 100 or 1000, the decimal point in the answer shifts accordingly to the left side by 1, 2 or 3 places respectively.

Example 9 : **Divide 7578.9 by 1000**

Solution : Number of zeros in the divisor (1000) is 3. So, the decimal point will move three places to the left.

$7578.9 \div 1000 = 7.5789$

Example 10 : **Divide 27.89 by 100**

Solution : The decimal point will shift to 2 places to the left.

$27.89 \div 100 = 0.2789$

Division of a Decimal by a Whole Number or a Decimal Fraction

- Move the decimal point in the divisor to the right until it changes into a whole number.
- Move the decimal point in the dividend to the right by the same number of places as the decimal point was moved to make the divisor a whole number.

- Divide the new dividend by the new divisor.
- After division, count the number of decimal places in the new dividend from right handed and place the decimal point accordingly in the quotient so obtained.

Example 11 : **Calculate 4.2625 ÷ 0.05**

Solution : Number of decimal places in divisor is 2. We make divisor a whole number. Now, shift the decimal point 2 places to the right in the dividend. So, new dividend is 426.25.

```
5)426.25(85.25
 −40
   26
  −25
    12
   −10
     25
    −25
      ×
```

Thus, 4.2625 ÷ 0.05 = 85.25

Example 12 : **Divide 72.5 by 29**

Solution :

```
29)72.5(2.5
  −58
   145
  −145
     ×
```

Thus, 72.5 ÷ 29 = 2.5

In the above example, we counted the number of decimal placed in the new dividend and placed the decimal point accordingly in the quotient so obtained.

Exercise 4.3

1. **Calculate the following :**
   (a) 17.28 ÷ 100 (b) 9.26 ÷ 10 (c) 5.685 ÷ 1000 (d) 75.399 ÷ 1000
2. **Divide the following :**
   (a) 0.14 ÷ 0.7 (b) 0.12 ÷ 1.2 (c) 1.44 ÷ 0.8
   (d) 1.5813 ÷ 2.51 (e) 10.9326 ÷ 2.74 (f) 11.2 ÷ 5.6
3. The product of two decimals is 5832.8222. If one decimal is 427.94, find the other.
4. Calculate the area of a rectangle whose breadth is 2.5 cm and length is 0.38 cm.
   (Hint : Area of rectangle = length × breadth)
5. An electrician earns ₹ 75.25 per hour. If he worked 200 hours this month, how much did he earn ?

Different Units and their Conversion

We use several methods to measure quantities in our day-to-day life. We use mainly metre (or kilometre), gram (or kilogram), millilitre (or litre), paise (or rupees) etc., to make our measurement easy.

We use metric system to measure length, mass and capacity.

To convert from bigger unit to smaller unit, we multiply by 10, 100 or 1000 and to convert from smaller unit to bigger unit, we divide by 10, 100 or 1000 as the case may be.

We spell out the abbreviation as:

kg means kilogram
g means gram
km means kilometre
m means metre
kl means kilolitre
l means litre
ml means millilitre
cm means centimetre

Example 13 : **Convert the following :**

(a) 7 kg into grams (b) 500 cm into metres (c) 12 km into metres
(d) 3 m into centimetres (e) ₹ 36 rupees into paise

Solution :
(a) 7 kg = 7 × 1000 g = 7000 g (Since, 1 kg = 1000 g)
(b) 500 cm = $\frac{500}{100}$ m = 5 m (Since, 100 cm = 1 m)
(c) 12 km = 12 × 1000 m = 12000 m (Since, 1 km = 1000 m)
(d) 3 m = 3 × 100 cm = 300 cm (Since, 1 m = 100 m)
(e) ₹ 36 = 36 × 100 paise = 3600 paise (Since, ₹ 1 = 100 paise and ₹ is an abbreviation for Rupees)
(f) 500 Paise = $\frac{500}{100}$ ₹ = 5 ₹ (Since, ₹ 1 = 100 paise and ₹ is an abbreviation for Rupees)

Example 14 : **Convert the following :**

(a) 4 km 600 m into m (b) 6 *l* 500 *ml* into *ml*

Solution :
(a) 4 km 600 m = 4 × 1000 + 600 m
= 4000 *m* + 600 *m* = 4600 *m*
(b) 6 *l* 500 *ml* = 6 × 1000 *ml* + 500 *ml*
= 6000 *ml* + 500 *ml* = 6500 *ml*

Exercise 4.4

1. **Fill in the blanks.**

(a) 7 *l* = ☐ *ml* (b) 25 km = ☐ m
(c) 3.5 kg = ☐ g (d) 2900 g = ☐ kg
(e) 5 kl = ☐ *l* (f) 2945 g = ☐ kg
(g) 7.25 *ml* = ☐ *l* (h) 200 cm = ☐ m

2. A bookbinder requires 1.7 m of thread to bind a book. How many such books can be binded from a bundle of thread of 44.2 m in length ?
3. 5590.35 kg wheat is distributed among 45 persons. How much wheat did each person get ?
4. 87 cartons of toys weigh 3936.75 kg. Find the weight of 100 such cartons of toys.
5. The thickness of one calculator is 1.2 cm. Find the thickness of 89 calculators altogether. Also convert the unit of result so obtained into metre (m).

Points to Remember

- The word "Decimal" comes from Latin word decima which means **tenth part.**
- Decimals are based on 10 and contain a decimal point.
- A decimal number always represents Whole number part and Decimal part, separated by a decimal point.
- We can add and subtract like decimals; but unlike decimals can not be added and subtracted.
- Unlike decimals can be added and subtracted only after their conversion into like decimals.
- When we multiply a decimal by 10, 100, 1000, ... the decimal point shifts to 1, 2, 3....... places to the right side.
- We ignore the decimal point while multiplying the decimal number with whole number, and place the decimal point in the answer so that it has same number of decimal places as that of decimal number.

- When we divide a decimal by 10, 100, 1000... the decimal point in the answer shifts accordingly to the left side by 1, 2, 3........ place respectively.
- To divide a decimal number by another decimal fraction, make the divisor a whole number, and in dividend shift the decimal point to the right by as many places as the number of decimal places in the divisor. Now, divide new dividend by the whole number.
- In our daily life, we use units such as metre, gram, millilitre and convert them into their corresponding bigger or smaller units.

EXERCISE

1. MULTIPLE CHOICE QUESTIONS (MCQs):

Tick (✓) the correct options :

(a) Which one is the value of 0.89 × 100 ?

(i) 980 ☐ (ii) 89 ☐

(iii) 8.9 ☐ (iv) 0.089 ☐

(b) How many grams are 37 kg 28 g ?

(i) 37280 grams ☐ (ii) 3728 grams ☐

(iii) 37028 grams ☐ (iv) None of these ☐

(c) Which of the following is the result of 2.93 + 17.1 + 15.002 ?

(i) 34.023 ☐ (ii) 37.302 ☐

(iii) 33.023 ☐ (iv) 35.032 ☐

(d) If we multiply a decimal fraction by 1000, how many places to the right a decimal point move ?

(i) 2 ☐ (ii) 1 ☐

(iii) 3 ☐ (iv) 4 ☐

(e) The sum of two decimals is 79.37. If one of them is 37.542, then the other is

(i) ₹ 40.582 ☐ (ii) ₹ 41.727 ☐

(iii) ₹ 41.828 ☐ (iv) ₹ 40.826 ☐

(f) One set of spoon and bowl costs ₹ 36.75. How much would 15 such sets cost ?

(i) ₹ 550.22 ☐ (ii) ₹ 551.25 ☐

(iii) ₹ 505.25 ☐ (iv) ₹ 55.125 ☐

(g) Kitty saves ₹ 34.50 per month (30 days). How much will she save for 18 days ?

(i) 22.5 ☐ (ii) 18.75 ☐

(iii) 20.7 ☐ (iv) 22.25 ☐

2. A photocopy costs ₹1.2 per sheet. If a shopkeeper makes 12000 copies, what is their cost ?

3. A fruit vendor takes 4 rounds of a street in 15 minutes. How many rounds will he take in 2 hours ?

4. Ambuj wants to buy the following items : A DVD player for ₹ 1597.50, a DVD holder for ₹ 75.25 and a personal stereo for ₹ 1090.35. How much extra money is required to purchase all these items, if he has only ₹ 1550 in his wallet ?

5. An old man gets retirement pension of ₹ 2470.50 for 6 months. How much will he get for 2 month ?

6. Find the area of a rectangle whose breadth is 4.65 m and length is 10.38 m ?

7. If 30 drums of water weigh 21765 kg, find the weight of 1 drum of water.

8. A household requires 2.94 liters of kerosene oil per day for cooking purpose. How much oil is used in one week.

9. If Patra earn ₹ 26850 per month (i.e., 30 days), how much will he earn in 9 days ?

10. A scooter driver covers 46.8 km in 1.6 liter of petrol. How much distance will it cover in one litre of petrol ?

HOTS

1. If one inch = 2.54 cm, then 1 yard = ... metres.
2. A flagpole of 20 m casts a shadow of 5 m. If another flagpole is only 12 m high, what is the length of its shadow?

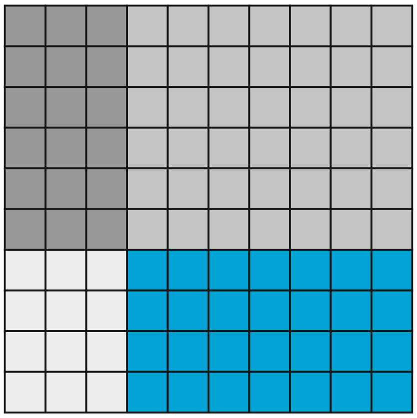
Lab Activity

Objective : To multiply two decimals

Materials Required : White chart paper, scissors, black sketch pen, yellow sketch pen, green sketch pen.

Procedure :

Step 1 : Take a white colored chart paper. Cut it in squared shape.



Step 2 : Draw grids with black sketch pen in 10 equals rows and 10 equal columns as shown above. Thus it makes 100 squares of equal length and breadth. Suppose we want to find the product of 0.6 and 0.3 i.e., 0.6 × 0.3

Step 3 : Shade the first 6 rows with yellow sketch colour.

Step 4 : Shade the first 3 columns with green sketch colour.

Count the boxes which are dark shaded and common in both.

You will see 18 such boxes out of 100 boxes i.e., $\frac{18}{100}$ = 0.18. Hence 0.6 × 0.3 = 0.18

Following the similar method, you could find the product

(i) 0.7 × 0.2 (ii) 0.9 × 0.3 (iii) 0.5 × 0.8

and many more ....

5 Exponents and Powers

In our daily life we come across many large numbers. For example, like the velocity of light is 300, 000, 000 m/s, mass of the earth is 5, 970, 000, 000, 000, 000, 000, 000, 000 kg, the distance of Sun from the Earth is 150, 000, 000. It is difficult to read these numbers in above form. But which we write them as the velocity a light is 3×10^8 m/s, mass of the earth is 597×10^{22} kg it becomes easy to read.

We have written above the short form of large numbers using exponent. In this chapter, we shall learn about exponent and its laws.

Exponents

We use exponent to write large numbers in their short form or exponential form.

Let us see, $100,000 = 10 \times 10 \times 10 \times 10 \times 10 = 10^5$

Here 10^5 has written for $10 \times 10 \times 10 \times 10 \times 10$. In 10^5, 10 is called the base, 5 is the exponent and 100, 000 is the exponential value. However the base can any numbers also.

For example, $243 = 3 \times 3 \times 3 \times 3 \times 3$ is written as in exponential form as 3^5. In 3^5, 3 is base and 5 is the exponent or power. $58786 = 5 \times 10000 + 8 \times 1000 + 7 \times 100 + 8 \times 10 + 6$. We can write it as $5 \times 10^4 + 8 \times 10^3 + 7 \times 10^2 + 8 \times 10 + 6$. The base of exponential form of exponential value can be negative also. For example,

(a) $(-2) \times (-2) \times (-2) = (-2)^3 = -8$

(b) $(-2) \times (-2) \times (-2) \times (-2) = (-2)^4 = 16$

(c) $\left(-\frac{2}{5}\right) \times \left(-\frac{2}{5}\right) \times \left(-\frac{2}{5}\right) \times \left(-\frac{2}{5}\right) \times \left(-\frac{2}{5}\right) = \frac{-32}{3125}$

Let us consider for example,

$a \times a = a^2$	;	We read it as *a* raised to the power of 2.
$m \times m \times m = m^3$	;	We read it as *m* raised to the power of 3.
$a \times a \times b \times b \times b = a^2 b^3$	;	We read it an *a* raised to the power of 2 and *b* raised to the power of 3.

Example 1 : Express 125 as a power 05.

Solution : We write 125 as

$125 = 5 \times 5 \times 5$

Hence, we can say that

$125 = 5^3$.

Example 2 : Which one of the following is greater.

(a) 2^4 or 4^2 (b) $(-3)^4$ or $(-4)^3$

Solution : (a) We write 2^4 as

$2^4 = 2 \times 2 \times 2 \times 2 = 16$

and 4^2 as $4^2 = 4 \times 4 = 16$

Both numbers are equal.

(b) We write $(-3)^4$ as

$(-3)^4 = (-3)\times(-3)\times(-3)\times(-3) = 81$

We write $(-4)^3$ as

$(-4)^3 = (-4)\times(-4)\times(-4) = -64$

It is clear that, $(-3)^4 > (-4)^3$.

Example 3 : **Find the exponential value of the following :**

(a) m^2n^3 (b) n^3m^2 (c) m^3n^2 (d) n^2m^3

Solution :
(a) $m^2n^3 = m\times m\times n\times n\times n$

(b) $n^3m^2 = n\times n\times n\times m\times m$

(c) $m^3n^2 = m\times m\times m\times n\times n$

(d) $n^2m^3 = n\times n\times m\times m\times m$

Example 4 : **Express the following numbers as a product of powers of prime factors :**

(a) 32 (b) 3125 (c) 729 (d) –1331

Solution :
(a) $32 = 2\times2\times2\times2\times2 = 2^5$

(b) $3125 = 5\times5\times5\times5\times5 = 5^5$

(c) $729 = 3\times3\times3\times3\times3\times3 = 3^6$

(d) $-1331 = (-11)\times(-11)\times(-11) = (-11)^3$

Example 5 : **Express the following numbers as product of powers of prime factors :**

(a) 1800 (b) 25000

Solution :
(a) $1800 = 2\times2\times2\times3\times3\times5\times5$

$= 2^3\times3^2\times5^2$

(b) $25000 = 2\times2\times2\times5\times5\times5\times5\times5$

$= 2^3\times5^5$

Example 6 : **Simplify the following :**

(a) 1^{10} (b) 100^1 (c) $(-1)^5$

(d) $5^2\times3^3$ (e) $(-2)^3\times(-10)^3$

Solution :
(a) $1^{10} = 1\times1\times1\times1\times1\times1\times1\times1\times1\times1 = 1$

(b) $100^1 = 100$

(c) $(-1)^5 = (-1)\times(-1)\times(-1)\times(-1)\times(-1)$

$= -1$

(d) $5^2\times3^3 = 5\times5\times3\times3\times3$

$= 25\times27 = 675$

(e) $(-2)^3\times(-10)^3 = (-2)\times(-2)\times(-2)\times(-10)\times(-10)\times(-10)$

$= (-8)\times(-1000) = 8000$

Exercise 5.1

1. **Find the value of each of the following :**

(a) 2^6 (b) 3^7 (c) 5^3
(d) 4^4 (e) 3^6 (f) $(-7)^3$

2. **Write the base and exponent (power) of the following:**

(a) $(5)^3$ (b) $(-5)^4$ (c) $(-1)^{11}$
(d) $(y)^m$ (e) m^y (f) $(-100)^5$

3. **Write the following in exponential form:**

(a) $y \times y \times y$ (b) $9 \times 9 \times 9 \times 9 \times 9$ (c) $a \times a$
(d) $7 \times 7 \times 7 \times 3 \times 3$ (e) $n \times n \times n \times m \times m$ (f) $x \times x \times x \times y \times y \times z \times z$

4. **Which is greater in the following pairs:**

(a) 1^9 or 9^1 (b) 3^6 or 6^3 (c) 10^9 or 9^{10}
(d) $(-2)^5$ or 5^2 (e) $(-1)^{10}$ or $(10)^1$ (f) $(7)^3$ or $(3)^7$

5. **Write the following numbers as a product of power of prime factor :**

(a) 75 (b) 9000 (c) 625
(d) 405 (e) 3600 (f) 675

6. **Simplify the following :**

(a) 3×10^3 (b) 3×4^5 (c) $5^2 \times 4^4$
(d) $2^6 \times 6^2$ (e) $3^2 \times 5^2$ (f) 4×5^3

7. **Simplify the following:**

(a) $(-5)^2 \times (-2)$ (b) $(-1)^{10} \times (-4)^3$ (c) $(-2)^3 \times (-1)^{80}$
(d) $2^3 \times 5$ (e) $3^5 \times 5^3$ (f) $2^4 \times 4^2$

Law of Exponents

Laws of exponents are learnt through observation method. These laws are as follows.

1. **Multiplying power with the same base**

When the base of numbers are same. The exponents are to be added.

(a) Let us calculate, $3^2 \times 3^3 = (3 \times 3) \times (3 \times 3 \times 3)$
$= 3 \times 3 \times 3 \times 3 \times 3 = 3^5 = 3^{2+3}$

Here, the base in 3^2 and 3^3 are same and the sum of exponents is $2+3 = 5$

(b) $m^2 \times m^5 = (m \times m) \times (m \times m \times m \times m \times m)$
$= m \times m \times m \times m \times m \times m \times m$
$= m^7$ i.e., m^{2+5}

(c) $(-4)^3 \times (-4)^2 = (-4 \times -4 \times -4) \times (-4 \times -4)$
$= (-4) \times (-4) \times (-4) \times (-4) \times (-4)$
$= -4^5$ i.e. $(-4)^{3+2}$

Hence, $5^2 \times 5^4 = 5^{2+4}$
$6^3 \times 6^2 = 6^{3+2}$
$10^{11} \times 10^9 = 10^{11+9}$ and so on.

Therefore then, we can establish that non-zero integer p when q and r are whole numbers.

$$p^q \times p^r = p^{q+r}$$

Power of a power

Some patterns with power of a power like $(2^3)^2$, $(3^2)^4$, $(4^2)^3$, we simplify these as follows.

(a) $(2^3)^2 = 2^3 \times 2^3 = 2^{3+3}$
$= 2^6 = 2^{2\times3}$

(b) $(3^2)^4 = 3^2 \times 3^2 \times 3^2 \times 3^2 = 3^{2+2+2+2}$
$= 3^8 = 3^{2\times4}$

(c) $(4^2)^3 = 4^2 \times 4^2 \times 4^2 = 4^{2+2+2}$
$= 4^6 = 4^{2\times3}$

From observing the above examples, we can establish the result in the form of another law as :

Law 1: It p is any integer ($p \neq 0$) and q and r are two natural number, then

$$(p^q)^r = p^{q\times r} = p^{qr}$$

Example 7 : Simplify the following as a single exponent.

(a) $(2^3)^4$ (b) $[(-20)^2]^5$ (c) $(5^2)^{30}$ (d) $(3^5)^4$ (e) $(10^{10})^{10}$

Solution : (a) $(2^3)^4 = 2^{3\times4} = 2^{12}$

(b) $[(-20)^2]^5 = (-20)^{2\times5} = (-20)^{10}$

(c) $(5^2)^{30} = 5^{2\times30} = 5^{60}$

(d) $(3^5)^4 = 3^{5\times4} = 3^{20}$

(e) $(10^{10})^{10} = 10^{10\times10} = 10^{100}$

Examples 8 : Find that which one is greater : $(2^3)^4$ or $(2^3)\times 4$

Solution : We know that $(2^3)^4 = 2^{3\times4} = 2^{12} = 4096$

$(2^3)\times 4 = 2^3\times4 = 8\times4 = 32$

It is clear that $(2^3)^4 > (2^3)\times 4$

2. Dividing powers with the same base

Let us simplify $2^5 \div 2^3$

$$2^5 \div 2^3 = \frac{\cancel{2\times2\times2}\times2\times2}{\cancel{2\times2\times2}}$$

$= 2^2$ *i.e.*, 2^{5-3}

Thus, $2^5 \div 2^3 = 2^{5-3}$

Here, the base of 2^5 and 2^3 are same. Therefore, $2^5 \div 2^3 = 2^{5-3}$.

Similarly $7^6 \div 7^5$

$$= \frac{\cancel{7\times7\times7\times7\times7}\times7}{\cancel{7\times7\times7\times7\times7}} = 7^1 = 7^{6-5}$$

Let m be a non-zero integer ($m \neq 0$), then find the value of $m^8 \div m^2$.

we know that, $m^8 \div m^2 = \dfrac{\cancel{m\times m}\times m\times m\times m\times m\times m\times m}{\cancel{m\times m}} = m^6$ i.e., m^{8-2}

Now, Let us see some examples :

$$10^9 \div 10^7 = 10^{9-7} = 10^2$$
$$9^{10} \div 9^3 = 9^{10-3} = 9^7$$
$$p^8 \div p^5 = p^{8-5} = p^3$$

Law - 2: If P is any non-zero integer and q and r are whole numbers such that $q > r$.

Then, $p^q \div p^r = \dfrac{p^q}{p^r} = p^{q-r}$

Multiplying Powers with the Same Exponents

Now, we will learn to multiply the terms having some exponent but different base. For example :

(a) $3^3 \times 5^3 = (3 \times 3 \times 3) \times (5 \times 5 \times 5)$

$= (3 \times 5) \times (3 \times 5) \times (3 \times 5)$

$= 15 \times 15 \times 15$

$= 15^3$ i.e., $(3 \times 5)^3$

(b) $4^5 \times 3^5 = (4 \times 4 \times 4 \times 4 \times 4) \times (3 \times 3 \times 3 \times 3 \times 3)$

$= (4 \times 3) \times (4 \times 3) \times (4 \times 3) \times (4 \times 3) \times (4 \times 3)$

$= 12 \times 12 \times 12 \times 12 \times 12$

$= 12^5$ i.e., $(4 \times 3)^5$

(c) $m^3 \times n^3 = (m \times m \times m) \times (n \times n \times n)$

$= (m \times n) \times (m \times n) \times (m \times n)$

$= (m \times n)^3$ i.e., $(mn)^3$

From observing above examples we can establish the result as a law that. If p and q are two non-zero ($p \neq 0, q \neq 0$) integers and r is any whole number, then

$p^r \times q^r = (pq)^r$ [$p \times q = pq$]

Dividing Powers with the Same Exponents

When dividing powers having same exponents, we simplify them as following.

(a) $\frac{2^5}{3^5} = \frac{2\times2\times2\times2\times2}{3\times3\times3\times3\times3} = \frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3} = \left(\frac{2}{3}\right)^5$

(b) $\frac{4^7}{5^7} = \frac{4\times4\times4\times4\times4\times4\times4}{5\times5\times5\times5\times5\times5\times5} = \frac{4}{5}\times\frac{4}{5}\times\frac{4}{5}\times\frac{4}{5}\times\frac{4}{5}\times\frac{4}{5}\times\frac{4}{5} = \left(\frac{4}{5}\right)^7$

(c) $\frac{p^3}{q^3} = \frac{p\times p\times p}{q\times q\times q} = \frac{p}{q}\times\frac{p}{q}\times\frac{p}{q} = \left(\frac{p}{q}\right)^3$

From observing the above examples, we establish a law that if p and q are two non-zero integers and r is a whole number, then

$$p^r \div q^r = \frac{p^r}{q^r} = \left(\frac{p}{q}\right)^r$$

Numbers with Exponent Zero

We observe some following examples in which numerator and denominator having same exponent and base. For examples.

(a) $2^5 \div 2^5 = \frac{2\times2\times2\times2\times2}{2\times2\times2\times2\times2} = 1$

or, 2^{5-5} *i.e.* 2^0

Thus, we know that, $2^0 = 1$

Let us observe the other examples :

(b) $5^5 \div 5^5 = 5^{5-5}$

$= 5^0 = 1$

(c) $7^4 \div 7^4 = 7^{4-4}$

$= 7^0 = 1$

From observing above examples, we reach at conclusion that $p^0 = 1$ for all $p \neq 0$.

Now, summaries the above laws of exponents.

(a) $p^q \times p^r = p^{q+r}$ (b) $(p^q)^r = (p)^{qr}$

(c) $p^q \div p^r = p^{q-r}$ (where $q > r$) (d) $p^r . q^r = (pq)^r$

(e) $\frac{p^r}{q^r} = \left(\frac{p}{q}\right)^r$

Example 9 : Write the following in the exponential form.

(a) $20 \times 20 \times 20 \times 20 \times 20$ (b) $32 \times 81 \times 12$

Solution : (a) We have $20 \times 20 \times 20 \times 20 \times 20 = (20)^5$

$= (4 \times 5)^5$

$= 4^5 \times 5^5 = 2^{10} \times 5^5$

(b) We have $32 \times 81 \times 12 = (2 \times 2 \times 2 \times 2 \times 2) \times (3 \times 3 \times 3 \times 3) \times (2 \times 2 \times 3)$

$= (2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2)(3 \times 3 \times 3 \times 3 \times 3)$

$= 2^7 \times 3^5$

Example 10 : Simplify and write in the exponential form.

(a) $4^3 \times p^3 \times 7p^4$ (b) $[(3^2)^5 \times 15^6] \div 5^3$

(c) $15^5 \div 3^4$ (d) $(3^5 \times 10^5 \times 25) \div (5^7 \times 6^5)$

Solution : (a) $4^3 \times p^3 \times 7p^4 = 4^3 \times p^3 \times 7 \times p^4$

$= 4^3 \times 7 \times p^3 \times p^4$

$= 64 \times 7 \times p^{3+4} = 7 \times 2^6 \times p^7$

(b) $[(3^2)^5 \times 15^6] \div 5^3 = [(3^2)^5 \times (3 \times 5)^6] \div 5^3$

$= [3^{10} \times 3^6 \times 5^6] \div 5^3$

$= \frac{3^{16} \times 5^6}{5^3} = 3^{16} \times 5^3$

(c) $15^5 \div 3^4 = (3 \times 5)^5 \div 3^4$

$= \frac{3^5 \times 5^5}{3^4}$

$= \frac{3^5}{3^4} \times \frac{5^5}{1}$

$= 3^{5-4} \times 5^5 = 3^1 \times 5^5$

(d) $(3^5 \times 10^5 \times 25) \div (5^7 \times 6^5)$

$= [3^5 \times (2 \times 5)^5 \times 5^2] \div [5^7 \times (2 \times 3)^5]$

$= [3^5 \times 2^5 \times 5^5 \times 5^2] \div [5^7 \times 2^5 \times 3^5]$

$= (3^5 \times 2^5 \times 5^{5+2}) \div [5^7 \times 2^5 \times 3^5]$

$= \frac{3^5 \times 2^5 \times 5^7}{2^5 \times 3^5 \times 5^7} = 1$

Exercise 5.2

1. Simplify and express in exponential form :

(a) $\dfrac{2\times3^4\times2^5}{9\times4^2}$ (b) $[(5^3)^2\times5^4]\div5^{14}$ (c) $\dfrac{9^3\times a^5\times b^2}{3^6\times a^3\times b}$

(d) $(3^0+4^0)\div7^0$ (e) $\left(\dfrac{p^6}{p^3}\right)\times p^9$ (f) $(2^4\times2)^2$

(g) $\dfrac{5^3}{5^2+5}$ (h) $3^2\times\left(\dfrac{p^{10}}{p^8}\right)$

2. Using law of exponents, simplify in exponential form :

(a) $5^2\times5^4\times5^3$ (b) $(-3)^5\times(-3)^4$ (c) $p^4\times p^5$

(d) $(5^3)^5$ (e) $10^x\times10^3$ (f) $(2^2)^3\div4^3$

(g) $m^7\times n^7$ (h) $(3^7\div3^4)\times3^2$ (i) $8^x\div8^3$

3. Write the following as a product of prime factors in exponential form :

(a) 432×48 (b) 6000 (c) 729×81

(d) 512×300 (e) 2187×625

4. Simplify the following :

(a) $\dfrac{(2^8)^2\times5^3}{7^3\times4}$ (b) $\dfrac{6^5\times10^5\times125}{5^5\times3^5}$ (c) $\dfrac{3^7\times81\times7^3}{3^{10}\times7^6}$

5. Justify and write true or false :

(a) $10\times10^9=10^9$ (b) $3^4<4^3$ (c) $7^3\times4^3=28^2$

(d) $5^1=(5000)^0$ (e) $10^5>50000$

6. Find the value of x in each of the following :

(a) $2^x=256$ (b) $2^x\div2^5=2^{16}$ (c) $5^{x-7}=5^{12}$

(d) $2^3\times(-5)^3=(-10)^x$

Decimal Number System

In decimal number system we write a number in the expanded form. For example,

$579876=5\times100000+7\times10000+9\times1000+8\times100+7\times10+6$

We can write the expanded form in exponential form as

$=5\times10^5+7\times10^4+9\times10^3+8\times10^2+7\times10^1+6\times10^0$

We have written the expanded form in product 10, which is in descending order as i.e., 5, 4, 3, 2, 1, 0.

Expressing Large Number in the Standard Form

In the beginning of this chapter, we have discussed that we use exponential form of the large numbers to read them conveniently. For example, the mass of earth is 5930,000,000,000,000,000,000,000 kg . It is not convenient to read it. Its exponential form is given

$=593\times10^{22}$ kg

$=59.3\times10^{23}$ kg

$=5.93\times10^{24}$ kg

It becomes easy to read after converting large number in exponential form.

Similarly we can convert the mass of Uranus.

= 86,800,000,000,000,000,000,000,000 kg

$= 8.68 \times 10^{25}$ kg.

Speed of light is

= 300,000,000 m/s

$= 3 \times 10^{8}$ m/s.

We have expressed the numbers above in standard form. A number can be expressed as decimal number between 1 and 10.0 is called its standard form.

Exercise 5.3

1. **Express the following numbers in the expanded form :**
   (a) 934657 (b) 80080807 (c) 3210312 (d) 70018
2. **Write the number, their expanded term are given below.**
   (a) $7 \times 10^{6} + 5 \times 10^{3} + 3 \times 10^{0}$ (b) $5 \times 10^{4} + 7 \times 10^{3} + 8 \times 10^{2} + 9 \times 10^{0}$
   (c) $4 \times 10^{5} + 3 \times 10^{3} + 5 \times 10^{1}$ (d) $7 \times 10^{4} + 6 \times 10^{3} + 5 \times 10^{1} + 5 \times 10^{0}$
3. **Express the following numbers in standard form :**
   (a) 90000000 (b) 4156900000 (c) 590789 (d) 590658
4. **Write the following in standard form :**
   (a) Speed of light in vacuum is 300,000,000 m
   (b) Diameter of the earth is 1,27,56,00 m
   (c) Our universe is estimated to be about 12,000,000,000 years old.
   (d) The population of India was approximately 1,027,000,000 in April 2001.
   (e) The distance between the earth and the moon is 384,000,000 m.
   (f) Diameter of sun is 1,400,000,000 m.

Points to Remember

- In x^m, x is called the base and m is called the power or exponent or the index.
- $x^m = x \times x \times x$ .... m times, where x is a rational number and m is a positive integer.
- x^m is read as x raised to the power of m, where x is called the base and m is called the exponents or power or index.
- If x and y are rational numbers and m and n are positive integers, then,

$$x^m \times x^n = x^{m+n}$$

$$x^m \div x^n = x^{m-n} \quad \text{if } m > n$$

$$= x^{n-m} \quad \text{if } n > m$$

$$(x^m)^n = x^{mn}$$

$$x^m \times y^m = (xy)^m$$

$$x^{-a} = \frac{1}{x^a}$$

$$x^0 = 1, \text{ where } x \neq 0$$

EXERCISE

1. MULTIPLE CHOICE QUESTIONS (MCQs):

Tick (✓) the correct options :

(a) The numbers, whose expanded form is $7\times10^5+5\times10^3+9\times10^0$ is

(i) 70509 (ii) 705009

(iii) 7005090 (iv) 75090

(b) The expanded form of 204305 is

(i) $2\times10^6+4\times10^4+3\times10^3+5\times10^1$ (ii) $2\times10^5+4\times10^4+3\times10^2+5\times10^1$

(iii) $2\times10^5+4\times10^3+3\times10^1+5\times10^0$ (iv) $2\times10^5+4\times10^3+3\times10^2+5\times10^0$

(c) 1.234×10^7 is the exponential form of

(i) 1234000 (ii) 12340000

(iii) 12340000000 (iv) 0.0001234

(d) If x and a are positive integers such that $x^a = 49$, then a^x equals to

(i) 49 (ii) $\frac{1}{4^9}$

(iii) 128 (iv) 64

(e) $a^x \div a^y = a^{y-x}$ when

(i) $x > 0$ (ii) $y > 0$

(iii) $x > y$ (iv) $y > x$

(f) The value of $14641 \div 11^2$ is

(i) 11 (ii) 111

(iii) 121 (iv) 1212

(g) $(4^0+8^0)\div3^0$ equals to

(i) 4 (ii) 2

(iii) $\frac{2}{3}$ (iv) $\frac{32}{3}$

(h) $(x^3)^4$ when expressed as a single exponent is

(i) x^7 (ii) x^{81}

(iii) x^{12} (iv) $\frac{x}{12}$

(i) Express $\frac{2^3\times9}{64\times27}$ in exponential form

(i) $\frac{1}{2^3\times3}$ (ii) $\frac{3}{8}$

(iii) $2^3\times3$ (iv) $\frac{3}{2^3}$

2. Find the value of the each of the following :

(a) 2^8 (b) 4^5 (c) 5^3

(d) 6^6 (e) 3^5 (f) $(-7)^3$

3. Write the base and exponent (power) of the following :

(a) $(7)^7$ (b) $(-11)^{10}$ (c) $(xy)^a$

(d) $(m)^0$ (e) $(101)^1$ (f) x^y

4. Write following in exponential form :

(a) $m \times m \times m$ (b) $(-1) \times (-1) \times (-1) \times (-1)$

(c) $3 \times 3 \times 3 \times 3 \times 3$ (d) $2 \times 2 \times 2 \times m \times n$

(e) $5 \times 5 \times 5 \times 7 \times 7$ (f) $m \times m \times m \times n \times n \times p \times p \times p$

5. Which is lesser in the following pairs:

(a) 3^5 and 5^3 (b) 2^4 and 4^2

(c) $(-2)^3$ and $(-3)^2$ (d) 2^7 and 7^2

(e) 4^5 and 5^4 (f) $(-1)^4$ and $(-1)^3$

6. Write the following numbers as a product of powers of prime factor :

(a) 540 (b) 6125 (c) 2400 (d) 648

7. Simplify the following :

(a) 5×10^5 (b) $3^2 \times 5^2$ (c) $(-1)^{11} \times (-4)^3$

(d) $(-4)^{20} \times (-4)^{100}$ (e) $m^2 \times m^2 \times m^5$ (f) $2^8 \div 2^{10}$

8. Using law of exponents and simplify in exponential form :

(a) $[(2^2)^3 \times 3^5] \times 5^7$ (b) $(2^{20} \div 2^{10}) \times 2^5$

(c) $(3^5 \times 5^5 \times 25) \div (5^3 \times 3^5)$ (d) $[(5^2)^5 \times 15^{10}] \div 5^3$

9. Justify and write true or false :

(a) $1^0 \times 1^{10} = (-1)^3$ (b) $10^2 < 2^5$

(c) $5^2 \times 2^5 > 2^5 \times 2^3$ (d) $11^{10} \times 11^1 > 675$

10. Find the value of *y* each of the following :

(a) $3^y = 243$ (b) $5^{y+3} = 5^{3+2y} + 3 \times 5^3$

(c) $2^y = 64$ (d) $3^y \div 3^5 = 3^9$

11. Express the following numbers in expanded form :

(a) 507345 (b) 5000785 (c) 495781 (d) 679001

12. Write the following numbers whose expanded form are given below :

(a) $5 \times 10^7 + 8 \times 10^5 + 3 \times 10^4 + 2 \times 10^2 + 1 \times 10^0$ (b) $4 \times 10^5 + 6 \times 10^4 + 9 \times 10^1 \times 4 \times 10^0$

13. Express the following in standard form :

(a) 7,00,00,000

(b) 3, 16, 78, 00, 000

(c) The earth has 1,353,000,000 cubic km of sea water.

HOTS

If $\frac{p}{q} = \left(\frac{5}{6}\right)^2 \div \left(\frac{5}{6}\right)^2$, find the value of $\left(\frac{p}{q}\right)^2$

Lab Activity

Objective : To understand exponents (power) by folding paper.

Materials Required : Squared paper, pencil, etc.

Procedure : Let us take exponents of 2, i.e., $2^0, 2^1, 2^2, 2^3, 2^4$, ... We represent the exponent by crease of folding.

Step 1 : Take the squared paper as shown in fig (i).

Fig. (i)

This squared paper has no fold. So, no any crease. It represents $2^0 = 1$.

Step 2 : Fold the squared paper into two halves making a vertical crease as shown in fig. (ii). Then, unfold the paper as shown in fig. (iii).

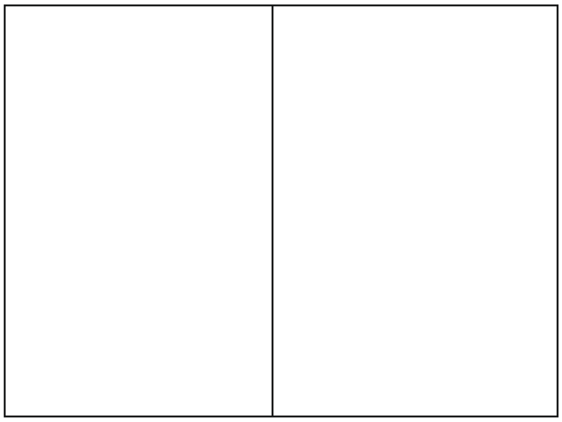


Fig. (iii)

The squared paper has one fold. So, one crease. It represents $2^1 = 2$.

Step 3 : Now, fold the squared paper into two halves making a horizontal crease as shown in fig. (iv)

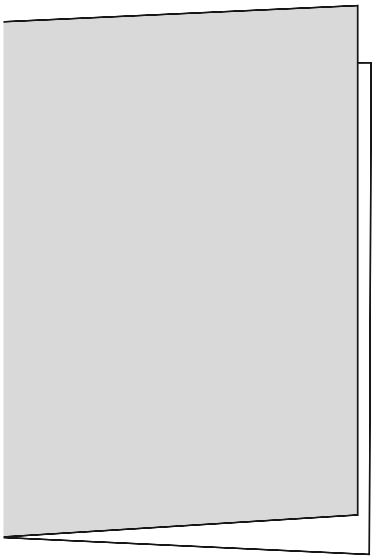


Fig. (ii)

Fig. (iv)

The squared paper now has two folds as shown in fig. (v). So, two crease. It represents $2^2 = 4$.

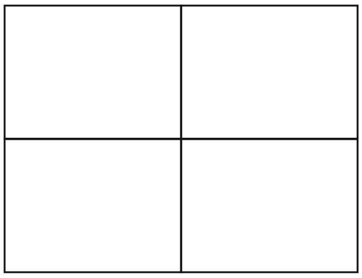


Fig. (v)

Step 4 : In the same manner, make horizontal creases with the help of three folds and vertical creases with the help of four folds as shown in fig. (vi) and fig. (vii) respectively.

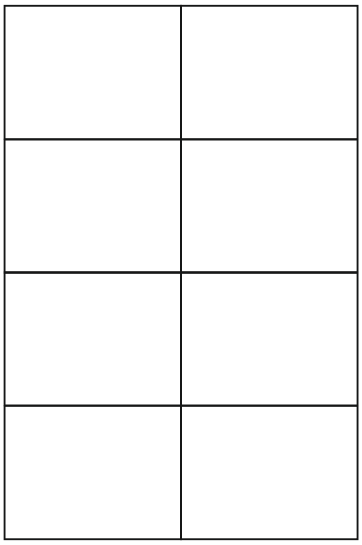


Fig. (vi)

In Fig. (vi), number of folds or crease = 3; Number of parts = 8 So, $2^3 = 8$

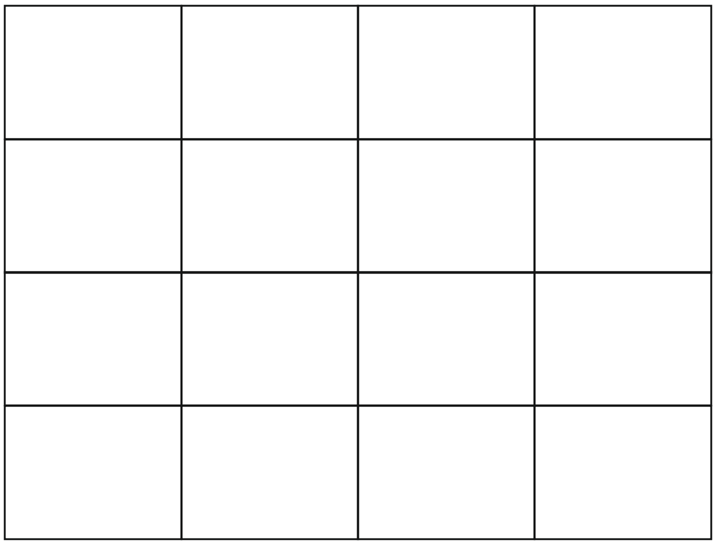


Fig. (vii)

In fig. (vii), number of fold or crease = 4;

Number of parts = 16

So, $2^4 = 16$

Conclusion : We can understand the exponents or powers as

$2^0 = 1$

$2^1 = 2$

$2^2 = 4$

$2^3 = 8$

$4^2 = 16$ and so on.

6 Algebraic Expression

In Class VI, we have learnt about the basic concepts on algebraic expression. A symbol having a fixed numeric value is called a constant and a symbol which takes various numerical values is called a variable. A combination of constants and variables connected by the signs of addition, subtraction, multiplication and division is called an algebraic expression. In this chapter, we shall learn more about algebraic expression its terms, constants, coefficients etc. Then, we shall learn to apply operations on algebraic expressions, linear equations and solve some practical problems using these operations.

Algebraic Expressions

To understand algebraic expression we need to know about constant and variables.

Constant : A quantity having a fixed numeric value is called constant. For example : 1, 15, –6, $\frac{7}{11}$, $4\frac{2}{5}$.

Variable : A quantity which can have different numerical values is called variable or literal. We use small letters a, b, c, p, q, r, m, n, *x*, y, z ... etc. to denote variables.

Types of Algebraic Expressions

A collection of constants and literals (variables) connected by one or more fundamental operations (+, –, ×, ÷) is called an algebraic expression.

In algebraic expression different parts are connected by signs. These parts are called **terms**.

The algebraic expressions containing only one term is called *simple algebraic expression.* While a compound algebraic expression contains two or more than two terms.

Monomial : An algebraic expression containing only one term is called a **monomial.**

Binomial : An algebraic expression containing two terms is called a **binomial.**

Trinomial : An algebraic expression containing three terms is called a **trinomial.**

Polynomial : An algebraic expression containing two or more terms is called a **polynomial.**

Algebraic Expression	No. of Terms	Name	Terms
(a) $7x^2y$	1	Monomial	$7x^2y$
(b) $5x^2-7y^2z$	2	Binomial	$5x^2, -7y^2z$
(c) $2x^2+\frac{3}{x}$	2	Binomial	$2x^2, \frac{3}{x}$
(d) $10x^2y-3xy^2+7$	3	Trinomial	$10x^2y, -3xy^2, 7$
(e) x^3+5x^2-9x+7	4	Polynomial	$x^3, 5x^2–9x$

We must remember that the terms of an algebraic expression do not separate by multiplication and division.

Factors **:** Each literal or constant quantity multiplied together to form a product is called a factor of the product. A constant factor is called 'numerical factor' and a factor containing only literals is called a literal factor.

In an algebraic expression $-10x^2y$, the numerical factor is -10 and x^2y is the literal factor.

Constant term **:** The term of an algebraic expression containing no literal factor is called its constant term, i.e., in the expression $4x^2 + 5z + 10$, the term 10 is the constant term.

Coefficients **:** In the literal factor of algebraic expression the numerical part is called the coefficient of the remaining factor of the term.

In particular, the constant part is called the numerical coefficients or generally the coefficient of the term and the remaining part is called the literal coefficient of the term.

Consider the expression $7x^2 - 5x^2y + 10$. In the term $-5x^2y$:

the literal coefficient $= x^2y$

the numerical coefficient $= -5$

the coefficient of $5y = -x^2$

the coefficient of $x^2 = -5y$

the coefficient of $-5xy = x$

Like and Unlike Terms

The terms containing same literal coefficients with same degree are called like terms while the terms containing different literal coefficients are called unlike terms.

Example :

(i) $4xy, -7xy, \frac{5}{2}yx$ are like terms because these terms have same literal coefficient xy.

(ii) $7m, 3n, 6x, -5y$ are unlike terms because terms have different literal coefficients.

(iii) $6p^2qr, 11qp^2r, -7qrp^2$ are like terms because terms have same literal coefficient p^2qr.

(iv) $5xy^2, 9x^2y$ are unlike term because these terms have different literal coefficient xy^2 and x^2y.

Polynomial in One Variables

An algebraic expression having one variable is said to be a polynomial in that particular variable, if the power of variable in each term is non-negative integer.

The greatest power of a variable in a polynomial is called its degree. For example :

(i) $8 + 5x$ is a polynomial in x of degree 1.

(ii) $7y^2 - 3y - \frac{4}{3}$ is a polynomial in y of degree 2.

(iii) $-10-5p-4p^3$ is a polynomial in p of degree 3.

(iv) $9x^6 - 8x^3 + 5x - \frac{5}{7}$ is a polynomial in x of degree 6.

Facts to Know

- A polynomial in one variable is called a univariate polynomial, a polynomial in more than one variable is called a multivariate polynomial.

Polynomial in Two or More Variables

An algebraic expression having two or more variables is said to be a polynomial in those variables, if the power of variable in each term is non-negative integer. Take the sum of the powers of variables in each term, the greatest sum is the degree of the polynomial. For example:

(i) $8 - 10x^2 - 12x^3y^2 - 6x^2y^2$ is a polynomial in two variables x and y. The degree of its terms are 0, 2, 3 + 2, 2 + 2. Therefore, the degree of the polynomial is 5.

(ii) $9p - 11m^2n - 21n^3p + \frac{1}{4}$ is a polynomial in 3 variables m, n, p, the degree of its terms are 1, 2 + 1, 3 + 1, 0. Therefore, the degree of polynomial is 4.

(iii) $5x^2 - \frac{6}{x} + 2x^zy + 7$ is not a polynomial. Because the power of variable in second term is a negative integer.

Example 1 : **Consider the algebraic expression $9x^3y^2 - 7xy^3 - 5x^2y^2 - \frac{5}{2}$**

(a) What is the total number of terms ? List all the term.

(b) What is the numerical coefficient of the term $-7xy^3$?

(c) What is the literal coefficient of the term $-5x^2y^2$?

(d) Is the given expression a polynomial? So, write its degree.

(e) What is the coefficient of $7x$ in the term $-7xy^3$?

(f) What is the coefficient of $-7xy$ in the term $-7xy^3$?

Solution : (a) There are 4 terms : $9x^3y^2, -7xy^3, -5x^2y^2, -\frac{5}{2}$

(b) The numerical coefficient of the term $-7xy^3$ is -7.

(c) The literal coefficient of the term $-5x^2y^2$ is x^2y^2.

(d) Given algebraic expression is a polynomial of two variable x and y.

The degree of its terms are 3+2, 1+3, 2+2, 0. So, the degree of the polynomial is 5.

(e) The coefficient of $7x$ in the term $-7xy^3$ is $-y^3$.

(f) The coefficient of $-7xy$ in the term $-7xy^3$ is y^2.

Exercise 6.1

1. **Write the following in algebraic form using variables, constants signs and symbols.**
   (a) Meera have ₹ 250 less than me.
   (b) Moyan have ₹ 670 more than her friend.
   (c) Product of numbers a and b subtracted from 100.
   (d) The square of sum of the three consecutive numbers.
   (e) The sum of squares of two consecutive numbers.

2. **Identify the terms in each expression and write their numerical factor and literal factor.**
   (a) $5x - 6$ (b) $-5x + 8$ (c) $5x^2 - 6x$ (d) $x^2y^2 - 6xy$

3. **In the term $15p^3q^4r^5z$, find the coefficient of the following:**
   (a) p^3q^4 (b) q^4r^5 (c) p^3r^5 (d) $p^3q^4r^5$

4. **Identify terms containing x^2 and give the coefficient of x^2.**
   (a) $4x^2y$ (b) x^2+5y (c) $x^2y^2 - 11x$

5. **Identify the like terms and unlike terms in the following :**
   (a) $ab^2, 6a^2b, 5b^2a, 11ab$ (b) xyz, zyx, zy^2x, zx^2b
   (c) $14a, 17b, 15b, 18bc$ (d) $7x^2y^2z^2, 5y^2z^2x^2, 3xyz, -5xy^2z$

6. **Classify the following into monomials, binomials and trinomials.**
   (a) 21 (b) $5x-4y$
   (c) $4x-7x^2+3z$ (d) $x^2+11x^2-8x^2$

7. **Identify the constant term of each of the following expression:**

(a) $5x^2 - 11$ (b) $ax^2 + bx + c$

(c) $x^2 + y^2 + 4xy$ (d) $a^2b - ab^2 + 15$

8. **Write the literal coefficient of each of the following :**

(a) $\frac{7xy}{2}$ (b) $-\frac{3mn^2}{8}$

(c) $\frac{5}{4}xyz^2$ (d) $\frac{5}{9}xyz^2$

Operation on Algebraic Expression

Now, we will learn the fundamental mathematical operations of addition and subtraction of algebraic expressions.

Addition of like terms

The basic principle of addition is that we can add like terms only. Unlike terms can not be added.

For Example :

(a) Sum of $7p, -9p, 8p, -p$ and $-11p$

$= 7p - 9p + 8p - p - 11p$

$= (7 - 9 + 8 - 1 - 11)p = -6p$

(b) Sum of $7xy, -5xy$ and $9xy$

$= 7xy - 5xy + 9xy$

$= (7 - 5 + 9)\,xy = 11xy$

(c) Sum of $11\,x^2yz, -9x^2yz, 5x^2yz$ and $-13x^2yz$

$= 11x^2yz - 9x^2yz + 5x^2yz - 13x^2yz$

$= (11 - 9 + 5 - 13)x^2yz = -6x^2yz$

(d) Sum of $7\,mn, \frac{1}{2}\,mn$ and $-\frac{3}{2}\,mn$

$= 7\,mn + \frac{1}{2}\,mn - \frac{3}{2}\,mn$

$= \left(7 + \frac{1}{2} - \frac{3}{2}\right)mn$

$= \frac{14 + 1 - 3}{2}mn = \frac{12}{2}mn = 6\,mn$

Addition of Algebraic Expression

To add two or more algebraic expressions, we take together the like terms and then add them. There are two methods of addition :

1. Horizontal Method : In this method all the algebraic expressions are written in a horizontal line and then the terms are arranged to collect all the groups of Like terms and then added.

2. Column Method : In this method, each algebraic expression is written in a separate row such that their Like terms are arranged one below the other in a column. Then the addition of Like terms is done column-wise.

Example 2 : **Add $x-2y+7z$, $3y-5x+2z$ and $y+7z-6x$ by horizontal method.**

Solution : Addition by Horizontal method.

$(x - 2y + 7z) + (3y - 5x + 2z) + (y + 7z - 6x)$

$= (x - 5x - 6x) + (-2y + 3y + y) + (7z + 2z + 7z)$

$= (1 - 5 - 6)x + (-2 + 3 + 1)y + (7 + 2 + 7)z$

$= -10x + 2y + 16z$

Example 3 : **Add $2x - 7y + 5z$, $4x + 2y + 3z$ and $-3x + 8y - 15z$ by Column method.**

Solution :

$$\begin{array}{r} 2x - 7y + 5z \\ 4x + 2y + 3z \\ -3x + 8y - 15z \\ \hline 3x + 3y - 7z \\ \hline \end{array}$$

Example 4 : **Add the following :**

(a) $4x^2 - 5xy + 3y^2$, $-6x^2 - 4xy + 2y^2$ and $-3x^2 - 2x - 4y^2$.

(b) $6mn + np - 7pm$, $3np + 2pm + mn$ and $pm - 4np - 9mn$.

Solution : (a) $=(4x^2 - 5xy + 3y^2) + (-6x^2 - 4xy + 2y^2) + (-3x^2 - 2xy - 4y^2)$

$=(4x^2 - 6x^2 - 3x^2) + (-5xy - 4xy - 2xy) + (3y^2 + 2y^2 - 4y^2)$

$=(4 - 6 - 3)x^2 + (-5 - 4 - 2)xy + (3 + 2 - 4)y^2$

$= -5x^2 - 11xy + y^2$

(b)

$$\begin{array}{r} 6mn + np - 7pm \\ mn + 3np + 2pm \\ -9mn - 4np + pm \\ \hline -2mn - 0np - 4pm \\ \hline \end{array}$$

$= \quad -2mn - 4pm$ is the required sum.

Example 5 : **Add $10p - 9q + 15r$, $9r - 12p$, $16q - 17r$ and $11r + 18p - 10q$**

Solution :

$$\begin{array}{rrr} 10p & -9q & +15r \\ -12p & +0 & +9r \\ 0 & +16q & -17r \\ +18p & -10q & +11r \\ \hline 16p & -3q & +18r \\ \hline \end{array}$$

Subtraction of Algebraic Expression

Like addition, subtraction is also possible between two Like term only. In subtracting an algebraic expression from another, we change the sign (from '+' to '–' or from '–' to '+' of all terms of the expression which is to be subtracted and then the two expressions are added. We can subtract by horizontal or column method.

Example 6 : **Subtract $10x - 9y - 3z$ from $7x - 6y + 8z$.**

Solution : Subtraction by **Horizontal method**

$(7x-6y+8z)-(10x-9y-3z)$

$= 7x-6y+8z-10x+9y+3z$

$= 7x-10x-6y+9y+8z+3z$

$= (7-10)x+(-6+9)y+(8+3)z$

$= -3x+3y+11z$

Subtraction by **Column method.**

$$\begin{array}{r} 7x-6y+8z \\ 10x-9y-3z \\ -\quad+\quad+ \\ \hline -3x+3y+11z \\ \hline \end{array}$$

Example 7 : Subtract the following : by column method and horizontal method.

$(x^2-3xy+7y^2-2)$ from $(6xy-4x^2-y^2+5)$

Solution : (By column method)

$$\begin{array}{r} +6xy-4x^2-y^2+5 \\ -3xy+x^2+7y^2-2 \\ +\quad-\quad-\quad+ \\ \hline 9xy-5x^2-8y^2+7 \\ \hline \end{array}$$

(By horizontal method)

$(6xy-4x^2-y^2+5)-(x^2-3xy+7y^2-2)$

$= 6xy-4x^2-y^2+5-x^2+3xy-7y^2+2$

$= (6xy+3xy)+(-4x^2-x^2)+(-y^2-7y^2)+(5+2)$

$= 9xy-5x^2-8y^2+7$

Example 8 : Subtract $8p+9q-10r$ from the sum of $7p-8q+12r$ and $-10p+7q+9r$

Solution : There are three expression one of them have to be subtracted from other two. So, we change the sign of third expression and then add them as follows greater.

$$\begin{array}{rcrcr} 7p & - & 8q & + & 12r \\ -10p & + & 7q & + & 9r \\ 8p & + & 9q & - & 10r \\ - & & - & & + \\ \hline -11p & - & 10q & + & 31r \\ \hline \end{array}$$

Example 9 : How much is $16x^3-17x^2+12x-18$ greater than $13x^3+10x^2-11x+12$?

Solution : We have to subtract $13x^3+10x^2-11x+12$ from $16x^3-17x^2+12x-18$

$$\begin{array}{r} 16x^3-17x^2+12x-18 \\ 13x^3+10x^2-11x+12 \\ -\quad-\quad+\quad- \\ \hline 3x^3-27x^2+23x-30 \\ \hline \end{array}$$

Example 10 : **How much is $13x^3 - 10x^2 + 15x + 11$ less than $15x^3 + 12x^2 - 13x + 14$?**

Solution : We have to subtract $13x^3 - 10x^2 + 15x + 11$ from $15x^3 + 12x^2 - 13x + 14$.

$$\begin{array}{r} 15x^3 + 12x^2 - 13x + 14 \\ 13x^3 - 10x^2 + 15x + 11 \\ - \quad + \quad - \quad - \\ \hline 2x^3 + 22x^2 - 28x + 3 \\ \hline \end{array}$$

Addition/Subtraction of Unlike Terms

We have read that we cannot add or subtract the unlike terms. But they can be expressed with appropriate sign *of* '+', of '–' . For example, we express the addition of $5x$ and $7y$ as :

$5x + 7y$

Similarly, we express the subtraction of 99 m from $100n$ as :

$100n - 99m$ etc.

Exercise 6.2

1. **Add the following by both horizontal and column methods :**
   (a) $10x + 9y$ and $x + y$
   (b) $x + y + 5$ and $5x + 4y + 9$
   (c) $5x + 6y + z$ and $5x - y - z$
   (d) $5x - y - 2z$ and $3x - 2y - 3z$
2. **Add the following algebraic expressions :**
   (a) $x^3 - 2x^2y + 5xy^2 - y^3$ and $3x^3 - 4xy^2 + 5x^2y - y^3$
   (b) $m^4 - 3m^3n + 4mn^3 + 7m^2n^2 + 5n^2$ and $-4m^4 - 7mn^3 + 9m^3n - 8m^2n^2 + 6n^2$
3. **Subtract by using column method.**
   (a) $x^2 + y^2$ from $7x^2 - 5y^2$
   (b) $m^2 - 5mn$ from $2m^2 - 9mn$
4. Subtract $8x + 9y - 4z$ from the sum of $5x + 4y + 5z$ and $5x - 6y + 7z$.
5. What should be added to $m^2 + 2mn + n^2$ to obtain $5mn + n^2$?

Power of a Term

Let m is a literal with different powers such as m^2, m^3, m^4 ...m^8. m^2 is called second power of m, m^3 as the third power of m, m^4 as the fourth power of m and so on. Here, m is called the base and 2, 3, 4, ... 8 are all exponents of m. Thus, power is actually the exponent of the base.

Facts to Know

- When variables are multiplied, the powers are added to decide the degree of the term.

Degree of an Expression

The degree of an expression is the highest power of the variable present in the expression. Thus, the degree of expression $x^2 + 5x + 7$ is 2, the degree of the expression $x^3 - 5x^7 - 7x^9 + 8$ is 9 and that of $x^7 + x^{16} + x^{13} + 4x - 8$ is 16. The degree of expression $x^2y^2 + xy^3 + x^2y^4 + 8xy + 9$ is also 6 as the highest power of the variable xy is $(2 + 4)$ i.e., 6.

Evaluation of Algebraic Expressions

An algebraic expression consists of one or more literal numbers. The literal numbers may have numerical value. We evaluate the algebraic expression by putting the numerical value or the literal numbers.

When we use the formulae from geometry algebra or mensuration with the help of literal numbers, we need to find the value of an algebraic expression. For example :

If we denote the length and breadth of a rectangle as *l* and *b* then, we express the area of a rectangle by formula as :

Area $= l \times b$

Suppose, length and breadth is given as 6 cm and 4 cm, we can find the area by putting the values of *l* and *b*, we get :

$6 \text{ cm} \times 4 \text{ cm} = 24 \text{ cm}^2$

This process of replacing the literal numbers in the algebraic expression by their numerical value is called the substitution.

Example 11 : **Find the value of following expressions by putting the value of $x = 5$:**

(a) $5x + 10$ (b) $6x - 7$ (c) $18 - 6x^2$ (d) $2x^2 - x + 1$

Solution : (a) $5x + 10 = (5 \times 5) + 10 = 25 + 10 = 35$

(b) $6x - 7 = (6 \times 5) - 7 = 30 - 7 = 23$

(c) $18 - 6x^2 = 18 - 6\,(5)^2 = 18 - 150 = -132$

(d) $2x^2 - x + 1 = 2\,(5)^2 - (5) + 1 = 50 - 5 + 1 = 51 - 5 = 46$

Example 12 : **Find the value of the following expressions when value of literal numbers are p = 1, q = –2, r = 3.**

(a) $p^2 + q^2 - r^2$ (b) $p^2 + (-q)^2 + (-r)^2$

(c) $p^2q + pq^2 - r^2$ (d) $4pqr$

(e) $p^2q + qr - rp$ (f) $pq - qr^2 - p^2q$

Solution : For $p = 1, q = -2$ and $r = 3$

(a) $p^2 + q^2 - r^2 = 1^2 + (-2)^2 - (3)^2$

$= 1 + 4 - 9 = -4$

(b) $p^2 + (-q)^2 + (-r)^2 = (1)^2 + (+2)^2 + (-3)^2$

$= 1 + 4 + 9 = 14$

(c) $p^2q + pq^2 - r^2 = 1^2\,(-2) + 1\,(-2)^2 - (3)^2$

$= -2 + 4 - 9 = -7$

(d) $4pqr = 4 \times 1 \times (-2) \times 3$

$= -24$

(e) $p^2q + qr - rp = 1^2\,(-2) + (-2)3 - 3 \times 1$

$= -2 - 6 - 3 = -11$

(f) $pq - qr^2 - p^2q = 1\,(-2) - (-2)3^2 - 1^2\,(-2)$

$= -2 - (-18) - (-2)$

$= -2 + 18 + 2 = 18$

Exercise 6.3

1. **If x = 3, find the value of following :**
   (a) $x + 7$ (b) $2x - 5$
   (c) $15 - 4x$ (d) $x^2 + 3x + 3$
2. **If x = –2, find the value of :**
   (a) $4 - 2x$ (b) $x^3 + 3$
   (c) $x^2 + 5x + 5$ (d) $x^3 + 2x^2 + 2x - 1$
3. **If p = 4, q = – 4 find the value of following :**
   (a) $p^2 + q^2$ (b) $p^2 - q^2$
   (c) $p^2 + 2pq + q^2$ (d) $pq^2 + p^2q + pq$
4. Simplify and evaluate the following expression. The value of different literals are $x = 5$ and $y = -3$.
   $5(x^2 + xy) - 3(xy - x^2) + y^2$
5. If $a = -7$, find the value of $3a^2 + 4a - 55$
6. If $x = -1$, find the value of $x^4 - 5(x - 5)$

Algebraic Expressions - Formulae and Rules

We have learned to use algebraic expression making of mathematical formula. Let us learn more about it.

Number System

1. Let *n* is the natural number. It's successor will be :

   $n + 1$ (Successor is 1 more than that number)

   It's predecessor will be :

   $n - 1$ (Predecessor is 1 less than the number)

2. Now we see some number of series in which numbers follow an expression.

   (a) 3, 5, 7, 9 it can be expressed in terms of algebraic expressions as $(2n + 1)$, where n = 1, 2 ....

   (b) –10, –7, – 4, – 1, expression as $(3n - 13)$, where n = 1, 2 ....

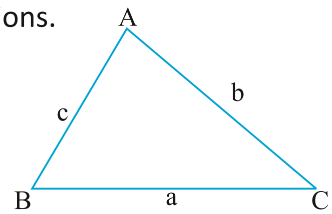
   (c) 4, 8, 12, 16, expression as 4n, where n = 1, 2, 3 ....

   (d) $1^2, 2^2, 3^2$, ... expression as n^2, where n = 1, 2, 3 ....

   We can verify the truth by putting a particular value of *n* in the above expressions.

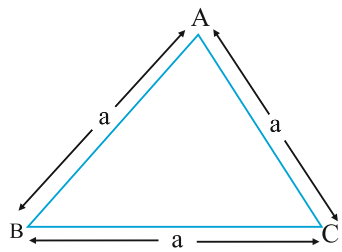
Perimeter formulae

We can represent perimeter of different geometrical shapes *as* algebraic expressions.



1. Perimeter of a triangle.

   Whose sides are *a*, *b*, *c* $= a + b + c$



2. Perimeter of an equilateral triangle :

   Whose each side is *a*

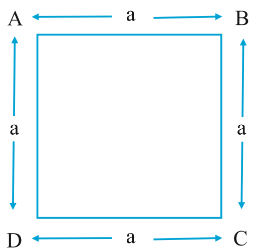
   $= a + a + a$

   $= 3a$

3. Perimeter of a square :

Whose each sides are equal.

= $4a$

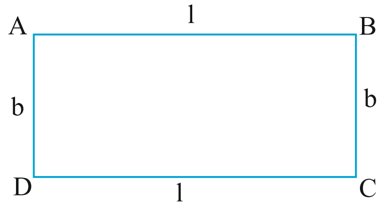


4. Perimeter of a rectangle :

Whose sides are l and b

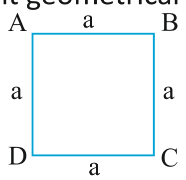
= $2l + 2b$

= $2(l + b)$



Area Formula

As like perimeter formula, we can also express the area of different geometrical shapes as algebraic expression.

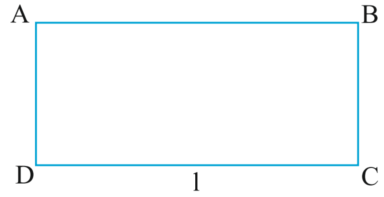


1. Area of a square whose each side is a

   $= a \times a = a^2$

2. Area of a rectangle whose length and width are l and b respectively

   $= l \times b = lb$



3. Area of a triangle, where b is the base and h be the height

   $\frac{1}{2} \times b \times h$

Exercise 6.4

1. Find 3rd, 7th, 10th and 20th term by using following expressions represent the terms.

(a) $3n+1$ (b) $4n-3$

(c) $5n+5$ (d) n^2+5 [Hint : $n = 1, 2, 3, ...$]

2. A square having side x. Express it's perimeter as algebraic expression.

3. Length and breadth of a rectangle are m and n respectively. Express its area as expression.

4. Perimeter of a rectangle is given 30 cm. If breadth is given as 'a' and length is 9 cm. Find its breadth ?

Points to Remember

- A quantity which can have different values is said to be variable or literal.
- A quantity having a fixed or definite numerical value is called constant.
- A collection of variables and constants connected by one or more basic operations (+, –, ×, ÷) is called an algebraic expression.
- A term is a product of factors. These are the parts of an algebraic expression.
- The term of an algebraic expression containing no literal factors is called constant term.
- An algebraic expression having one variable is said to be a polynomial in one variable.
- The degree of an algebraic expression in the highest power of the variable present in the expression.

EXERCISE

1. MULTIPLE CHOICE QUESTIONS (MCQs)

Tick (✓) the correct options :

(a) A quantity having a fixed numerical value is called

(i) variable ☐ (ii) constant ☐

(iii) term ☐ (iv) degree ☐

(b) A variable may have

(i) only one value ☐ (ii) only two values ☐

(iii) no values ☐ (iv) different values ☐

(c) An algebraic expression different parts are connected by

(i) addition ☐ (ii) subtraction ☐

(iii) addition and subtraction ☐ (iv) None of these ☐

(d) Which of the following is a monomial ?

(i) 5x ☐ (ii) 5x + y ☐

(iii) 3x + y = 0 ☐ (iv) p + q ☐

(e) A constant factor is also known as :

(i) coefficient ☐ (ii) numerical factor ☐

(iii) literal factor ☐ (iv) all of these ☐

(f) Coefficient of y in term $-10x^2 y$ is :

(i) x^2 ☐ (ii) $-10x$ ☐

(iii) $10 x^2$ ☐ (iv) $-10x^2$ ☐

(g) Polynomial $9y^3 - 4y - \frac{2}{3}$ is in y of degree :

(i) 2 ☐ (ii) 3 ☐

(iii) 3 + 1 ☐ (iv) 3 + 0 ☐

(h) We change the sign of every term of the expression then

(i) subtract ☐ (ii) Add ☐

(iii) multiply ☐ (iv) none of these ☐

(i) When we subtract a from b we write it as :

(i) a – b ☐ (ii) b – a ☐

(iii) a + b ☐ (iv) we can't subtract ☐

(j) $2xy + x^2 + y^2$ is an example of

(i) monomial ☐ (ii) binomial ☐

(iii) trinomial ☐ (iv) none of these ☐

2. Write the following in algebraic form using variables, constant, signs and symbols.

(a) The sum of 3 consecutive even number is 111.

(b) Ranjan has ₹ 1000 more than his friend

(c) Tapan has ₹ 90 less than Palak

(d) Product of numbers m and n subtracted from 676.

3. **Identify the numerical coefficient of terms (other than constants) in each of the following expressions given below:**

(a) $11 - 10x^2$ (b) $p + 9qr + 11q$

(c) $8x^2 - 5x$ (d) $a^2b^2 - 7ab$

4. **Identify the like terms of the following :**

(a) $pq^2, 3p^2q, 7q^2p, 9pq$ (b) abc, cab, x^2yz, x^2yz^2

(c) $11a, 111b, 11c, 11b$ (d) $5x^2y^2z^2, 9y^2z^2x^2, 7x^2yz, 6zy^2z^2$

5. **Write the literal coefficient of each of the following :**

(a) $\frac{5}{9}xy^2z$ (b) $-\frac{1a^2b}{4}$

(c) $\frac{3}{4}x^2a^2m^2$ (d) $-121x^2y^5$

6. **Add the following algebraic expressions by horizontal and column methods.**

(a) $p^3 - 5p^2q + 7pq^2 - q^3$ and $4p^3 - 7p^2q + 9pq^2 + 2q^3$

(b) $x^4 + 5xy^3 - 4x^3y + 8x^2y^2 + 7y^2$ and $-5x^4 - 9xy^3 + 11x^3y - 10x^2y^2 + 9y^2$.

7. **Subtract the following by column method.**

(a) $p^2 + q^2$ from $9p^2 - 10q^2$ (b) $x^2 - 7xy$ from $5x^2 - 10xy$

8. How much is $m^3 - 2m^2 + m + 9$ greater than $3m^3 + 5m^2 - 6m + 7$?

9. **If $x = 3$, $y = 2$, find the value of each of the following.**

(a) $x^2 + y$ (b) $x^2 - xy + y^2$

(c) $7(5x + 4) - 5(2 - 5x)$ (d) $3x^3 - 7(5x - 4)$

10. If $m = 7$, find the value of $m^3 - 7(m - 5)$.

11. Length and breadth of a rectangle is m and n respectively. Find its area and perimeter.

HOTS

The perimeter of a triangle is $14x^2 + 20x + 13$. Two of its sides are $3x^2 + 5x + 1$ and $x^2 + 10x - 6$. Find its third side.

Lab Activity

Objective : To evaluate an algebraic expression for different values by the activity method.

Materials Required : White chart paper, coloured chart paper, a pair of scissors, sketch pen, gum.

Procedure : Let us try to evaluate $3y + 2$ for $y = 1$, $y = 2$ and $y = 3$

Follow these steps :

Step 1 : We cut out 24 strips, each of 1 cm × 1 cm, from coloured chart paper.

Step 2 : Draw a 12 cm × 12 cm grid on a white chart paper.

Step 3 : Let y = 1. So, paste 3 strips on the grid and colour 2 unit boxes (since the constant term is 2) using sketch pen as shown in Fig. (ii) (case 1.)

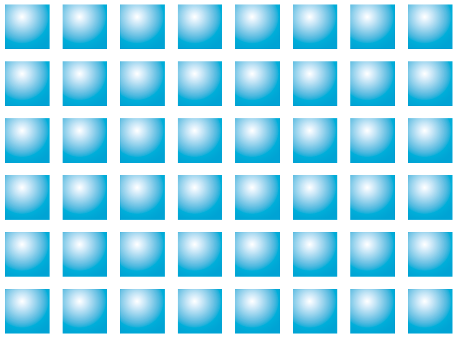


Fig. (i)

Step 4 : Let y = 2. So, take 2 strips at a time and paste 2 sets of 3 strips on the grid and colour 2 unit boxes as shown in Fig (*ii*) (case 2).

Step 5 : Let y = 3. So, take 3 strips at a time and paste 3 sets of 3 strips on the gird and colour 2 unit boxes as shown on the grid as shown in Fig. (ii) (case 3).

Step 6 : Count the number of coloured boxes in each case. You will find the value of $3y + 2$ for different value of y.

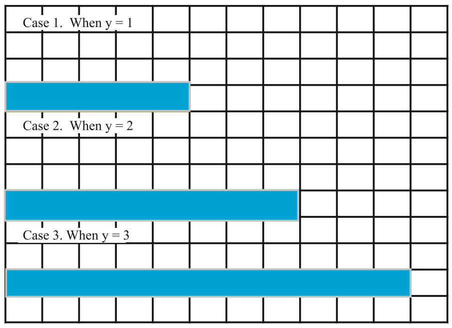


Fig. (ii)

Conclusion :

When $y = 1$, then

$3y + 2 = 3 \times 1 + 2$

$= 5$

When $y = 2$, then

$3y + 2 = 3 \times 2 + 2$

$= 8$

When $y = 3$, then

$3y + 3 = 3 \times 3 + 2 = 11$

7 Simple Linear Equations

You are well aware of algebraic expressions. You know that any algebraic expression has some terms. These terms have constants and variables. Now, let us move further. Assume that we have a situation. We want to multiply a variable with 100 and then we have to add 20 to that result.

Or, we can write:

$100x + 20$

This is an algebraic expression (you have read these in chapter 6). We are dealing with algebraic expressions that have only one variable.

Read another example : $\frac{4}{3}x + 36$

Here, $\frac{4}{3}$ and 36 are constants. The variable x can be given any value of a real number.

Therefore, algebraic expressions in one variable may have one or more terms. They have only one variable which shows certain conditions of scientific, commercial or mathematical nature. We have read about algebraic expressions in the previous class.

Equation

Let us consider the following situations.

Example : 6 subtracted from one third of y gives 5.

This can be written as :

$\frac{1}{3}y - 6 = 5$ .... (i)

Example : p multiplied by itself is 6 less than 5 times the number q.

This can be written as :

$p^2 = 5q - 6$ .... (ii)

Example : 7 added to double of m gives 10.

This can be written as :

$2m + 7 = 10$ .... (iii)

Example : The sum of number x and twice the number y is 20.

This can be written as:

$x + 2y = 20$ .... (iv)

Don't you think that the expressions shown in (i), (ii), (iii) and (iv) are different from the ones that we had studied in class VI ? Yes, they have the equality sign. Morever, there are a few terms (at least one) on the right side of equality. They all are equations.

Definition of Equation

A statement of equality that involves some variables is called equation. We can also call it algebraic equation.

Examples :

$4x^2 + 30 = 7$

$4x + 3 = 7$

$2pqr = 7q + 5p$

$8 + (4x + 3y) = 120$

Linear Equation

It is an equation in which the maximum power of a variable is 1. Have a look at the given examples :

(a) $7x^2 + 8x = 18$

(b) $15y^3 + 36 = 112$

(c) $7q + 88 = 0$

Out of these three examples only equation (c) has a variable q, whose power is 1. So, it is a linear equation. Others two are not linear equations.

Further, note that if the variables are more than, they one have to be considered independently to judge whether the algebraic expression is a linear equation or not.

Example : $6x + 9y = 15z - 4y$

The power of x, y and z is 1each. So, it is also a linear equation, even if the number of different variables is 3.

Facts to Know

- The value of any number or radical with exponent 0 is always 1. Example : $x^0 = 1$

Linear Equation in One Variable

It is an equation, which has :

(a) the equality sign;

(b) a single variable; and

(c) only one as power (1) of that variable.

These equations are also called simple equation.

Example :

$$25 = 18x + 7x$$
$$14y - 7 = 0$$
$$6z + 3z - 81 = 0$$
$$15p + 36 = 12p$$

These all are linear equations in one variable.

Example : $6x^2 + 7x + 6 = 0$

This is not a linear equation because its highest power is not 1.

Example : $5x^2 + 6y^2 = 118$

This is not a linear equation, as it has two variables and the maximum power of each variable is 2.

Facts to Know

- The general equation of a linear equation in one variable is : $ax + b = 0, a \neq 0$.

Example 1 : **Write the following statements in the form of equations:**

(a) Subtract 18 from 9 times a number r and you get 18.

(b) One sixth of p is subtracted from 35 and the result is –1.

(c) Add 100 to a and divide it by 4 to get 25.

(d) The sum of three times y is added to 8 and we get 38.

Solution : (a) 9 times a number r = $9r$

Subtract 18 from $9r$ = $9r - 18$

So, $9r - 18 = 18$

(b) One sixth of $p = \frac{p}{6}$

Subtracted from 35 $= 35 - \frac{p}{6}$

So, $35 - \frac{p}{6} = -1$

(c) Add 100 to $a = a + 100$

Divide it by 4 $= \frac{a+100}{4}$

So, $\frac{a+100}{4} = 25$

(d) Three times $y = 3y$

$3y$ added to 8 $= 3y + 8$

So, $3y + 8 = 38$

Facts to Know

- The inequalities > (greater than) and < (less than), are not used to form equations.

Example : $6x + 17 > 0$ (Not an equation)

$15x + 28 = 0$ (Equation)

$22r + 65 < 0$ (Not an equation)

Example 2 : Write the following statements in the equation form:

(a) Six added to one fourth of a number n gives 101.

(b) 5 added to 7 times y gives 102.

(c) 25 subtracted from y is 60.

Solution : (a) One fourth of $n = \frac{n}{4}$

Add six to $\frac{n}{4} = \frac{n}{4} + 6$

So, $\frac{n}{4} + 6 = 101$

(b) Seven times $y = 7y$

Add 5 to $7y = 7y + 5$

So, $7y + 5 = 102$

(c) Subtract 25 from $y = y - 25$

So, $y - 25 = 60$

Example 3 : Check whether the value given in the brackets satisfies the given equation :

(a) $4m - 3 = 9$ $(m = 6)$

(b) $5r + 3 = 13$ $(r = 2)$

(c) $10x + 25 = 135$ $(x = 11)$

(d) $19a + a + 1 = 21$ $(a = 3)$

Solution : (a) For $m = 6$

LHS $= 4m - 3$

$= (4 \times 6) - 3$

$= 24 - 3$

$= 21$

RHS $= 9$

LHS $\neq$ RHS

So, $m = 6$ does not satisfy the equation.

(b) For $r = 2$

LHS $= 5r + 3$

$= (5 \times 2) + 3$

$= 10 + 3$

$= 13$

RHS $= 13$

LHS $=$ RHS

Hence, $r = 2$ satisfies the equation.

(c) For $x = 11$

LHS $= 10x + 25$

$= 10 \times 11 + 25$

$= 110 + 25$

$= 135$

RHS $= 135$

LHS $=$ RHS

So, $x = 11$ satisfies the equation.

(d) For $a = 3$

LHS $= 19a + a + 1$

$= (19a + a) + 1$

$= 20a + 1$

$= (20 \times 3) + 1$

$= 60 + 1 = 61$

RHS $= 21$

LHS $\neq$ RHS

Hence, $a = 3$ does not satisfy the equation.

Example 4 : In a test, the highest marks obtained by a student is four times the lowest marks plus 23. The highest score is 120. Form an equation in one variable with the help of this data.

Solution : Let lowest marks obtained by student $= x$

4 times lowest marks $= 4x$

Add 23 $= 4x + 23$

As per the statement, the term $4x + 23$ is equal to the highest score.

$\Rightarrow \quad 4x + 23 = 120$

Exercise 7.1

1. Write the following equations in the statement form :

(a) $6m + 32 = 62$

(b) $\frac{7}{8}x + 31 = 120$

(c) $\frac{t}{2} - 10 = 103$

(d) $5l = 125$

(e) $d - 30 = -20$

(f) $5a + 35 = 0$

(g) $\frac{x}{3} - 4 = 4$

(h) $\frac{15}{16}q = 225$

2. Form equations with the help of the following statements :

(a) In an isosceles triangle, the base angles are equal. The vertex angle is twice of the base angle. Form an equation for the sum of all angles. (Take base angle as p).

(b) A number is multiplied by 80. When 60 is added to it, the result becomes 365. Assuming the variable as t form an equation with zero 0 on RHS of equality.

(c) A number is taken. Its $\frac{1}{200}$ value is obtained. When 40 is added to it the result is 1000. Form an equation with the help of this data. Assume the variable on your own.

(d) Three times of a number reduced by 100 gives 330.

(e) Suzen's father is thrice as old as Suzen. After 12 years, he will be twice as old as his daughter. Make an equation with the help of this data. The present age of Suzen is h years.

3. Check if the given values are the solutions of the respective equations:

Equation	Value	Status	
		Solution - (ü)	Not a Solution (û)
(a) $2(a + 5) = 8$	$a = -1$		
(b) $3r - 2 = 13$	$r = 5$		
(c) $25x = 625$	$x = 26$		
(d) $\frac{17}{t} - 1 = 16$	$t = \frac{1}{2}$		

4. Answer the following questions in yes or no :

(a) Is equation $x - 3 = 2$ the same as equation $2 = 3 - x$?

(b) Is $\frac{5x}{60} = 12x$?

(c) Do we get the same value of r from $36r = 180$ and $\frac{r}{36} = \frac{1}{180}$?

Solution of Linear Equation in One Variable

The solution of an algebraic equation is also known as root of the equation. The root of an algebraic equation is the value of the variable that satisfies the equation.

We can start from a simple example.

Let $x + 19 = 20$ .... (i)

$\Rightarrow x = 20 - 19 = 1$

If we put $x = 1$ in equation (i)

We get :

L.H.S. $= x + 19$

$= 1 + 19 = 20 =$ R.H.S

Hence $x = 1$ is the solution or root of the equation given to us. We solved it with the help of the method of transposition.

Methods of Solving Linear Equations in One Variable

Hit and Trial Method

In this method, we guess a number which may satisfy the given equation.

Example 5 : **If $6t = 96$, find the value of t.**

Solution : We think of a number in place of t, so that its multiplication with 6 gives us 96. The guesswork starts as per your convenience. You can start from $t = 2$ also. But it would take long time that way. So, start from $t = 11$.

$6 \times 11 = 66$

$6 \times 12 = 72$

$6 \times 13 = 78$

and so on

$6 \times 16 = 96$

So, $t = 16$

This method is no longer followed.

Transposition

It is the most commonly used method for solving linear equations. The meaning of 'transpose' is to "change the side of the number". When we do transposing, the following changes are to be made.

(a) $-$ becomes $+$ on changing the side.

(b) $+$ becomes $-$ on changing the side.

(c) $\div$ becomes $\times$ on changing the side.

(d) $\times$ becomes $\div$ on changing the side.

Example 6 : **Solve by transposition.**

$$\frac{x}{2} - 5 = 2$$

Solution : Let us take -5 to RHS of equality. Thus we have :

$\frac{x}{2} = 2-(-5)$ (–5 is subtracted)

$\Rightarrow \frac{x}{2} = 2+5=7$

Now let us take 2 to the other side of equality,

$x = 7 \times 2$ (2 is multiplied)

$\Rightarrow x = 14$ is the solution.

Facts to Know

- "Changing the side" means we are going to either side of the equality sign. Left to right or right to left.

Verification :

We have

$\frac{x}{2} - 5 = 2$ .... (i)

Put $x = 14$ in equation (i), we get

LHS $= \frac{14}{2} - 5$

$= 7 - 5$

$= 2 =$ RHS, which is true.

Hence, $x = 14$ is the solution of the equation $\frac{x}{2} - 5 = 2$.

Facts to Know

- "Once a term is transferred from RHS of equation to its LHS, it is no longer in RHS.

Balancing an Equation

In this method, we use the concept that LHS and RHS of an algebraic equation are always equal to each other. If LHS seems to be light (less) than RHS, it means some weight must be added to it (in the unknown variable) to balance both sides.

The rules to be followed in this method are as follows :

(a) The same quantity can be added to both the sides of an equation without changing the equality.
(b) The same quantity can be subtracted from both the sides of an equation without changing the equality.
(c) The same quantity can be multiplied to both sides of an equation without changing the equality.
(d) Both sides of an equation can be divided by the same quantity without changing the equality.

Example 7 : Solve the following equation with the method of balancing the equation : $\frac{(x+1)}{3} = 2$

Solution : We have to eliminate 1 and 3 from LHS in this equation. First of all, let us remove 3 from LHS. Multiplying both sides by 3, we get:

$$\frac{(x+1)}{3} \times 3 = 2 \times 3$$

$$\Rightarrow \quad (x+1) = 2 \times 3$$

$$\Rightarrow \quad x + 1 = 6$$

Now subtract 1 from LHS. Naturally, we have to subtract 1 from RHS as well. We get,

$$x + 1 - 1 = 6 - 1$$

$$\Rightarrow \quad x = 5$$

Hence, $x = 5$ is the solution of the given equation.

If the method to be used to solve an equation is not mentioned, you can use any one of the three methods explained in this chapter.

Construction Equations from a Given Solution

If we have been provided an algebraic equation, we can find out its unique solution. But if the situation is just the opposite (we have a unique solution), can we make algebraic equations? The answer is yes. We can create infinite number of equations from a given solution of an algebraic equation. Read table that follows:

Equation to Solution	Solution to Equation
1. There is a unique solution for an equation.	1. There are infinite numbers of equations corresponding to a given solution.
2. In order to solve an equation we eliminate the numbers written with the variable one by one by transposing or balancing method.	2. We introduce a number and also initiate an arithmatic operation (+ , –, ×, ÷) to both sides of the equation to form many equations.

Example 8 : **Construct three equations with the unique solution $x = 7$.**

Solution : We have been given

$$x = 7 \qquad \text{.................................... (i)}$$

Add 10 to both sides of equation,

$$x + 10 = 7 + 10$$

$$\Rightarrow \quad x + 10 = 17 \qquad \text{.................................... (ii)}$$

The solution of this equation is $x = 7$

Multiply both sides of equation (i) with 21. Then, subtract 8 from both sides,

$$x \times 21 = 7 \times 21$$

$$\Rightarrow \quad 21x = 147$$

$$\Rightarrow \quad 21x - 8 = 147 - 8$$

$$\Rightarrow \quad 21x - 8 = 139 \qquad \text{.................................... (iii)}$$

The solution of this equation is $x = 7$

Divide equation (iii) by 71,

$$\Rightarrow \quad \frac{21x - 8}{71} = \frac{139}{71} \qquad \text{.................................... (iv)}$$

The solution of this equation is $x = 7$

The three equations are : $x + 10 = 17$; $21x - 8 = 139$ and $\frac{21x - 8}{71} = \frac{139}{71}$.

Exercise 7.2

1. Three times a certain number increased by 26 gives 227. Find out the number.
2. The difference of two angles of a triangle is 39.8°. The sum of the same angles equals 89.2°. Find the measure of all angles of this triangle.
3. Seema's age is two third of Reema's age. If 7 years from now, Reema will be 6 years older than Seema, what is the present age of Seema and Reema ?
4. Abraham has five times money as that of Jonathan. After giving ₹ 25 to Jonathan, Abraham has double the money Jonathan has now. How much money did Abraham have in the beginning ?
5. The denominator of a common fraction exceeds the numerator by 10. If 8 is added to the numerator and denominator the new fraction obtained is $\frac{5}{6}$. Find the original fraction.
6. **Write the following statements in the form of equations :**
   - (a) Ten added to one fifth of a number *t* gives 14*t*.
   - (b) The number *y* divided by 88 gives 4.
   - (c) Ten taken away from 19 times a number *q* gives 180.
   - (d) The sum of seven times *r* and 11 is 81.

(e) A number x is multiplied by 7 and 110 is added to it. If this entire term is divided by 100, we get $\frac{12}{13}$.

(f) A number m is subtracted from 200 and then this term is divided by 17, the result is $\frac{1}{6}$.

7. Some equations have been given below. Check whether the value given in the brackets is a solution for the equation or not :

(a) $4m-3=8$ $(m=3)$

(b) $2x+18=38$ $(x=11)$

(c) $3x+10=\frac{10}{7}$ $(x=\frac{-20}{7})$

(d) $\frac{12y+12}{12}=1$ $(y=0)$

8. Write the following equations in the statement form :

(a) $6x=139$

(b) $\frac{17x+21}{121}=87$

(c) $5r-108=12$

(d) $\frac{l}{8}-25=75$

(e) $87p+100=13$

(f) $\left(\frac{a}{6}+7\right)3=3$

9. Solve for x : $x-5=3$. Verify your answer also.

10. Solve for x : $\frac{2x}{3}-\frac{x}{6}=\frac{1}{2}$. Verify your answer also.

11. Find the value of z by the method of balancing the equation.

$\frac{2}{3}(z-4)=6$

12. Solve for p. Verify the solution also.

$\frac{p-5}{10}-\frac{2(p-3)}{5}=\frac{3(p-4)}{15}$

13. Find the values of the following equations if $y=3$.

(a) $100-10y$

(b) $\frac{6y+32}{5}$

(c) $\frac{5}{3}(y-3)+\frac{7}{3}(-6+y)$

(d) $\left(\frac{7y+19}{4}\right)+\left(\frac{18y-4}{5}\right)$

14. The sum of three consecutive numbers is 54. Find out the numbers.

15. Fill in the blanks :

(a) An equation having only one variable and only one power of that variable is called a ______________ equation.

(b) When a term is transposed to the either side of the equality sign, its sign is __________.

(c) If the double of a number is 106, the number is __________.

(d) If $\frac{x}{21}=3$, then the value of x is _________.

(e) If $\frac{75}{y}=25$, then the value of y is _________.

16. Misha has ₹p with her. Her money is two-third of the money Mona has. If Mona has ₹1224, what is the value of p?

17. A man travels one-third of the total distance by foot. He rides in a bus and covers two-third of the distance by bus. If the distance covered by bus was 100 km, what was the total distance travelled by him?

18. Find out two numbers whose difference is 18 and the ratio of the larger number to the smaller one is 7 : 4.
19. One-sixth of Poly's income is equal to one-fifth of Mouly's income. If Poly earns ₹ 3000 more than Mouly, how much are they earning separately ?
20. A student was asked to multiply a number by 11. By mistake, he multiplied the number by 101. If the difference between the wrong number and the right number is 900, find out the number.

Points to Remember

- An **equation** is a statement of equality involving variables and constants. The equality sign is a must in every algebraic equations.
- Equations contain algebraic expressions.
- A **linear equation in one variable** is an equation in which only one variable is used and the maximum power of the variable is 1.
- The value of the variable that satisfies the equation is called **solution of the equation**. A linear equation in one variable has only one solution.
- We can solve linear equations through three methods — hit and trial, transposition and balancing the equation.
- In the method of transposition, the terms can be taken to either side of the equality sign. If we do so, + becomes –, – becomes +, × becomes ÷ and ÷ become ×. The terms are processed according to their new signs in the method of transposition.
- In the method of balancing the equation, some process is done on both of the equality sign.
- After calculating the value of the unknown variable, we must verify the solution by putting the value of variable in the equation designed by us (or in the one given to us). The verification of answer is a must in all questions involving linear equations.

EXERCISE

1. **MULTIPLE CHOICE QUESTIONS (MCQs):**

Tick (✓) the correct options.

(a) x, y, z, p, q, r ..... etc are called _______ in algebraic equations.

(i) constants ☐ (ii) powers ☐
(iii) radicals ☐ (iv) None of these ☐

(b) All linear equations have the variables whose power is

(i) two ☐ (ii) three ☐
(iii) one ☐ (iv) cannot be determined. ☐

(c) In an isosceles triangle, the base angles are equal. The vertex angle is 40°. What is the measure of base angles?

(i) 40° ☐ (ii) 80° ☐
(iii) 60° ☐ (iv) 70° ☐

(d) In the method of balancing the equation, if we divide LHS by 21, what we would do the following on RHS?

(i) Multiply RHS by 21 ☐ (ii) Divide RHS by 42 ☐
(iii) Divide RHS by 21 ☐ (iv) Any one of these ☐

(e) The difference of two angles of a triangle is 21°. The sum of these angles is 105°. The angles have the following measure :

(i) 53°, 50°, 77° ☐ (ii) 60°, 60°, 60° ☐
(iii) 42°, 63°, 75° ☐ (iv) None of these ☐

(f) In the process of the verification of solution, which of the following is essentially true.

(i) RHS is non-zero ☐ (ii) LHS is non-zero ☐

(iii) LHS$\neq$RHS ☐ (iv) LHS = RHS ☐

(g) A is 3 years elder to B. Five years ago, three fifth of A's age was equal to three fourth of the age of B. What is the age (in years) of A and B now?

(i) A – 17, B – 20 ☐ (ii) A – 20, B – 17 ☐

(iii) A – 28, B – 25 ☐ (iv) A – 25, B – 28 ☐

(h) The number 80 is to be divided in such a manner that the bigger part is four times the smaller one. The numbers are :

(i) 60, 20 ☐ (ii) 68, 17 ☐

(iii) 64, 16 ☐ (iv) 60, 20 ☐

(i) The sum of three consecutive natural numbers is 75. The numbers are :

(i) 18, 19, 20 ☐ (ii) 23, 24, 25 ☐

(iii) 24, 25, 26 ☐ (iv) 25, 26, 27 ☐

(j) In the algebraic equation $\frac{s}{3} = 5$ the value of s is :

(i) 6 ☐ (ii) 12 ☐

(iii) 18 ☐ (iv) 15 ☐

2. Write the following statements in the form of equations :

(a) The number *k* divided by seventeen gives ten.

(b) Ten added to seven times a number gives three hundred ten.

(c) Seven-sixth of a number plus twenty-one is one hundred twenty one.

(d) Two taken away from seven times a number *b* gives one hundred twelve.

3. Write the following equations in the statement form :

(a) $7p + 111 = 201$ (b) $-12 = 20$

(c) $5x + 10 = 15$ (d) $\left(\frac{x+7}{3}\right) + 2 = 10$

4. Solve for *x* :

(a) $\frac{x-1}{3} + \frac{x+1}{2} + \frac{x+7}{2} = 1$ (b) $0.6x = 34 - 0.4x + 14$

(c) $\frac{x-2}{x+1} = \frac{3}{4}$ (d) $\frac{3(x-4)}{15} = \frac{x-5}{10} - \frac{2(x-3)}{5}$

(e) $\frac{2x}{3} - \frac{x}{6} = \frac{1}{2}$

5. Romi's father is 49 years of age. Her father is 24 years older than her. What is the present age of Romi?

6. Rashmi thinks of a number. If she takes away 7 from $\frac{5}{2}$ of that number she gets 8. What is the original number ?

7. Construct three linear equations with the following equations :

(a) $p = -\frac{11}{2}$ (b) $7x = 18$

(c) $14y = 84$ (d) $72l + 11 = 102$

8. When you multiply a number by 6 and subtract 5 from the product, you get 7. What is the number ?

9. Find the two numbers whose difference is 18 and the ratio of the larger number to smaller one is 5 : 4.

10. Think of a number. Add 13 to it and divide the sum by 5. You get 6. What is the number ?

11. Think of a variable. Now, multiply it by 2 and deduct 17 from it. This difference is multiplied by 6 and the result is 126. What is the value of the variable?

HOTS

If one side of a square is represented by $4x - 7$ and the adjacent side is represented by $3x + 5$. Find the value of x.

Lab Activity

In the chart given below, fill up the right-most column with the explanation of what is happening in the middle column. Do not miss any column.

S.N	Algebraic Process	What is Happening
1.	$\frac{19x - 50}{18} = 50$	
2.	$\frac{19x - 50}{18} \times 18 = 50 \times 80$	
3.	$19x - 50 = 50 \times 18$	
4.	$19x - 50 = 900$	
5.	$19x - 50 + 50 = 900 + 50$	
6.	$19x = 900 + 50$	
7.	$19x = 950$	
8.	$19x \times \frac{1}{19} = 950 \times \frac{1}{19}$	
9.	$x = \frac{950}{19}$	
10.	$x = 50$	

8 Ratio and Proportion

Ratio is a numerical relation of one quantity to another of the same kind. This is a comparison of two numbers. The number which form the ratio are called its terms.

In our daily routine, we are told to compare two quantities quite often. We can compare two quantities as follows :

(1) We can subtract one (small) quantity from the other (big) one. So, we learn the difference between the magnitude of two quantities. Such a comparison is comparison by difference. Eg. : The weight of Ram is 42 kg. The weight of John is 53.200 kg. The difference is 53.200 – 42.000 = 11.200 kg. So, John is 11.2 kg heavier than Ram.

(2) We can compare the two quantities and find out how many times one quantity is in comparison to the other quantity. In this process, we find out a ratio of the magnitude of one quantity with that of the other. Such a comparison is comparison by division.

Example: The height of person A is 160 cm. The height of person B is 80 cm. So, person A is $\frac{160}{80}$ = 2 times taller than person B.

Thus, we are better able to understand the concept when we divide quantities or make their ratios. Thus, we shall use ratios for the purpose of accurately comparing quantities.

Let us study a few more examples regarding ratios. They are as follows :

1. Ishita has ₹ 100 with her, Natasha has ₹ 80 with her. The ratio of Ishita to Natasha is $\frac{100}{80} = \frac{5}{4}$.

So, Ishita has $\frac{5}{4}$ times more money than Natasha.

2. The speed of car A is 120 km per hour. The speed of car B is 100 km per hour. So, the speed of car A is $\frac{120}{100} = 1.2$ times the speed of car B.

Defining Ratio

A ratio represents a relationship between two quantities and is obtained by dividing those two quantities.

It is written as follows : $\frac{a}{b}$ or a : b.

So, this ratio tells us how much two quantities of the same kind are with respect to each other. Here, the numerator or the first quantity of ratio is called antecedent. The denominator or the second quantity is called consequents. If we reverse the consequent and antecedent of a ratio, the ratio changes and is not equal to the previous ratio.

Hence, ratio is a fraction which shows how many times a quantity is of another quantity of the same type.

Facts to Know

Eg .: $\frac{800\,kg}{1000\,kg} = \frac{800}{1000} = \frac{8}{10} = \frac{4}{5}$

So, a ratio does not have units.

Further, the measures must always be in the same unit so that two quantities may be compared. If unit is not the same, the quantities cannot be compared.

Mixed Fractions

Quite often, we are given mixed fractions as ratios. We cannot solve such questions with making them simple fractions. So, we have to make them simple fractions and rationalise the denominators of both.

For Example: Two numbers are in the ratio of $3\frac{1}{2}$ and $5\frac{1}{3}$. We want to compare them.

The numbers are

$3\frac{1}{2}$ and $5\frac{1}{3}$

$3\frac{1}{2} = \frac{7}{2}$

$5\frac{1}{3} = \frac{16}{3}$

LCM of 2 and 3 is 6

We have to make denominators equal in case of both these simple fractions.

The final fractions are

$\frac{7}{2} = \frac{7}{2} \times \frac{3}{3} = \frac{21}{6}$

$\frac{16}{3} = \frac{16}{3} \times \frac{2}{2} = \frac{32}{6}$

Now, these fractions can be compared. Their base is common, i.e., 6.

So, $\frac{32}{6} > \frac{21}{6}$

$\frac{16}{3} > \frac{7}{2}$

Facts to Know

Ratio of two quantities, a and b, is $a \div b$ or $\frac{a}{b}$. It is denoted as $a : b$ and spoken as : "a is to b."

Example 1 : Divide ₹ 1280 between A and B in the ratio of 3 : 2.

Solution : Ratio of A and B = 3 : 2

Sum of ratios = 3 + 2 = 5

Sum received by A = $\frac{3}{5} \times 1280 = 3 \times 256$

= ₹ 768

Sum received by B = $\frac{2}{5} \times 1280$

= 2×256 = ₹ 512

Cross check $768 + 512$ = ₹ 1280 verified

Example 2 : Two numbers are in the ratio of 161 : 170. The difference between them is 900. Find out both the numbers.

Solution : Let the numbers be a and b.

So, as per the statement, we have.

$a : b :: 161 : 170$

$$\frac{a}{b}=\frac{161}{170} \Rightarrow a=\frac{161b}{170} \qquad \text{.................. (i)}$$

Also, $a-b=900$ (given) .... (ii)

Put the value of a from equation (i) in equation (ii)

$$a-b=900$$

or, $$\frac{161b}{170}-\frac{b}{1}=900$$

$$\frac{161b-170b}{170}=900$$

$$-9b = 153000$$

$$-b = \frac{153000}{9}$$

$$= 17000$$

$$b = -17000$$

Now, $$a = \frac{161}{170}b$$

Put the value of b here.

$$a = \frac{161}{170}b$$

$$= \frac{161}{170}\times(-17000)$$

$$= -\frac{161}{170}\times 17000$$

$$= -161\times 100$$

$$= -16100$$

Hence the two numbers are : –17000 and –16100.

Example 3 : **Write three equivalent ratios of 17:22.**

Solution : We have 17 : 22

$$=\frac{17}{22}$$

$$=\frac{17}{22}\times\frac{3}{3}$$

$$=\frac{51}{66}=51:66$$

Also

$$\frac{17}{22}=\frac{17}{22}\times\frac{4}{4}$$

$$=\frac{68}{88}=68:88$$

Also $$\frac{17}{22}=\frac{17}{22}\times\frac{5}{5}$$

$$=\frac{85}{110}=85:110$$

Example 4 : Compare the following ratios :

(a) 21 : 22 and 84 : 88

(b) 3 : 4 and 8 : 3

Solution :

(a) $21:22 = \frac{21}{22}$

(b) $84:88 = \frac{84}{88} = \frac{42}{44} = \frac{21}{22}$

Hence, the ratio 21 : 22 is equivalent to the ratio 84 : 88.

(b) $3:4 = \frac{3}{4}$

$8:3 = \frac{8}{3}$

The LCM of 4 and 3 is 12. We can rationalise the denominator now.

$$\frac{3}{4} = \frac{3\times3}{4\times3} = \frac{9}{12}$$

$$\frac{8}{3} = \frac{8\times4}{3\times4} = \frac{32}{12}$$

Now, denominator is the same for both fractions.

Hence, $\frac{32}{12} > \frac{9}{12}$

$\frac{8}{3} > \frac{3}{4}$

8 : 3 is larger than 3 : 4

Example 5 : If x : y = 7 : 8, find the value of $\frac{8x+7y}{7x+8y}$

Solution : x : y = 7 : 8

$$\frac{x}{y} = \frac{7}{8} \qquad \text{.......................... (i)}$$

Now, $$\frac{8x+7y}{7x+8y} = \frac{\frac{8x+7y}{y}}{\frac{7x+8y}{y}}$$

$$\frac{\frac{8x}{y}+\frac{7y}{y}}{\frac{7x}{y}+\frac{8y}{y}}$$

$$= \frac{\frac{8x}{y}+7}{\frac{7x}{y}+8} \qquad \text{.......................... (ii)}$$

But we can use equation (i) to put the value of $\frac{x}{y}$ in equation (ii)

$$= \frac{8\times\frac{7}{8}+7}{7\times\frac{7}{8}+8}$$

$$= \frac{7+7}{\frac{49}{8}+8}$$

$$= \frac{14}{\frac{49}{8}+8}$$

$$= \frac{14}{\frac{(49+64)}{8}}$$

$$= \frac{14\times 8}{(49+64)} = \frac{112}{113}$$

Example 6 : If $(8x+7y):(7x+8y) = 112:113$, find out the value of $x:y$.

Solution : We have :

$$(8x+7y):(7x+8y) = 112:113$$

$$\frac{8x+7y}{7x+8y} = \frac{112}{113}$$

Cross multiplying, we get

$$113\,(8x+7y) = 112\,(7x+8y)$$

⇨ $(113\times 8x)+(113\times 7y) = (112\times 7x)+(112\times 8y)$

⇨ $904x+791y = 784x+896y$

⇨ $904x-784x+791y = 896y$

⇨ $904x-784x = 896y-791y$

⇨ $120x = 105y$

⇨ $x = \frac{105}{120}y$

⇨ $\frac{x}{y} = \frac{105}{120}$

$= \frac{21}{24} = \frac{7}{8}$

⇨ $x:y = 7:8$

Exercise 8.1

1. Divide ₹ 225000 among Rojy, Jeny and Simi in the ratio of 7 : 3 : 5.
2. The sides of a triangle are in the ratio of 2 : 3 : 4. If the perimeter of the triangle is 36 cm, find out the length of each side of the triangle.
3. (a) Compare 5 : 7 and 4 : 5.
   (b) Arrange the following ratios in the descending order:
   3 : 2, 5 : 7, 11 : 13

(c) Find the ratio of 1 km and 10 m.

(d) Find the ratio of ₹ 1.36 and 85 paise.

4. Write three equivalent ratios of 111 : 323.

5. Find the ratio between $3\frac{1}{9}$ kg and $5\frac{1}{3}$ kg.

Proportion

Consider the following fraction :

$$\frac{39}{78}=\frac{1}{2}$$

We can simplify this fraction and write it as $\frac{1}{2}$. So, both $\frac{39}{78}$ and $\frac{1}{2}$ are equivalent fractions. You have read about equivalent fractions in class VI. From the concept of equivalent fractions, the concept of proportion can be derived.

Continuing the previous example, We have:

$39 : 78 = 1 : 2$

or, We may write.

$39 : 78 :: 1 : 2$

Hence, there is equivalence in these two ratios. Hence, we can state that:

$$\frac{39}{78}=\frac{1}{2}$$

Hence, they are said to be in proportion.

Defining Proportion

A statement showing the equivalence of two ratios is known as **proportion**. If a, b, c and d are four real numbers, Then, we state that they are in proportion if

$a : b :: c : d$

$\frac{a}{b}=\frac{c}{d}$

$ad = bc$

The terms b and c are called **middle terms**. The terms a and d are called **extremes**.

Thus, Product of Extremes = Product of Middle Terms.

Further, $a : b :: c : d$

We state, a is to b as c is to d." The double colon in the middle denotes the equivalence of two ratios, i.e., $\frac{a}{b}=\frac{c}{d}$.

Continued Proportion

A continued proportion is one in which the ratio between the first and second terms is equal to the ratio between the second and the third term.

Thus, we have,

$a : b :: b : c$

⇨ $a : b = b : c$

$\Rightarrow \quad \frac{a}{b} = \frac{b}{c}$

$\Rightarrow \quad b^2 = ac$

The first term is known as first proportional. Here, it is a.

The second term is known as mean proportional. Here, it is b.

The third term is known as third proportional. Here, it is c.

Facts to Know

A ratio has no limit. A ratio should be expended in the simplest form.

Direct and Inverse Proportions

In real life, we face some natural situations. In these situations, there are two types of unit involved. Let us take some examples:

1. Time taken and work done
2. Speed and distance covered
3. Number of items purchased (in dozens or scores) and money paid in rupees

In such cases, the change in one measure leads to a change in the other. For example, when more money is spent more number of bananas comes home. If money spent is less, then, less number of bananas comes home. Further, if speed of the car is high, the time taken by it to cover a particular distance is less and vice-versa. The case of money spent and number of bananas coming home is a positive (direct) relationship.

The case of increased speed and less time taken is a negative (inverse) relationship.

The extent of change in one measure as a result of change in the other is known as **variation**. As already explained, the variation can have two types of relationship : **direct** and **inverse**. In case of a **direct variation**, an increase or decrease in one measure results in a corresponding increase or decrease in the other measure. Then, the two measures are said to be in a direct : proportion.

Thus, we have.

$a : b = c : d$

$ad = bc$

If three variables are known, fourth one can be calculated.

Eg : $5 : 6 = p : 180$ (directly proportional)

$\Rightarrow \quad 6p = 5 \times 180$

$\Rightarrow \quad p = \frac{5 \times 180}{6} = \frac{900}{6} = 150$

Further, in case of an inverse variation, an increase in one measure results in a corresponding decrease in the other measure. Similarly, a decrease in one measure would lead to a corresponding increase in the other measure.

Thus, we have,

$a : b = d : c$ ($d : c$ is inverse of $c : d$)

$\Rightarrow \quad \frac{a}{b} = \frac{d}{c}$

$\Rightarrow \quad ac = bd$

If three variables are known, then fourth one can be calculated.

Eg : $5:6 = 180:p$

$\Rightarrow \frac{5}{6} = \frac{180}{p}$

$\Rightarrow 5p = 180 \times 6$

$\Rightarrow p = \frac{180 \times 6}{5} = 36 \times 6 = 216$

Example 7 : **Find x : y : z given that**

$x:y = 4:7$ and $y:z = 49:50$

Solution : Take LCM of 7 and 49. It comes out to be 49.

Multiply first ratio by 7

$x:y = 28:49$

$y:z = 49:50$

$x:y:z = 28:49:50$

Example 8 : **Find the value of *x* if $x:18 = 12:108$**

Solution : $x:18 = 12:108$

$$\frac{x}{18} = \frac{12}{108}$$

$$x \times 108 = 18 \times 12$$

$$x = \frac{18 \times 12}{108}$$

$$= \frac{6}{36} \times 12$$

$$= \frac{12}{6} = 2$$

Example 9 : **Find out the mean proportional between 21 and 84.**

Solution : Let x be the mean proportional between 21 and 84.

Hence, 21, x and 84 are in proportion.

Hence $21:x = x:84$

$$x^2 = 21 \times 84$$

$$x^2 = (7 \times 3) \times (7 \times 12)$$

$$= 7 \times 7 \times 3 \times 12$$

$$= 49 \times 36$$

$$x = \sqrt{49 \times 36}$$

$$= 7 \times 6$$

$$= 7 \times 6 = 42$$

Example 10 : **Sand and cement were mixed in a ratio of 1.7 : 1.9 to make a slab of concrete. If the slab of concrete weighs 31 kg 104 g, how much sand was used to make it ?**

Solution : Let S = quantity of sand used in grams

C = quantity of cement used in grams

Hence, we have,

$S : C = 1.7 : 1.9$

$$\frac{S}{C} = \frac{1.7}{1.9}$$

$$S = \frac{1.7}{1.9} \times C = \frac{17}{19}C \qquad \text{............................ (i)}$$

Also, we have,

$$\begin{aligned} \text{Weight of concrete slab} &= 31 \text{ kg } 104 \text{ g} \\ &= (31 \times 1000 \text{ g}) + (104 \text{ g}) \\ &= (31000 + 104) \text{ g} \\ &= 31104 \text{ g.} \end{aligned}$$

But $S + C = 31104$ .... (ii) (given in statement)

Put the value of S from equation (i) in equation (ii), we get

$$S + C = 31104$$

$$\frac{17}{19}C + C = 31104$$

$$\frac{17C + 19C}{19} = 31104$$

$$\frac{36C}{19} = 31104$$

$$C = \frac{31104 \times 19}{36}$$

$$= \frac{2 \times 15552 \times 19}{6 \times 6}$$

$$= \frac{2 \times 2 \times 7776 \times 19}{6 \times 6}$$

$$= 2 \times 2 \times \frac{19}{6} \times \frac{7776}{6}$$

$$= 2 \times 2 \times \frac{19}{6} \times 1296$$

$$= 2 \times 2 \times 19 \times \frac{1296}{6}$$

$$= 2 \times 2 \times 19 \times 216$$

$$= 4 \times 19 \times 216$$

$$= 76 \times 216 = 16416 \text{ g}$$

$$= 16 \text{ kg } 416 \text{ g}$$

$$S + C = 31104$$

$$S = 31104 - C$$
$$= 31104 - 16416$$
$$= 14688\text{ g} = 14\text{ kg } 688\text{ g}$$

Example 11 : Find the value of m if the ratio m : 27 and 81 : 981 are inversely proportional.

Solution : m : 27 and 81 : 981 are inversely proportional.

Hence, m : 27 = 981 : 81

$$\frac{m}{27} = \frac{981}{81}$$

$$\Rightarrow m = \frac{981 \times 27}{81}$$

$$= \frac{981 \times 27}{9 \times 9}$$

$$= \frac{981}{9} \times \frac{27}{9}$$

$$= 109 \times 3 = 327$$

Example 12 : Are the following numbers in direct proportion ?

5.6, 25.2, 3.4, 15.3

Solution : If these numbers are in direct proportion, then we have:

5.6 : 25.2 :: 3.4 : 15.3

$$\frac{5.6}{25.2} = \frac{3.4}{15.3}$$

$\Rightarrow 25.2 \times 3.4 = 5.6 \times 15.3$

$\Rightarrow 85.68 = 85.68$

Which is true.

Hence, these numbers are in direct proportion.

Example 13 : Are the following numbers in inverse proportion

$$\frac{8}{12},\ \frac{1}{2},\ 1,\ 1\frac{1}{3}$$

Solution : Let us rewrite the numbers.

$$\frac{8}{12},\ \frac{1}{2},\ 1,\ \frac{4}{3}$$

If these numbers are in the inverse proportion, then

$$\frac{8}{12} : \frac{1}{2} :: \frac{4}{3} : 1$$

$$\Rightarrow \frac{8}{12} : \frac{1}{2} = \frac{4}{3} : 1$$

$$\Rightarrow \frac{8}{12} \times 1 = \frac{1}{2} \times \frac{4}{3}$$

$$\Rightarrow \frac{8}{12} = \frac{2}{3}$$

$$\Rightarrow \frac{2}{3} = \frac{2}{3}$$

Which is true.

Hence, these numbers are in inverse proportion.

Example 14 : **A man and his wife earn in the ratio of. $\frac{3}{5}:\frac{11}{25}$ If the man earns ₹ 2200 more than his wife, then what are the man and wife earning separately ?**

Solution : Let, the earning of man = ₹ M

Let, the earning of wife = ₹ W

Hence, we have:

$$M : W = \frac{3}{5}:\frac{11}{25}$$

$$\Rightarrow \frac{M}{W} = \frac{\frac{3}{5}}{\frac{11}{25}}$$

$$= \frac{3}{5} \times \frac{25}{11}$$

$$= 3 \times \frac{5}{11} = \frac{15}{11}$$

$$M = \frac{15W}{11} \qquad \text{................................. (i)}$$

Also, the difference between their earnings is ₹ 2200.

Hence, M – W = 2200. .... (ii)

Insert the value of M from equation (i) in equation (ii), we have :

$$M - W = 2200$$

$$\frac{15}{11}W - W = 2200$$

$$\frac{15W - 11W}{11} = 2200$$

$$\frac{4W}{11} = 2200$$

$$W = \frac{2200 \times 11}{4}$$

$$= 550 \times 11 = ₹\,6050$$

$$M - W = 2200$$

Put the value of W in equation (ii)

$$M - 6050 = 2200$$

$$M = 6050 + 2200$$

$$= ₹\,8250$$

Exercise 8.1

1. The perimeter of a square is 26 cm. The area of another square is 121 cm^2. Find out the ratio of their sides.
2. A and B decided to share expenses in the ratio of 2 : 3. B and C decided to share expenses in the ratio of 5 : 7. How would they share a bill of ₹ 1104 ?
3. Find the third proportional to 6 and 18.

4. If $x : y = 2 : 3$, find out the value of $\frac{3x+2y}{2x+3y}$.
5. An, NGO gave 286 balls to three children. If the ratio of distribution was $\frac{1}{2}:\frac{1}{3}:\frac{1}{4}$, how many balls did each child get?
6. Find the mean proportional between 8 and 242.
7. If the weight of 4 oranges is 600 g, make a table to show the weight of :
   (a) 12 oranges (b) 48 oranges (c) 200 oranges
8. A car can go 180 km in 10 litres of petrol. How far would it be able to go in 35 litres of petrol ?
9. An army base has sufficient ration for 5 days and 60 soldiers are stationed in it. Then, 40 more soldiers arrive at the army base. How many days would be spent with this ration ?
10. A distance of 400 km is represented by 3 cm in a map. What is the distance PQ if P and Q are 7.5 cm apart on the same map ?

Unitary Method

This is a method of using the value of one unit to find the value of many units.

This method can be applied to the problems of direct and inverse proportion. We have already studied the basic concept of the unitary method in Class VI. Now, we will study some examples.

Example :

The cost of one pencil = ₹ 6

The cost of one 18 pencils = 6×18

= ₹ 108

So, when the first quantity increases, the second quantity also increases. So, we multiply in this case. This is an example of direct proportion.

Example :

The cost of 8 muffins = ₹48

The cost of 1 muffin = $\frac{48}{8}$ = ₹6

So, when the first quantity becomes less, the second quantity also becomes less. So, we divide in this case. This is also an example of direct proportion.

Example : A woman takes 2 hours to make a pizza. How much time will be taken by 6 women to cook the same pizza ?

One woman takes time to cook pizza = 2 hrs

6 woman takes time to cook pizza $= \frac{2}{6}$

$= \frac{1}{3} hr.$

Naturally, 6 woman would take for less time than 1 woman to make the pizza. So, when the first quantity becomes more the second quantity becomes less. In this case, we divide. This is an example of indirect proportion.

Example : 6 men can complete a task in 8 days. How much time is needed by 1 man to complete the same task ?

6 men complete 1 task in = 8 days

1 man completes 1 task in = 8×6 = 48 days

Naturally, one man would take much more time to complete that task. So, when the first quantity becomes less, the second quantity becomes more. In this case, we multiply. This is also an example of inverse proportion.

Facts to Know

- In direct proportions, for finding the value of more, we multiply and for finding the value of less, we divide.

Example 15 : The cost of 1 dozen eggs is ₹ 46.50. Find the cost of 144 eggs.

Solution : 144 eggs $= \frac{144}{12}$ dozen

$= \frac{12 \times 12}{12}$

= 12 dozen.

Cost of 1 dozen eggs = ₹46.50

Cost of 12 dozen eggs = 46.50 × 12 = ₹ 558

Facts to Know

- The questions related to time and work are to be done with the help of the unitary method.

Example 16 : The cost of 108 spoons is ₹ 1296. What is the cost of one spoon ?

Solution : Cost of 108 spoons = ₹ 1296

Cost of 1 spoon $= \frac{1296}{108}$

$= \frac{108 \times 12}{108}$

= ₹ 12

Example 17 : Nakul takes 5 days to paint a fence. If Nakul, John and Swati take up this task together, how much time would be taken to complete the painting task ?

Solution : 1 person paints fence in = 5 days

3 person paints fence in $= \frac{5}{3}$

$= 1\frac{2}{3}$ day.

Example 18 : 16 women can pick tea leaves from a tea garden in 6 days. How much time would be needed by 1 woman to do the same job ?

Solution : 16 women pick leaves from a tea garden in = 6 days

1 woman pick leaves from a tea garden in = 6 × 16

= 96 days.

Example 19 : A box having 6 kg of apricots costs ₹ 720. What would be the cost of 14.2 kg of apricots ?

Solution : Cost of 6 kg of apricots = ₹ 720

Cost of 1 kg of apricots $= \frac{720}{6}$

Cost of 14.2 kg of apricots $= \frac{720}{6} \times 14.2$

$= 120 \times 14.2 = ₹1704$

Example 20 : Twenty-five bags of wheat weighing 50 kg each cost ₹ 15000. Find out the cost of 15 bags of wheat if each bag weighs 30 kg.

Solution : Cost of 25 bags of wheat = ₹ 15000

Cost of 1 bag of wheat $= \frac{15000}{25}$

= ₹ 600

Hence, one bag having 50 kg wheat would cost is ₹ 600.

Cost of 50 kg wheat = ₹ 600

Cost of 1 kg of wheat $= \frac{600}{50}$

Cost of 30 kg of wheat $= \frac{600}{50} \times 30$

$= 600 \times \frac{3}{5}$

$= 120 \times 3 = ₹360$

So, if the packing is changed to 30 kg per bag, the cost comes out to be ₹ 360.

Cost of 1 bag = 30 kg = ₹ 360

Cost of 15 bags = 360×15

= ₹ 5400

Example 21 : A medicine is to be given to a patient at the rate of 12.5 mg per 10 kg of body weight. If the patient weighs 45 kg, what would be the requirement of this medicine for him ?

Solution : **(i) Through proportions**

The proportion here is :

$12.5 : x :: 10 : 45$

Where, x is the quantity of medicine needed in mg for a patient weighing 45 kg.

Or, $\frac{12.5}{x} = \frac{10}{45}$

Cross-multiply, we get:

$12.5 \times 45 = 10 \times x$

$562.5 = 10x$

$10x = 562.5$

$x = 56.25$

So, the quantity of medicine to be administered to the patient weighing 45 kg is 56.25 mg.

(ii) Through the unitary method

For 10 kg body weight, medicine needed = 12.5 mg

For 1 kg body weight, medicine needed $= \frac{12.5}{10}$

For 45 kg body weight, medicine needed $= \frac{12.5}{10} \times 45$

$$= \frac{125}{100} \times 45$$

$$= \frac{5}{4} \times 45$$

$$= \frac{225}{4}$$

$$= 56.25\ mg$$

Example 22 : **If a fan's blade turns 10,800 times in an hour, then how many times does it turn in one second ? Use the proportion method.**

Solution : The proportion will be as follows :

1 hour = 60 min.

= 60 × 60 = 3600 Sec.

We have :

10800 : x : : 3600 : 1

Where x is the number of times the blade turns in 1 second.

$$\frac{10800}{x} = \frac{3600}{1}$$

Cross-multiply, we get:

10800 × 1 = 3600 × x

3600x = 10800

$$x = \frac{10800}{3600}$$

$$= \frac{108}{36}$$

$$= 3 \times \frac{36}{36}$$

= 3 seconds

Exercise 8.1

1. Rajan can build a structure in 15 days. Sanjay can build the same structure in 18 days. Both Rajan and Sanjay start making the structure. But Sanjay goes after 6 days of work. How many days would be needed by Rajan to complete the structure ?
2. Three workers take 5 days to do a particular task. How many days will 6 workers take to do that very task ?
3. A can contains edible oil. It has 12 litres of oil. Its price is ₹ 660. Find the cost of 42 litres of this oil.
4. A ration shop sells wheat at the rate of ₹ 225 for 5 kg. Sonu and Monu bought 1.5 kg and 3.5 kg of wheat, respectively from that ration shop. How much was paid by them on an individual basis ?
5. A sketch maker was appointed by a person to make 400 sketches for an amount of ₹ 20,000. He made only 280 sketches and left. How much would he be paid ?
6. Donna can do a task in 15 days. Her friend, Tipsy, can do the same task in 12 days. They start doing the task together. But after 5 days, Donna quits because she is sick. How long would Tipsy take to complete the remaining task ?

7. Jumbo can repair a machine in 1 hour 30 minutes. He asks Rambo to give him support in the process of repair. Together, they start doing the repair and complete the task in 1 hour only. Had Rambo been given this task, how much time would he have taken to repair the machine ?

8. 15 boys can make 80 T-shirts in 4 days. Additional 5 boys are given to the group of 15 boys. How many days are needed now to make 80 T-shirts ?

9. A takes 40 minutes to do a job. B takes 1 hour to do the same job. A, B and C start doing that job together. They complete it in only 15 minutes. Had C been alone, how much time would he have taken to do that job ?

10. A car uses 50 litres to cover 950 km. What is the output of the car per litre ?

Points to Remember

- The ratio of two numbers a and b is a fraction $\frac{a}{b}$ and is denoted as a : b (pronounced as "a is to b.")
- Ratios do not have any units.
- The first quantity of the ratio is called antecedent and the second quantity is called consequent.
- $\frac{a}{b} \neq \frac{b}{a}, a \neq 0, b \neq 0.$
- The numbers a, b, c, d are in proportion if $\frac{a}{b} = \frac{c}{d}$ or if a : b : : c : d.
- If $\frac{a}{b} = \frac{c}{d}$ ⇨ ad = bc. Therefore product of extremes is equal to product of means.
- If a, b, c are in continued proportion, we have $b^2 = ac$. Here, a is the first proportional, b is the middle proportional and c is the third proportional.
- If a : b is directly proportional to c : d, then a : b : : c : d when means ad = bc.
- Unitary method a method of finding the value of one unit to find the value of many units.
- If a : b is inversely proportional to c : d, then we have:
  a : b = d : c which means ac = bd.
- In direct proportions, for finding more, we multiply and for finding less, we divide.
- In inverse proportions, for finding more, we divide and for finding less, we multiply.
- The problems of the unitary method can also be calculated with the help of the method of proportions. Both are similar to each other.

EXERCISE

1. **MULTIPLE CHOICE QUESTIONS (MCQs):**

Tick (✓) the correct options.

(a) If x : y = 5 : 3 , then $(x^2 + y^2) : (x^2 - y^2)$ is equal to

(i) 4 : 1 ☐ (ii) 16 : 1 ☐

(iii) 17 : 8 ☐ (iv) 17 :1 ☐

(b) If $\frac{a}{b} = \frac{c}{d}$, then

(i) ac = bc ☐ (ii) ad = cd ☐

(iii) ad = bc ☐ (iv) cd = ab ☐

(c) If a : b : : b : c, then

(i) $a^2 = b^2$ ☐ (ii) a = bc ☐

(iii) $b^2 = ca$ ☐ (iv) $c^2 = ba$ ☐

(d) What is the mean proportional between 21 and 84 ?

(i) 42 ☐ (ii) 49 ☐

(iii) 51 ☐ (iv) 28 ☐

(e) If you have to divide ₹ 5000 in the ratio of 12 : 13, what are the two amounts obtained after this bifurcation of the given amount ?

(i) ₹ 2400, 2500 ☐ (ii) ₹ 2400, 2800 ☐

(iii) ₹ 2800, 3000 ☐ (iv) ₹ 2400, 2600 ☐

(f) If 805 apples can be packed in little packs of 5 apples each, how many big packs would be made out of these little packs if one big pack has 23 little packs ?

(i) 7 ☐ (ii) 23 ☐ (iii) 161 ☐ (iv) 5 ☐

(g) A 121 notebooks have 2178 pages. How many pages are there in 21 such notebooks ?

(i) 370 ☐ (ii) 379 ☐ (iii) 380 ☐ (iv) 378 ☐

2. Divide ₹ 25,50,000 among A, B and C in the ratio of 3 : 7 : 5.

3. Compare $\frac{121}{7}$ and $\frac{222}{8}$.

4. 100 soldiers have sufficient food for 30 days in a garrison. Then, 25 soldiers left the garrison. Now, how long would this food last ?

5. A bus travels 450 km in 9 hours. How far will it go in 21 hours ?

6. Which ones of the following are in direct proportion and which are in inverse proportion ?

(a) $5\frac{1}{3}, 4, \frac{3}{4}, \frac{9}{16}$ (b) 5.04, 252, 304, 6.08

7. Find the value of m if the following are in inverse proportion.

(a) 1 : m and 729 : 27

(b) $\frac{5}{7} : 1$ and $m : \frac{25}{49}$

8. Process the following ratios and put them in an ascending order.

$\frac{2}{5}, \frac{5}{7}, \frac{9}{13}, \frac{1}{2}, \frac{3}{5}$

9. Find out the mean proportional between 9 r and 16 r if the value of $r = \frac{17}{12}$.

10. If 21 : 147 : 1029 is a continue proportion, put the proportion in the $\frac{a}{b}$ form.

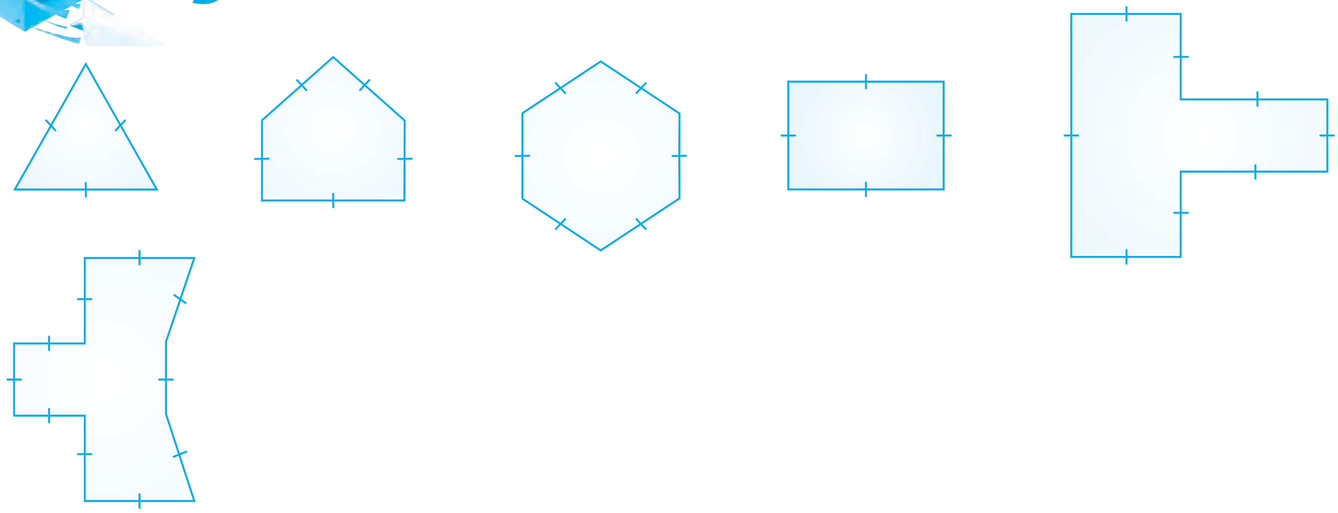
11. A government agency has to construct 30 km of air strip in 6 months. It takes 450 workers to do the job. But the order is extended by another 10 km (of strip). The agency must also finish the task in 4 months (instead of the earlier limit of 6 months). How many more workers would the agency hire to complete the strip on time.

HOTS

A 100 kg pack of Basmati rice costs ₹ 4500. In retail the same quality of rice is available at 5 kg for ₹ 240. Compare the retail and wholesale prices of 3 kg of rice.

Lab Activity

On a piece of chart paper draw the following figures.



Now, count the number of sides of each one of these figures. Arrange these figures in such a manner that the ratio of the sides of one to second comes out to be 1 : 2 in all cases. When you have found out the right combinations, cut out the figures and paste them here:

Paste your figures here			
			← Antecedent
			← Consequent

9 Percentage and its Applications

Latin termology 'Per-centum' whichever Per means "out of" and centum means "one hundred". Hence Percent means out of a hundred and denoted by %.

We come across various situations in our day-to-day life when the concept of percentage is used. See these examples:

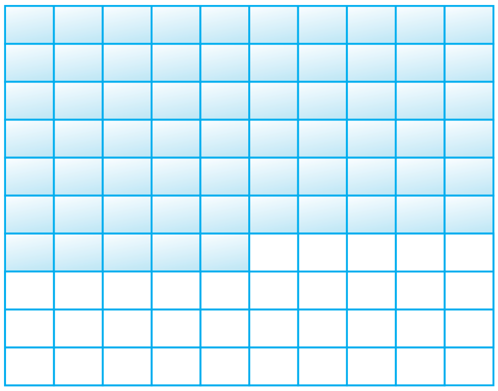
(i) Alisha scored 85 per cent marks in class VI.
(ii) Bank gives 3.5 per cent interest on saving bank accounts.
(iii) Bata has announced 50 per cent off during festive season.
(iv) More than 30 per cent population of India fall below the poverty line.
(v) The government allocates 3 per cent of GDP to education sector.

The common term in above statements is per cent. In this chapter, we shall discuss about percentage, percentage as a fraction and also as a ratio. We shall also learn conversion of fractions and decimals into percentage and vice-versa.Other than this, we shall work on the applications of percentage in problems related with profit and loss and simple interest.

Percentage

The word 'per cent' is taken from the Latin term 'per centum'. Per means out of and centum means one hundred. So, per cent means 'out of hundred'. The symbol used to denote per cent is '%'. Percentages are the fractions whose denominators are equal to 100. Per cent is used for comparison. When we say 70% students got A grade, it means out of 100 students 70 got A grade.

Let us consider a square divided into 100 equal parts, out of which 65 small squares are shaded. The shaded part comprises 65 out of 100, i.e. 65 per cent or 65%.



Let us understand the concept of percentage through an example. Adyant and Aditi showed their report card to their parents. Adyant got 450 marks out of 600 while Aditi got 400 out of 500. Adyant claimed that he deserved a reward as he had secured more marks than Aditi. Do you agree? Perhaps, no. Just by comparing the marks secured by each one can't reach at judgement, as the maximum marks out of which they got marks, are different.

Adyant got 450 marks out of 600 means $\frac{450}{600}=\frac{3}{4}$

Aditi got 400 marks out of 500 means $\frac{400}{500}=\frac{4}{5}$

Now, we have to compare the two ratios or two fractions,

3:4 and 4:5 or $\frac{3}{4}$ and $\frac{4}{5}$

Let us convert both the fractions with common denominator 100.

$$\frac{3}{4}=\frac{3\times25}{4\times25}=\frac{75}{100}=75\%$$

$$\frac{4}{5}=\frac{4\times20}{5\times20}=\frac{80}{100}=80\%$$

Hence, Aditi's performance is better than Adyant because she secured 80%, whereas Adyant secured75%.

Facts to Know

- The word "percentage" is often a misnomer in the context of sports statistics, when the referenced number is expressed as a decimal proportion, not a percentage. The winning percentage of a team that has. 500 winning percentage has won 50% of their matches.

Converting Fractions, Ratios and Decimals into Percentage

1. To convert a fraction into per cent

When a fraction is to be converted into percent, we multiply the fraction by 100 and then attach % symbol.

Example 1 : **Convert the following fractions into per cent :**

(a) $\frac{3}{5}$ (b) $4\frac{3}{4}$ (c) $\frac{18}{25}$ (d) $\frac{1}{50}$

Solution :

(a) $\frac{3}{5} = \frac{3}{5}\times 100 = 60\%$

(b) $4\frac{3}{4} = \frac{19}{4}\times 100 = 475\%$

(c) $\frac{18}{25} = \frac{18}{25}\times 100 = 72\%$

(d) $\frac{1}{50} = \frac{1}{50}\times 100 = 2\%$

2. To convert a ratio into per cent

To convert a ratio into per cent, first write the ratio as fraction, and then multiply it by 100 and attach % symbol.

Example 2 : **Express the following ratios as per cent :**

(a) 5:8 (b) 3:8 (c) 2:5 (d) 1:10

Solution :

(a) 5:8 = $\frac{5}{8}\times100=62.5\%$

(b) 3:8 = $\frac{3}{8}\times100=37.5\%$

(c) 2:5 = $\frac{2}{5}\times100=40\%$

(d) 1:10 = $\frac{1}{10}\times100=10\%$

3. To convert a decimal into per cent

To convert a decimal into per cent, multiply it by 100 and attach % symbol or move the decimal point two places to the right and attach % symbol.

Example 3 : Express the following decimals as per cent :

(a) 2.75 (b) 0.0312 (c) 125.1 (d) 11.011

Solution :

(a) $2.75 = 2.75 \times 100 = 275\%$

(b) $0.0312 = 0.0312 \times 100 = 3.12\%$

(c) $125.1 = 125.1 \times 100 = 12510\%$

(d) $11.011 = 11.011 \times 100 = 1101.1\%$

Converting Percentage into Fractions, Ratios and Decimals

1. To convert a per cent into fraction

To convert a per cent into fraction, divide the number by 100 and drop % symbol. Express the fraction in its simplest form.

Example 4 : Express the following per cent into fraction :

(a) 75% (b) 48% (c) 4% (d) 175%

Solution :

(a) $75\% = \frac{75}{100} = \frac{3}{4}$

(b) $48\% = \frac{48}{100} = \frac{12}{25}$

(c) $4\% = \frac{4}{100} = \frac{1}{25}$

(d) $175\% = \frac{175}{100} = \frac{7}{4} = 1\frac{3}{4}$

2. To convert a per cent into ratio

To convert a per cent into ratio, divide it by 100 and drop % symbol. Express the obtained fraction as ratio in its simplest form.

Example 5 : Express the following per cent into ratio :

(a) 32% (b) 12% (c) 75% (d) 325%

Solution :

(a) $32\% = \frac{32}{100} = \frac{8}{25} = 8:25$

(b) $12\% = \frac{12}{100} = \frac{3}{25} = 3:25$

(c) $75\% = \frac{75}{100} = \frac{3}{4} = 3:4$

(d) $325\% = \frac{325}{100} = \frac{13}{4} = 13:4$

3. To convert a per cent into decimal

To convert a per cent into decimal, divide the number by 100 and drop % symbol or shift the decimal point by two places to the left and drop % symbol.

Example 6 : **Express the following per cent into decimals :**

(a) 0.015% (b) 0.25% (c) 7.6% (d) 10.5%

Solution : (a) 0.015% $= \frac{0.015}{100} = 0.00015$

(b) 0.25% $= \frac{0.25}{100} = 0.0025$

(c) 76% $= \frac{76}{100} = 0.76$

(d) 10.5% $= \frac{10.5}{100} = 0.105$

To Find Percentage of a Number

To find the percentage (say x %) of a number (say y), we multiply y by $\frac{x}{100}$, i.e.

$$x\% \text{ of } y = \frac{x}{100} \times y$$

Example 7 : **Find the value of the following.**

(a) 80% of ₹ 120 (b) 25% of 6l (c) 12.5% of 400 (d) 50% of 210 km

Solution :

(a) 80% of ₹ 120 $= \frac{80}{100} \times 120 = ₹96$

(b) 25% of 6l $= \frac{25}{100} \times 6 = 1.5l$

(c) 12.5% of 400 $= \frac{12.5}{100} \times 400 = 50$

(d) 50% of 210km $= \frac{50}{100} \times 210 = 105\ km$

Example 8 : **Vasu secured 360 marks out of 600 in the half yearly examination. Find the per cent score.**

Solution : Let the percent score $= x$

According to the question,

$$x\% \text{ of } 600 = 360$$

$$\frac{x}{100} \times 600 = 360$$

$$x = \frac{360 \times 100}{600} = 60$$

Hence, Vasu secured 60% marks in the half yearly examination.

Example 9 : **The price of commodity rises by 25%. How much per cent should a man reduce his consumption of the commodity so that his expenditure remains same?**

Solution : Assume the man bought 100 units of commodity for ₹ 100

Increase in price = 25%

∴ Cost of 100 units after increase = 100 + 25 = ₹ 125

Now, for ₹ 125, the quantity of commodity purchased is 100 units but the amount to be spent on commodity is only ₹ 100

∴ Quantity of commodity that will be bought for ₹100= $\frac{100}{125} \times 100 = 80$ units

Hence, consumption of commodity is to be reduced by = 100 – 80 = 20%

Example 10 : **In an election contested by Sania and Mahi. Sania got 48% of the votes cast. If the total number of votes cast is 60,000, find the votes obtained by Mahi.**

Solution : Votes obtains by Sania = 48% of 60,000

= 28, 800

∴ Votes obtained by Mahi = 60,000 – 28,800

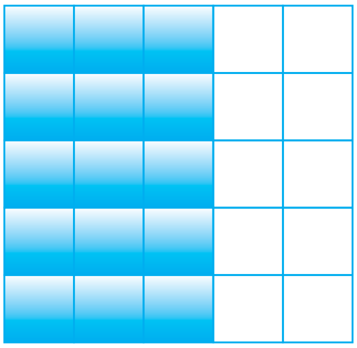
= 31,200

∴ Hence, votes obtained by Mahi = 31,200

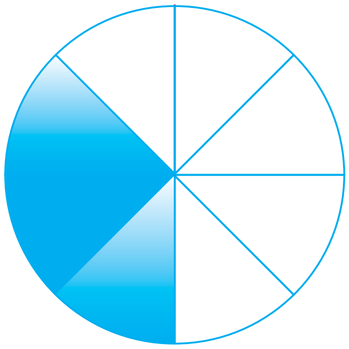
Exercise 9.1

1. Find the percentage of shaded portion of each figure:

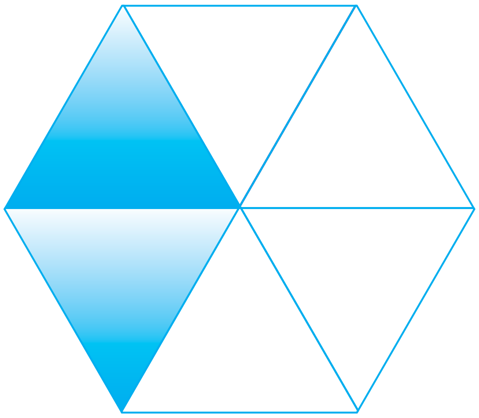
(a)



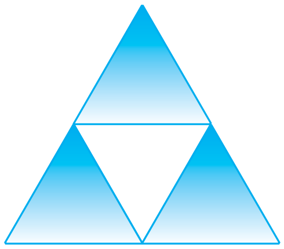
(b)



(c)



(d)



2. Express the following fractions as percentages:

(a) $\frac{1}{8}$ (b) $\frac{6}{25}$ (c) $2\frac{3}{4}$ (d) $2\frac{2}{5}$

3. Express the following ratios as per cent:

(a) 1 : 4 (b) 12 : 5 (c) 5 : 4 (d) 11 : 5

4. Express the following decimals as per cent:

(a) 0.16 (b) 1.25 (c) 0.625 (d) 265.25

5. Express the following per cent as fraction:

(a) 36% (b) 125% (c) 12.5% (d) 8%

6. **Express the following per cent as ratios in the simplest form :**

(a) 1.6% (b) 65% (c) $37\frac{1}{2}\%$ (d) 10.5%

7. **Express the following per cents as decimals:**

(a) 18% (b) 22.5% (c) 225% (d) 1.123%

8. **Find the value of the following :**

(a) 40% of 25ℓ (b) 8.5 % of ₹ 75,000 (c) 60% of 500 kg

(d) $37\frac{1}{2}\%$ of 200 km (e) 6% of 1 hr (f) 15% of 90

9. **Find the whole quantity if**

(a) 15% is 75 (b) 125% is 600 (c) 5% is 12 (d) 2.5% is 7

10. Avantika travelled 25 km by bus and 50 km by train. What per cent of the total journey did she travel by bus?
11. In a society 54% are adult, 26% children and the rest are old. If there are 26000 children, find the number of old in the society.
12. Mahesh scored 375 marks out of 400 and Naaz scored 450 marks out of 500. Who performed better?
13. A candidate must get 40% to pass in an examination. Pinky gets 250 marks and fails by 70 marks. Find the maximum marks.
14. In an examination, 85% of the candidates passed and 60 candidates failed. Find the number of total candidates who appeared in the examination.
15. A man spent 72% of his salary and remaining were his savings. If he saved ₹ 7,200 per month, find his monthly income.
16. Roshan gave 50% of the amount he had to his wife, 30% to his son and the remaining ₹ 50,000 to his daughter. Find the amount he had.
17. In a school, 55% of students are boys and the number of girls is 3600. Find the total number of students in the school.

Profit and Loss

Profit = SP - CP, when SP > CP ; loss = CP - SP, when CP > SP.

We can learn basics of profit and loss from our day-to-day experiences. We buy articles from nearby shapes. The shopkeepers purchase it either from the whole salers or directly from manufacturers after paying a certain price.

The price at which an article is purchased is called the cost price and is written as CP.

The price at which an article is sold is called the selling price and is written as SP.

If the selling price of an article is more than the cost price, then the shopkeeper makes a profit.

So, Profit = Selling Price – Cost Price

= SP – CP

If the cost price of the article is more than the selling price, then the shopkeeper makes a loss.

So, Loss = Cost Price – Selling Price

= CP – SP

If the cost price and the selling price of the article is same, then there is no profit no loss in the transaction. Here, CP = SP

Usually, a shopkeeper has to bear some additional expenditures like freight, wages, maintenance charges etc. These extra spendings are called overhead charges. Overhead charges are an essential part of cost price.

Thus,

Cost Price = Payment made while purchasing the articles + Overhead charges

Keep in mind that the profit and loss are always calculated on the cost price.

$$\text{Profit\%} = \frac{\text{Profit}}{\text{CP}} \times 100$$

$$\text{Loss\%} = \frac{\text{Loss}}{\text{CP}} \times 100$$

Facts to Know

- At each stage, the selling price of one becomes the cost (buying) price for the other.
- Profit and loss is always calculated on CP.

Example 11 : Viru bought a bicycle for ₹ 4,250 and sold it to Shyam for ₹ 3,750. Find his gain or loss.

Solution : CP of cycle = ₹4,250

SP of cycle= ₹ 3,750

Since CP > SP, so there is a loss.

Loss = CP – SP

= ₹ 4,250 – ₹ 3,750 = ₹ 500

Hence, Viru suffered a loss of ₹ 500.

Example 12 : If the profit made on a toy is ₹ 24 and the selling price of the toy is ₹ 504, then find the profit %.

Solution : It is given that Profit = ₹ 24 and SP = ₹ 504

CP = SP – Profit

= ₹ 504 – ₹ 24

= ₹480

$$\text{Profit \%} = \frac{\text{Profit}}{\text{CP}} \times 100$$

$$= \frac{24}{480} \times 100 = 5\%$$

Hence, the profit per cent on toy is 5 %.

Example 13 : Dinesh bought 15 dozen balloons at ₹24 a dozen for his birthday. He spent ₹40 on his transportation. Due to fight with his friends he cancelled the celebration and sold the balloons at ₹ 2.5 each. What was his profit or loss per cent.

Solution : Cost of 1 dozen balloons = ₹ 24

Cost of 15 dozen balloons = ₹ (24×15)

= ₹ 360

Overhead expense on transportation = ₹ 40

∴ Cost Price = ₹ (360 + 40)

= ₹ 400

Selling Price of 1 balloon = ₹ 2.5

Selling Price of 1 dozen balloons = (2.5×12)

= ₹ 30

Selling Price of 15 dozen balloons = ₹ (15×30) = ₹ 450

As SP > CP, there is a profit on the cost price

Profit = SP – CP

= ₹ (450 – 400)

= ₹ 50

$$\text{Profit \%} = \frac{\text{Profit}}{\text{CP}} \times 100$$

$$= \frac{50}{400} \times 100$$

$$= 12.5\%$$

Hence, the profit per cent of Dinesh is 12.5%

Example 14 : **The selling price of 12 articles is the same as the cost price of 16 articles. Find gain per cent.**

Solution : Let the cost price of 1 article = ₹ 1

∴ The cost price of 12 articles = ₹ 12

But selling price of 12 articles = Cost price of 16 article = ₹ 16

As SP > CP, there is a profit.

Profit = SP – CP

= ₹ (16 – 12) = ₹ 4

$$\text{Profit\%} = \frac{\text{Profit}}{\text{CP}} \times 100$$

$$= \frac{4}{12} \times 100 = 33.\overline{3}\%$$

Hence, profit in the transaction is $33.\overline{3}\%$.

Example 15 : **Ashu bought two horses at ₹ 36,000 and ₹ 40,000 respectively. He sold first horse at a gain of 25% and second horse at a loss of 20%. Find the gain or loss per cent in the whole transaction.**

Solution : Cost price of two horses = ₹ (36,000 + 40,000)

= ₹ 76,000

Cost price of first horse = ₹ 36,000

Profit = 25% of ₹ 36,000

$$= \frac{25}{100} \times 36000 = ₹\ 9,000$$

Selling price of first horse = ₹ (36,000 + 9,000)

= ₹ 45,000

Cost price of second horse = ₹ 40,000

Loss = 20% of ₹ 40,000

$$= \frac{20}{100} \times 40,000 = ₹\ 8,000$$

Selling price of second horse = ₹ (40,000 – 8,000)

= ₹ 32,000

Selling price of two horses = ₹ (45,000 + 32,000) = ₹ 77,000

As SP > CP, there is a profit.

Profit = SP – CP

= ₹ (77,000–76,000)

= ₹ 1000

$$\text{Profit \%} = \frac{\text{Profit}}{\text{CP}} \times 100$$

$$= \frac{1000 \times 100}{76000} = 1.316\% \text{ (approx)}$$

Hence, Ashu made a profit of 1.316% (approx) in the transaction.

Exercise 9.2

1. **Find the SP of the following :**
   (a) CP=₹2250,Loss=₹75
   (b) CP =₹1235, Profit = ₹ 65
   (c) CP = ₹ 2390, Profit = ₹120.75
   (d) CP = ₹ 127, Loss = ₹12.25
2. **Find the CP of the following :**
   (a) SP = ₹ 1000, Profit = ₹200
   (b) SP = ₹2100, Loss = ₹ 137
   (c) SP = ₹1320, Loss = ₹75.5
   (d) SP = ₹ 778, Profit = ₹ 32
3. **Find the missing terms (wherever applicable) in each of the following:**

S. No.	Cost Price	Selling Price	Profit	Profit%	Loss	Loss%
(a)	₹ 450	₹ 540				
(b)	₹ 2200			20%		
(c)		₹ 660	₹ 60			
(d)		₹ 180	₹ 30			
(e)			₹ 108	15%		

4. Roney bought a mobile for ₹ 12,000. At what price should he sell it so as to get a profit of 30%.
5. A shopkeeper purchased 50 dozen eggs for ₹1800. Five dozen eggs could not be sold because they got broken. At what price per dozen should the shopkeeper sell the remaining eggs so that he made an overall profit of 20%.
6. Mr. Parag sold his scooter for ₹ 24,000 making a loss of 20%. What was the cost of the scooter?
7. By selling a computer for ₹ 16,000 Arnav lose 20%. At what price should he sell it so that he gets 30% profit.
8. Pammy bought 60 articles at the rate of ₹80 each article. She sold three-fourth of them at the rate of ₹ 75 each article and the rest at the rate of ₹ 100 each article. Find her gain or loss per cent.
9. The selling price of 8 apples is equal to the cost price of 10 apples. Find the gain per cent.
10. Trisha purchased a bicycle for ₹ 5000 and sold it to Tipu at a profit of 25%. Tipu sold it to Tina at a loss of 15%. For how much did Tina buy it ?
11. Aman bought five radio sets at rate of ₹ 320. He sold two radio sets at a loss of 25%. At what gain per cent should he sell the remaining radios to gain 25%, on the overall investment ?
12. Deepak buys an article and sells it to Raju at a profit of 20%, Raju sells it to Mahesh gaining 30%. If Mahesh pays ₹1872 for it, how much did Deepak pay?
13. A shopkeeper buys 12 kg tea of one quality at the rate of ₹ 250 per kg and 18 kg tea of another quality at the rate of ₹ 300 per kg. He mixes them and sells at the rate of ₹ 290 per kg. Find his gain or loss per cent.
14. Jagan sold a trouser at a profit of 16%. Had he sold it for ₹ 27 more, the profit would have been 20%. Find the cost price of the trouser.
15. A reduction of 25% in the price of commodity enables a purchaser to get 3 kg more for ₹ 180. Find
    (a) Reduced price per kg of commodity.
    (b) Original price per kg of commodity.

Simple Interest

Some expected and unexpected expenditure sudden comes like a in hospital, family function, education, Marriages etc, some saved money is not sufficient. They will perhaps borrow it from a friend, relative, a money lender or a bank. A friend or relative might not show interest in getting their money back, but others like money lenders and financial institutions lend us money, only if we agree to return the borrowed money within a specific period of time with some extra money for using their money.

The borrowed or invested money is called the Principal denoted by P. 'T' is the time for which the money is borrowed. The additional money to be paid after a specific period of time is called the Simple Interest and is denoted by I. The Rate of Interest (Interest Rate) is an agreed percentage of the sum borrowed at the time of taking the money throughout the loan period and is denoted by R. Generally the rate of interest is taken as "per cent per annum" which means per ₹ 100 per year. When the interest for a specific period is added to the principal, then the sum is called the Amount and is denoted by A.

Amount = Principal + Interest

Example : **Mr. Mathur bonrowed ₹ 8500 from a bank He Paid 10% per annual and returned the amount after 3 years. How much interest did Mr. Mathur Pay in all.**

P = 8500 Rs (hence, P = Principal

R = 10% PA (hence, R = ROI)

T = 3 year (hence, T = Time)

$$\text{Now } I = \frac{P \times R \times T}{100}$$

$$= \frac{8500X10X3}{100}$$

$$= 2550 \text{ ₹}$$

Hence Mr. Mathure Paid 2550₹ as interest

Example 16 : **Eliana borrowed ₹ 8000 from her friend at the rate of 15% for 3 years. Find the amount to be paid after 3 years.**

Solution : Here,

Principal = ₹ 8000

Rate = 15%

Time = 3 years

$$I = \frac{P \times R \times T}{100}$$

$$= \frac{8000 \times 15 \times 3}{100}$$

$$= \text{₹ } 3600$$

$$A = P + I$$

$$= \text{₹ } (8000 + 3600) = \text{₹ } 11600$$

Hence, Eliana will pay ₹ 11600 after 3 years.

Example 17 : **At what rate will ₹3000 amount to ₹3600 in 4 years ?**

Solution : Here,

Amount = ₹3600

Principle = ₹3000

Time = 4 years

Interest = Amount – Principal

= ₹ (3600 – 3000)

= ₹600

$$\text{Rate} = \frac{I \times 100}{P \times T}$$

$$= \frac{600 \times 100}{3000 \times 4}$$

= 5%

Hence, the rate of interest is 5%

Example 18 : At what rate of interest will a sum of money triple itself in 15 years ?

Solution : Let the principal (P) be x,

Amount = 3x

Interest = Amount – Principal

= $3x - x$

= $2x$

$$\text{Rate} = \frac{I \times 100}{P \times T}$$

$$= \frac{2x \times 100}{x \times 15}$$

= $13.\overline{3}\%$

Hence, the rate of interest is $13.\overline{3}\%$.

Example 19 : **Bhola borrowed ₹75,000 from his friend Bhalla. He gave ₹40,000 at the interest rate of 15% and the remaining amount at 18%. How much interest did he pay in 4 years ?**

Solution : Interest paid on ₹40,000 for 4 years at 15%,

$$I_1 = \frac{40,000 \times 15 \times 4}{100}$$

= ₹24, 000

Remaining amount = ₹ (75, 000 – 40, 000)

= ₹ 35, 000

Interest paid on ₹35,000 at 18% for 4 years

$$I_2 = \frac{35,000 \times 18 \times 4}{100}$$

= ₹25, 200

Total interest = $I_1 + I_2$

= ₹ (24,000 + 25, 200) = ₹49, 200

Thus, Bhola paid ₹49, 200 after 4 years as interest.

Example 20 : **In how many years will a sum of money double it self at 10% interest per annum?**

Solution : Let the principal (P) be x

Amount = 2x (double)

Rate = 10% P.A

I = Amount - Principal

= 2x - x

= x

Now $= \frac{1x100}{PxR}$

$= \frac{Xx100}{PxR}$

= 10 years

Exercise 9.3

1. Fill in the blanks:

S.No.	Amount	Principal	Simple Interest	Rate	Time
(a)		₹ 1800	₹ 216	6%	
(b)		₹ 5000	₹ 1200		4 Years
(c)	₹900			10 %	6 Years
(d)	₹ 5000		₹ 1000	10%	

2. What sum of money lent out at 5% per annum simple interest produces ₹ 500 as interest in 2.5 years ?
3. What sum of money will amount to ₹ 27930 at the rate of 10% per annum simple interest in 3 years ?
4. A sum of money doubles itself in 6 years. Find the rate of simple interest per annum.
5. Sophiya borrowed some amount at 18% per annum. He had to pay ₹ 225 as interest after 4 years. How much did he borrow?
6. In what time will the simple interest on a certain sum of money at 10% per annum be $\frac{2}{5}$ of itself?
7. At what rate per cent simple interest will a sum of money amount to $\frac{5}{4}$ of itself in 2 years ?
8. A certain sum of money amounts to ₹ 4800 in 4 years and ₹ 5200 in 6 years. Find the sum and the rate of simple interest.

9. A village moneylender wants one eighths of the amount loaned, every year as interest. What will be the rate of interest, if a farmer borrows ₹ 12000 for 1 year from the moneylender? What is the amount that he has to pay back altogether?

10. A sum of money lent at 8% per annum simple interest for 5 years yields a certain amount of interest . Had it been lent for 7 years, it would have yielded ₹ 960 more. Find the sum.

Points to Remember

- **Per cent** is a fraction with denominator 100 or a ratio with 100 as the second term.
- All fractions, ratios and decimals can be expressed as percentages and vice-versa.
- **Cost Price** of an article is the amount paid to purchase it.
- Overhead expenses like cartage, labour, taxes etc, are included in the cost price.
- **Selling Price** of an article is the amount at which it is sold.
- If SP > CP, there is profit. Profit = SP – CP
- If CP > SP, there is loss. .Loss=CP –SP
- Profit % $= \frac{\text{Profit} \times 100}{\text{CP}}$; Loss % $= \frac{\text{Loss} \times 100}{\text{CP}}$
- The money borrowed or invested is called the **Principal.**
- The extra money to be paid after specific period of time is called the **Simple Interest** or **Interest.**
- The rate of interest (R) is the agreed percentage of the sum borrowed at the time of taking the money throughout the loan period.
- Interest $= \frac{\text{Principal} \times \text{Rate} \times \text{Time}}{100} = \frac{\text{PRT}}{100}$
- Amount = Principal + Interest

EXERCISE

1. **MULTIPLE CHOICE QUESTIONS (MCQs):**

Tick (✓) the correct options :

(a) The word per cent is taken from Latin words

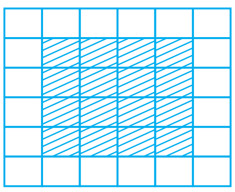
(i) per cent ☐ (ii) per centage ☐ (iii) per centum ☐ (iv) per santa ☐

(b) Converting $1\frac{1}{5}$ in per cent we get

(i) 12% ☐ (ii) 60% ☐ (iii) 120% ☐ (iv) 83.3% ☐

(c) Aditya scored 45% marks in Mathematics. How many marks she got if the maximum marks was 80 ?

(i) 36 ☐ (ii) 38 ☐ (iii) 40 ☐ (iv) 42 ☐

(d) 

The percentage of shaded portion in the given figure is

(i) 16% ☐ (ii) 20% ☐ (iii) 25% ☐ (iv) 45% ☐

(e) What per cent of 150g is 1kg?

(i) 15% ☐ (ii) 666.6% ☐ (iii) 115% ☐ (iv) 66.6% ☐

(f) Find the whole quantity if its 7.5% is 150.

(i) 20 ☐ (ii) 200 ☐ (iii) 2000 ☐ (iv) 20000 ☐

(g) Rinky traveled $\frac{3}{4}$ of the journey by car and the rest by bus. If she covered 120 km in total, what per cent of journey she travelled by bus.

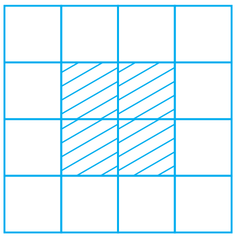
(i) 75 % ☐ (ii) 20% ☐ (iii) 10 % ☐ (iv) 25% ☐

(h) A sum of ₹1600 is lent for 2.5 yrs at rate of 4 % per annum. The amount to be returned after 2.5 years is

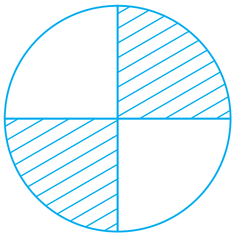
(i) ₹160 ☐ (ii) ₹1340 ☐ (iii) ₹1760 ☐ (iv) ₹2000 ☐

2. Find the percentage of unshaded portion of each figure :

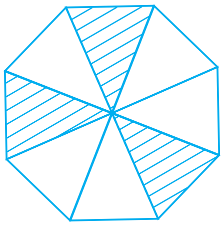
(a)



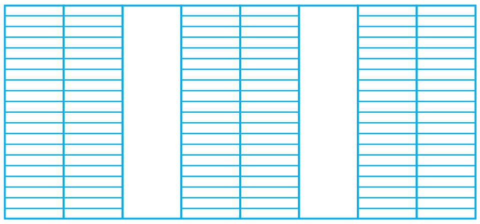
(b)



(c)



(d)



3. Find the value of the followings :

(a) 7.5% of ₹ 500 (b) 50% of 124 *l* (c) 8.0% of ₹ 1600 km (d) 12 % of 2 kg.

4. Shikhar hit six fours in two overs whereas Gambhir hit four fours in one over. Who has a better strike rate in terms of percentage?

5. The wholesale price of wheat rises by 12.5%. By how much per cent should a family reduce his consumption of commodity so that his expenditure remains same?

6. A candidate must score 60% marks to pass in an examination. Rahul gets 240 marks and fails by 60 marks. Find the maximum marks.

7. On selling a computer for ₹ 18,000. Mr. Pandey earned profit of ₹ 4500. Find his profit % .

8. Rakesh bought two electronic items at ₹ 800 and ₹ 1200 respectively. He sold the first item at profit of 20% and second item at loss of 20%. Find the profit or loss per cent in the whole transaction.

9. Tipu buys an article and sells it to Sindhu at a profit of 10%, Sindhu sells it to Amit gaining 20%. If Amit pays ₹1980 for it, how much did Tipu pay ?

10. Shreya sold his bicycle at gain of 10%. If he had bought it for 10% less and sold it for ₹ 45 more, he would have gained 25% . Find the cost price of the bicycle.

11. Oman borrowed ₹ 5000 from his uncle at the rate of 12% for 5 years . Find the amount to be paid after 5 years.

12. In how many years will ₹ 2500 amount to ₹ 3200 at rate of 4% ?

13. At what rate of interest will a sum of money doubles itself in 12.5 years.

14. What sum of money amounts to ₹ 8250 in 4 years at 8% per annum ?

HOTS

A factory grants raises at two different times a year. If 62% of the workers received raises in the first period and 10% of these did not receive a raise in the second period, what percentage of the people received raises in both the periods.

Lab Activity

Objective : To find the per cent of multi-coloured figure.

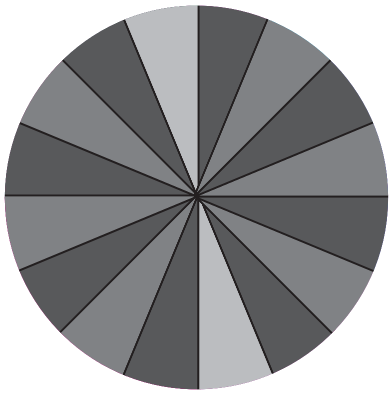
Materials Required : White circular chart paper, black sketch pen, red sketch pen, yellow sketch pen and green sketch pen.

Procedure :

Step 1 : Take a white coloured circular chart paper and shade it with different colours as shown below.

Step 2 : Find the coloured parts in fractions.

Yellow parts = $\frac{2}{16}$, Green parts = $\frac{6}{16}$, Red parts = $\frac{8}{16}$



Step : Convert the fractions into percentage.

Yellow parts = $\frac{2}{16}\times 100 = 12.5\%$

Green parts = $\frac{6}{16}\times 100 = 37.5\%$

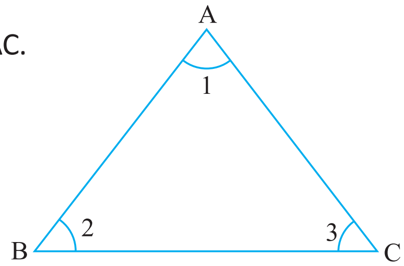
Red parts = $\frac{8}{16}\times 100 = 50\%$

10 Triangle and Its Properties

A triangle is a closed figure formed by there line Segments, and denoted by the symbols △.

We see many things which have the word '**tri**' embedded in their names. Eg: Tricycle, tri-series and so on. Similarly, we have a concept in geometry which is based on '**tri**'. The meaning of '**tri**' is **three**.

The word **triangle** literally means a figure having three angles. It also has three sides. It is a closed figure. It has been categorised as a polygon with three sides. Look at Figure given here, it shows triangle ABC. We can conclude about the following features of triangles:



- **Sides :** There are three sides of a triangle ABC. It has three sides, viz. AB, BC and AC.
- **Angles :** There are three angles of a triangle. In Figure given here, triangle ABC has three angles. They are

$$\angle BAC = \angle A = \angle 1$$
$$\angle ABC = \angle B = \angle 2$$
$$\angle ACB = \angle C = \angle 3$$

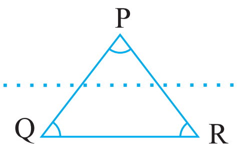
- **Vertices :** There are three vertices of a triangle. In Figure given here , the three vertices of triangle ABC. They are A, B and C.

Further, in △ABC, shown in Figure given above, vertex A is opposite side BC. Similarly, vertex B is opposite side CA, vertex C is opposite side AB.

In △ABC, ∠A is opposite side BC. Further, ∠B is opposite side CA. Finally, ∠C is opposite side AB.

Facts to Know

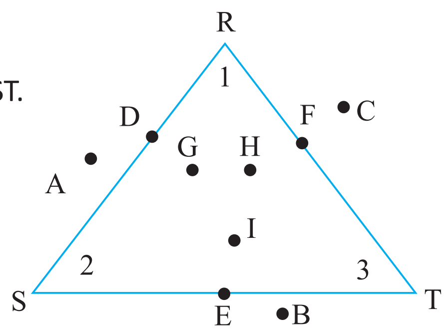
- The three sides and three angles of a triangle together are called six elements of triangle.



Interior and Exterior of Triangle

Let us draw △ RST.

In figure given here, we have shown some points **on, inside** and **outside** △ RST.



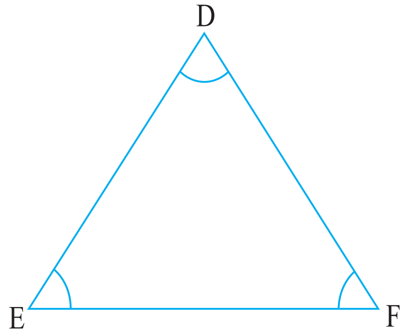
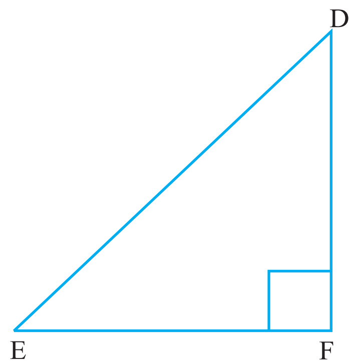
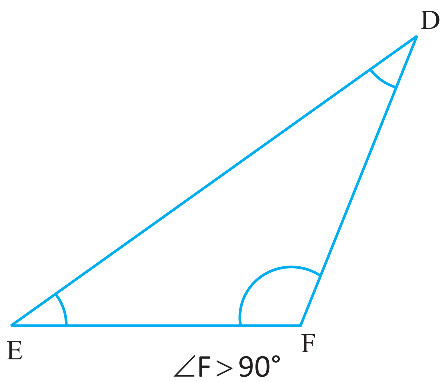
Note that :

- Points A, B, C are in the **exterior** of △RST.
- Points D, E, F are **on** the △RST
- Points G, H, I are in the **interior** of △ RST.

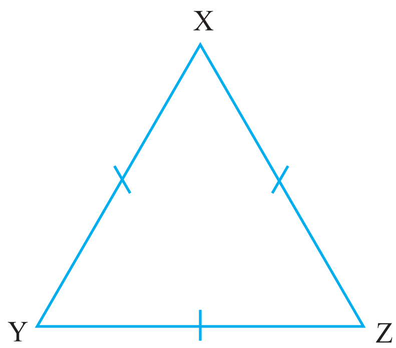
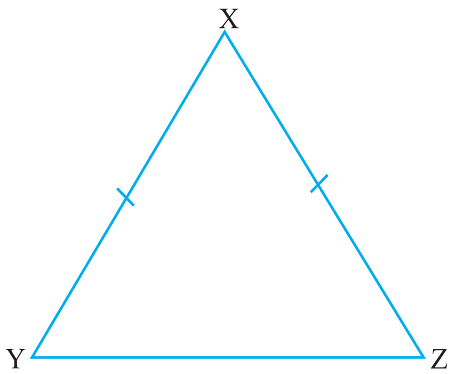
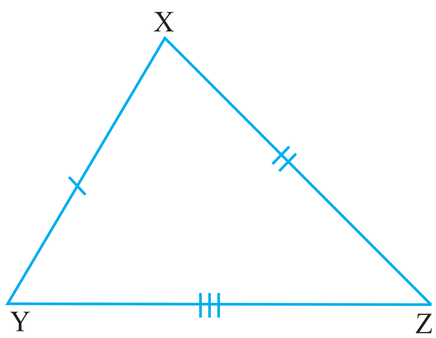
In sum the points lying in △ RST and inside the three sides of △RST are called **Triangle Interior**. All other parts are called **exterior**.

Classification of Triangles

Classification on the Basis of Angles

Name of Triangles	Sketch	Definition
1. Acute-angled triangle	 $\angle D < 90°$ $\angle E < 90°$ $\angle F < 90°$	A triangle with all acute angle is called **acute-angled triangle**.
2. Right-angled triangle	 $\angle F = \angle 90°$ $\angle E < 90°$ $\angle D < 90°$	A triangle with one angle of 90° and other two acute angles is called **right-angled triangle**.
3. Obtuse - angled triangle	 $\angle F > 90°$ $\angle E < 90°$ $\angle D < 90°$	A triangle with one angle more than 90° and other two acute angles is called **obtuse-angled triangle**.

Classification on the Basis of Sides

Name of Triangle	Sketch	Definition
1. Equilateral Triangle	 XY = YZ = ZX	A triangle having all three equal sides is called **equilateral triangle**.
2. Isosceles Triangle	 XY= XZ XY $\neq$ YZ XZ $\neq$ YZ	A triangle having two equal sides and one side different from two equal sides is called **isosceles triangle**.
3. Scalene triangle	 XY $\neq$ YZ $\neq$ ZX	A triangle having all sides unequal to one another is called **scalene triangle**.

Features Associated With Triangle

In this section, we shall discuss some features of triangles that are of great use in this class as well as higher classes. Consider the figure given here.

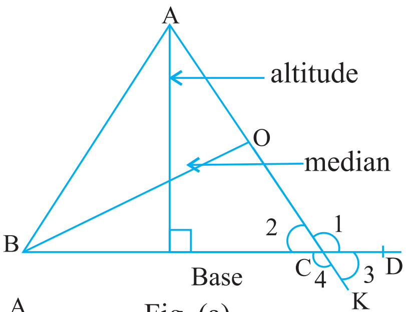


Fig. (a)

Base of Triangle

The horizontal line on which the triangle is constructed is called **base of triangle**. In Figure (a) BC is the base of △ABC.

Altitude of Triangle

The perpendicular drawn from any vertex to the opposite side that vertex is called **altitude of triangle**. Since there are three vertices and sides of a triangle, we have three altitudes, too.

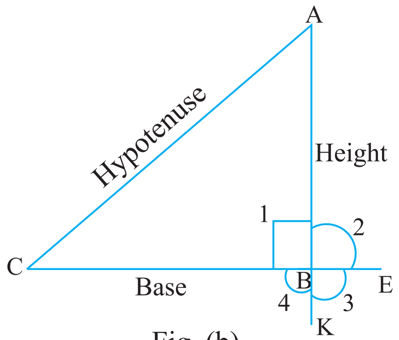


Fig. (b)

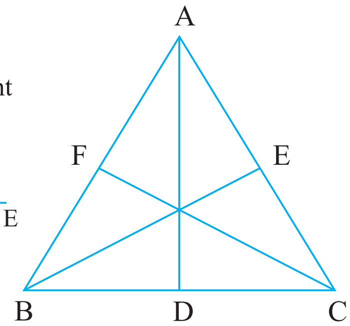


Fig. (c)

Median of Triangle

The line joining a vertex of the triangle to the mid-point of the side opposite to that vertex is called **median of triangle**. Since there are three vertices and sides of a triangle, we have three medians, too. △ABC, AD, BE and CF are three medians in Figure (c).

Interior Angle

An angle inside the triangle area is called **interior angle of triangle**. In the Figure (a) ∠2 is an interior angle. Note that ∠2 = ∠BCA. In Figure (b) ∠1 is the interior angle.

Exterior Angle

An angle outside the triangle area is called **exterior angle of triangle**. In Figure (a) ∠1 is an exterior angle. It was formed by extending BC to D. In Figure (b) ∠2 is the exterior angle. It was formed by extending CB to E.

Facts to Know

- All triangles have three interior angles and three exterior angles each. That is quite logical.

Right Angle

An angle of 90° is called **right angle**. The triangles having one angle of 90° are **called right-angled triangles**. Figure (b) is right-angled triangle.

Hypotenuse

In a right-angled triangle, the slant line which is also the longest side is called **hypotenuse** shown in Figure (b).

Height

In a right-angled triangle, the perpendicular to the base of triangle is called **height**. Note that perpendicular and height are the same in all triangles. Figure (b) shows the height BA of △ABC.

Interior Opposite Angle

In Figure (a) ∠A and ∠B are interior opposite angles. They are the interior angles which are opposite to each other. Similarly, ∠A and ∠2 are interior opposite angles. Finally, ∠B and ∠2 are also interior opposite angles.

Vertically Opposite Angles

Refer Figure (a). We have extended AC up to K. Thus, we have ∠3 and ∠4, which are exterior angles.

$$\angle 2 = \angle 3$$
$$\angle 1 = \angle 4$$

There two pairs of angles are equal since they are vertically opposite angles. If two unparallel lines intersect each other, the opposite pairs of angles are called vertically opposite angles. Moreover, these angles are equal to each other.

In Figure (b) we have extended AB up to K.

$\Rightarrow \quad \angle 1 = \angle 3$

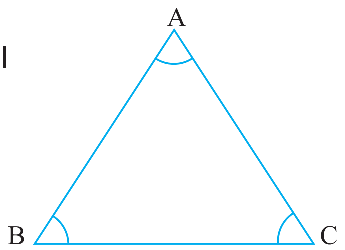
and $\quad \angle 2 = \angle 4$ | Vertically opposite angles

Facts to Know

- Angles are measured in degrees. The value of angle in degrees is called **measure of angle** and is denoted as *m*. For example, if an angle measures 107°, we would write m $\angle ABC = 107°$.

Properties of Triangle

Angle Sum Property of Triangle

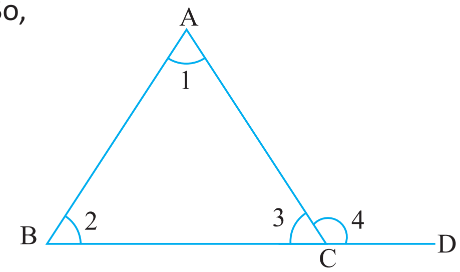


The angle sum property of triangle states that the sum of all angles of a triangle is equal to 180°. In $\triangle ABC$ shown in Figure , the sum of angles is equal to 180°.

$$\Rightarrow \quad m\ \angle A + m\ \angle B + m\ \angle C = 180°$$

Exterior Angle Property of Triangle

The exterior angle property of triangle states that the exterior angle of a triangle is equal to the sum of its opposite interior angles.



Refer to Figure given here, we have $\triangle ABC$. Base BC has been extended to D. So, $\angle 4$ is the exterior angle.

Now, $\angle 1$, $\angle 2$ and $\angle 3$ are the interior angles of $\triangle ABC$. According to the exterior angle property, we have :

$$\angle 4 = \angle 1 + \angle 2$$

This means that the exterior angle is equal to the sum of measure of two interior opposite angles.

The sum of all the exterior angles of triangle is always 360°.

Facts to Know

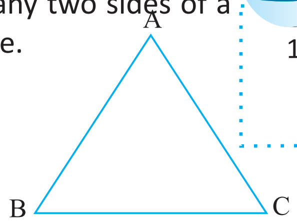
180° = 90° + 90°
= 1 right angle + 1 right angle
= 2 right angles.

Side Sum Property of Triangle

According to this property, the sum of the length of any two sides of a triangle is always greater than the length of the third side.

In the figure, $\triangle ABC$, we have :

$$AB + BC > AC$$
$$BC + AC > AB$$
$$AC + AB > BC$$



Equal Side Property of Triangle

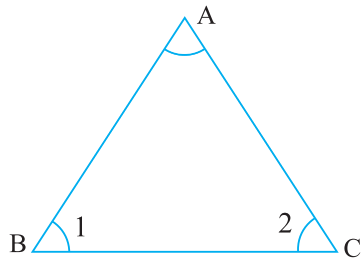
According to this property, the sides opposite to equal angles of a triangle are also equal. In $\triangle ABC$, if $\angle 1 = \angle 2$, then we have :

AB = AC

$\angle 1$ is opposite side AC.

$\angle 2$ is opposite side AB.

Since these angles are equal, these two sides are also equal to each other.



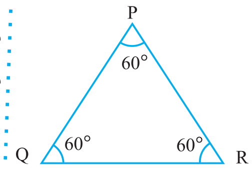
Equal Angle Property of Triangle

This property has been taken from the previous one. According to this property, if two sides of a triangle are equal to each other then the angles opposite to these two sides are also equal to each other. In above triangle ABC, if AB = AC, we have

$$\angle 1 = \angle 2$$

Facts to Know

In an equilateral triangle, all angles are equal. Each angle is 60°. Further, all sides are also equal to one another in this triangle.



So, if the two sides of a triangle are equal, the angles opposite these two sides are also equal to each other.

Theorem 1. Prove that the sum of the angles of a triangle is 180°.

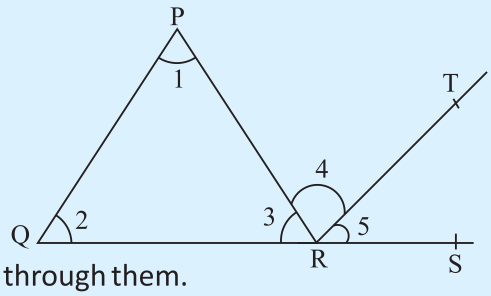
Given: A triangle PQR. It has three angles $\angle 1$, $\angle 2$ and $\angle 3$.

To Prove : $\angle 1 + \angle 2 + \angle 3 = 180° = 2$ right angles .

Construction: Draw RT parallel to PQ.

Also, extend QR to S.

Thus, $\angle 4$, $\angle 5$ are clearly visible in the figure shown here.



Proof:

PQ and RT are two parallel straight lines and PR is a transversal that cuts through them.

Hence, $\angle 1 = \angle 4$ .... (i) (alternate angles)

Again PQ and RT are two parallel straight lines and QS is a transversal that cuts through them.

Hence, $\angle 2 = \angle 5$ .... (ii) (corresponding angles)

Adding the equation (i) and equation (ii)

$\angle 1 + \angle 2 = \angle 4 + \angle 5$ .... (iii)

Adding $\angle 3$ to both sides of equation (iii), we get,

$\angle 1 + \angle 2 + \angle 3 = \angle 3 + \angle 4 + \angle 5$ .... (iv)

But $\angle 3$, $\angle 4$ and $\angle 5$ are the parts of a straight angle whose measure is 180°.

Thus, $m(\angle 3 + \angle 4 + \angle 5) = m\angle QRS$

$= 180°$

Hence, $\angle 1 + \angle 2 + \angle 3 = 180°$ (from equation iv)

Hence, the sum of three angles of a triangle is 180° = 2 right angles.

Example 1 : **In $\triangle PQR$, $\angle P = 67°$, $\angle Q = 77°$, what is the measure of $\angle R$?**

Solution : We have

$$\angle P + \angle Q + \angle R = 180°$$

Put values in this equation.

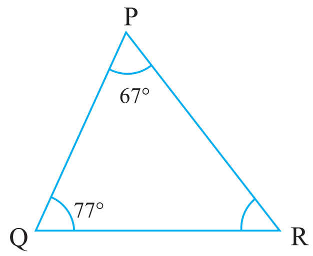
$$67° + 77° + \angle R = 180°$$

$$\angle R = 180° - 67° - 77°$$

$$= 180° - 144°$$

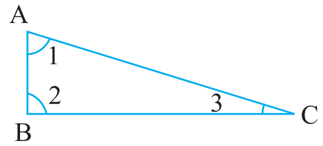
$$= 36°$$

So, $m\angle R = 36°$



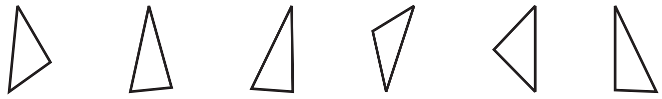
Example 2 : **Name six elements of the triangle given here.**

Solution : The 6 elements of $\triangle ABC$ are : AB, BC, CA, $\angle 1 = \angle A$, $\angle 2 = \angle B$ and $\angle 3 = \angle C$.



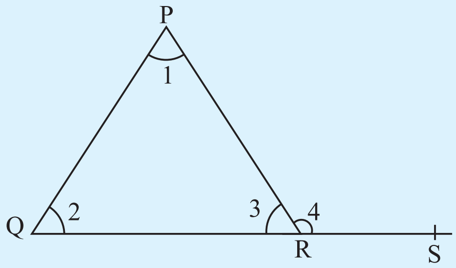
Facts to Know

The case study of triangle is well know as triangle geometry.



Theorem 2 : Prove that the exterior angle of a triangles is equal to the sum of its interior opposite angles.

Given : A triangle PQR. QR has been extended to S. The interior angles are $\angle 1, \angle 2, \angle 3$. The exterior angle is $\angle 4$.



To Prove : $\angle 4 = \angle 1 + \angle 2$

In $\triangle PQR$, we have

$\angle 1 + \angle 2 + \angle 3 = 180°$ .... (i) (Angle sum property)

Now, $\angle 3$ and $\angle 4$ are lying on a straight angle QRS. Thus, their sum is 180°.

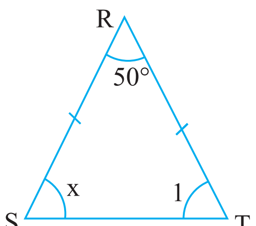
$\angle 3 + \angle 4 = 180°$ .... (ii)

Both equation (i) and equation (ii) are equal since their RHS are equal.

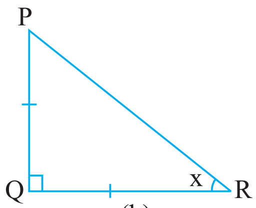
Hence, $\angle 1 + \angle 2 + \angle 3 = \angle 3 + \angle 4$

or, $\angle 1 + \angle 2 = \angle 4$ Hence proof

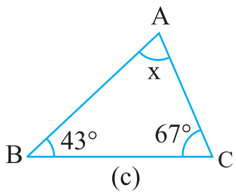
Example 3 : **Find the value of x in each one of the following figures.**



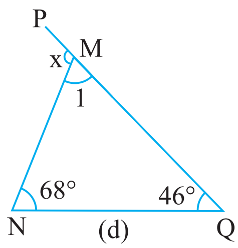
(a)



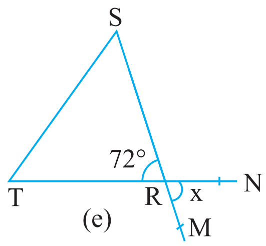
(b)



(c)



(d)



(e)

Solution : (a) $x + 50° + \angle 1 = 180°$ .... (i)

$RS = RT$

$x = \angle 1$ .... (ii)

Put (ii) in (i), we get,

$x + 50° + x = 180°$

$2x + 50° = 180°$

$2x = 180° - 50° = 130°$

$x = \frac{130°}{2} = 65°$

(b) $\angle Q = 90°$

So, $\triangle PQR$ is a right-angled triangle.

In a right-angled triangle, the angles other than the right angle are 45° each in measure.

Hence, $x = 45°$

(c) $x + 43° + 67° = 180°$

$x + 110° = 180°$

$x = 180° - 110° = 70°$

(d) $\angle 1 + 68° + 46° = 180°$

$\angle 1 + 114° = 180°$

$\angle 1 = 180 - 114 = 66°$

Now, $\angle 1$ and x are 2 parts of the straight angle QMN.

$\Rightarrow \qquad \angle QMN = 180°$

$\angle 1 + x = 180°$

$x = 180° - \angle 1$

$= 180° - 66° = 114°$

(e) It is clear that TN and SM are intersecting each other at R.

Hence, $x = 72°$ (vertically opposite angles)

Example 4 : **The angles of a triangle are in the ratio 2 : 3 : 4. Find out the angles.**

Solution : Let the angles be $2x$, $3x$ and $4x$. We have used the given ratio to define these angles.

$$2x + 3x + 4x = 180°$$

$$9x = 180°$$

$$x = \frac{180}{9} = 20°$$

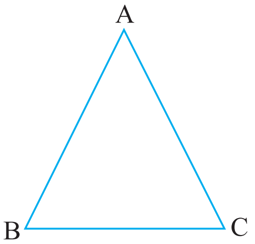
So, the angles are :

(i) $2x = 2 \times 20° = 40°$

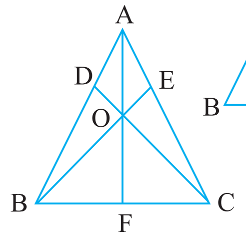
(ii) $3x = 3 \times 20° = 60°$

(iii) $4x = 4 \times 20° = 80°$

Example 5 : **A triangle has been shown here. Draw its medians. Use a scale or compass to find out the midpoints of sides.**

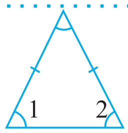


Solution : The mid-points can be obtained with the help of scale (ruler). Join mid-points of sides with opposite vertices.

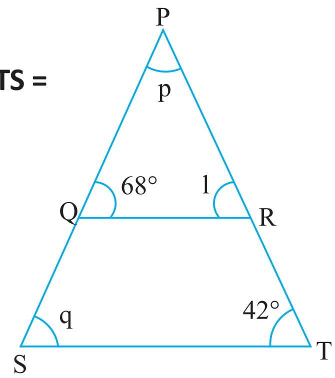


Facts to Know

An isosceles triangle has two equal sides. Since the sides are equal, the angles opposite to those sides are also equal. Hence in the adjacent figure, $\angle 1 = \angle 2$.



Example 6 : **In the figure shown here, QR is parallel to ST. ∠PQR = 68°. Also, ∠RTS = 42°. Find the values of *p* and *q*.**



Solution : QR is parallel to ST and PT is a transversal that cuts it at points R and T.

Hence, $\angle 1 = 42°$ (corresponding angles.)

In $\triangle PQR$, $\angle P + \angle Q + \angle 1 = 180°$

Putting the value of $\angle Q$ and $\angle 1$, we get,

$\angle P + 68° + 42° = 180°$

$\Rightarrow \qquad \angle P = 180° - 68° - 42°$

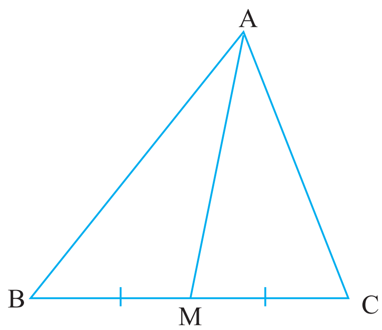
$= 70°$

QR and ST are parallel lines, and PS is a transverse at that cuts through them.

Hence, $\angle Q = \angle q$ (corresponding angles are equal)

So, $\angle q = 68°$

Example 7 : **AM is the median of △ABC. Is AB + BC + CA > 2 AM ?**



Solution : In $\triangle ABC$, AM is the median.

M is the mid-point of BC

Hence, BM = MC and BM + MC = BC

In $\triangle ABM$, we have.

AB + BM > AM .... (i)

In $\triangle ACM$, we have.

$AC + CM > AM$................. (ii)

Adding equation (i) and equation (ii) we get,

$AB + BM + AC + CM > AM + AM$

Or, $AB + AC + BM + CM > 2\,AM$

Or, $AB + AC + (BM + MC) > 2\,AM$

Or, $AB + AC + BC > 2\,AM$ ($\because BC = BM + MC$)

Hence proof

Example 8 : **The length of two sides of a triangle is 12 cm and 15 cm. Between what two measures should the length of the third side fall ?**

Solution : We know that the sum of length of two sides is greater than the length of the third side.

Two sides are 12 cm and 15 cm.

Add up, we get :

Sum = A = 12 + 15 = 27 cm.

The third side cannot be more than 27 cm.

Subtract now, we get :

Difference = 15 – 12 = 3 cm.

The third side cannot be less than the difference of length of other two sides.

Hence, the third side's minimum length would be more than 3 cm and its maximum length would be less than 27cm.

Example 9 : **Is there a triangle whose sides have the length 10.2 cm, 5.8 cm and 4.5 cm ?**

Solution :

$$a = 10.2 \text{ cm}$$
$$b = 5.8 \text{ cm}$$
$$c = 4.5 \text{ cm}$$
$$a + b = 10.2 + 5.8 = 16.0$$
$$16 > 4.5 \quad \text{true}$$
$$b + c = 5.8 + 4.5 = 10.3$$
$$10.3 > 10.2 \quad \text{true}$$
$$c + a = 4.5 + 10.2 = 14.7$$
$$14.7 > 5.8 \quad \text{true}$$

Since all three side sum properties are true, this triangle is possible.

Example 10 : **Classify the following triangles as acute-angled, obtuse-angled or right-angle triangles.**

(a) $\angle A = 90°$, $\angle B = 45°$, $\angle C = 45°$

(b) $\angle A = 30°$, $\angle B = 80°$, $\angle C = 70°$

(c) $\angle A = 0°$, $\angle B = 60°$, $\angle C = 60°$

(d) $\angle A = 107°$, $\angle B = 30°$, $\angle C = 43°$

Solution : (a) right - angled triangle

(b) Acute - angled triangle

(c) Acute - angled triangle (equilateral)

(d) Obtuse - angled triangle

Example 11 : **Find the values of *x*, *y* and *z* in the figure that follows.**

Solution : In $\triangle PQR$, we have

$$x + 50° + 110° = 180° \quad \text{(Angle sum property)}$$
$$x + 160° = 180°$$

$\Rightarrow \quad x = 180° - 160° = 20°$

Now, $\angle QRS$ is a straight angle.

Hence, $\quad 110° + y = 180°$

$y = 180° - 110° = 70°$

In $\triangle$ PRS, we have.

$\angle RPS + \angle PRS + \angle RSP = 180°$ (angle sum property)

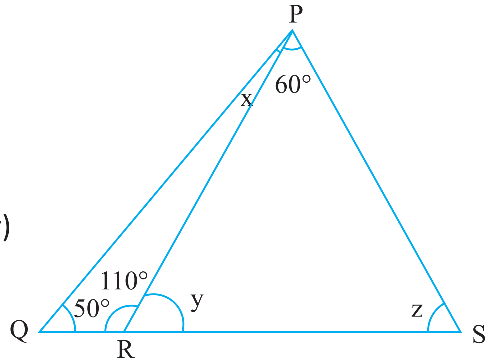
$60° + y + z = 180°$

$60° + 70° + z = 180°$ [$y = 70°$ calculated]

$130° + z = 180°$

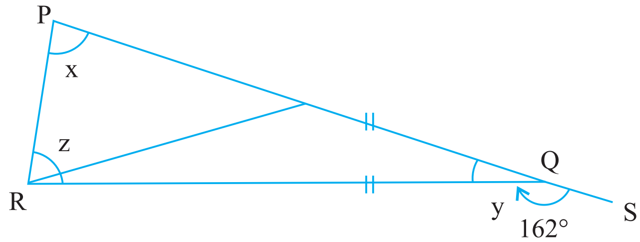
$z = 180° - 130° = 50°$

Therefore $x = 20°$, $y = 70°$ and $z = 50°$



Example 12 : In the figure shown here, PQ = QR

Further $\angle SQR = 162°$. Find the value of *x*.



Solution : $\angle$ PQS is a straight angle.

So, $\quad \angle SQR + \angle y = 180°$

$162° + \angle y = 180°$

$\angle y = 180° - 162° = 18°$

$\triangle$PQR is isosceles.

So, $\quad PQ = QR$ (given)

$x = \angle z$ (Equal sides have equal angles opposite them)

In PQR, We have

$\angle y + \angle z + x = 180°$

But $\quad x = \angle z$

$\angle y + x + x = 180°$

$\angle y + 2x = 180°$

But $\angle y = 18°$

So, $\quad 18° + 2x = 180°$

$2x = 180° - 18° = 162°$

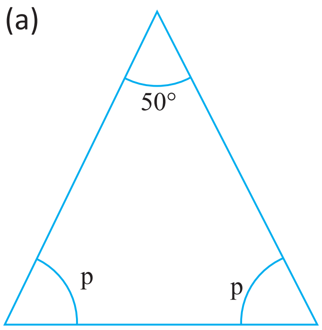
$= 81°$

$x = \dfrac{162°}{2}$

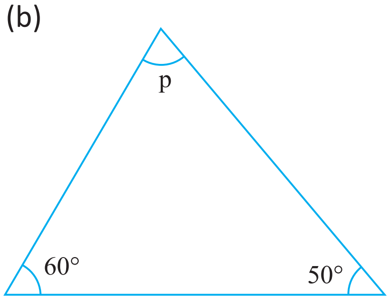
Exercise 10.2

1. The angles of a triangle are in the ratio 1 : 3 : 5. Find out all the angles of the triangle.
2. Find out the sum of four angles of a quadrilateral using the angle sum property of triangle.
3. **Find the value of *p* in the following figures.**

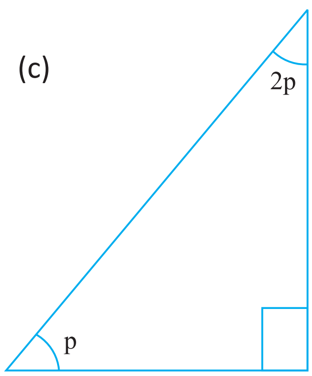
(a)



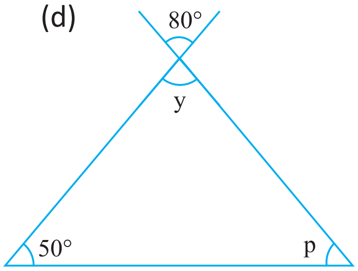
(b)



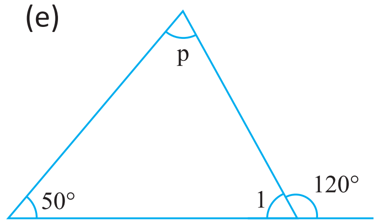
(c)



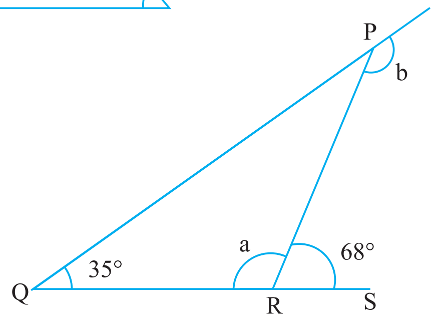
(d)



(e)

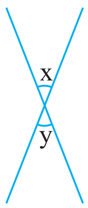


4. Find the value of *a* and *b*.

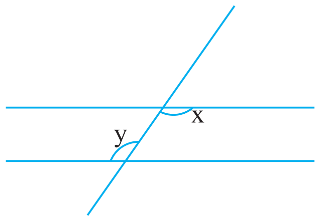


5. (a) Define the angle sum property of triangle. Can a triangle have two angles of 90° each ?
   (b) Can we have a triangle with sides 6 cm, 3 cm and 7 cm?
   (c) Identify the type of triangle if $\angle ABC = 30°$, $\angle BCA = 65°$ and $\angle CAB = 85°$
   (d) $\angle ABC = 2(x + 5)$, $\angle BCA = 2(x + 5)$ and $\angle CAB = 2(x + 5)$. Then, what type of triangle $\triangle ABC$ is ?
6. In an isosceles triangle, the vertex angle is 6° more than one base angle. Find out the measures of all angles of this triangle.

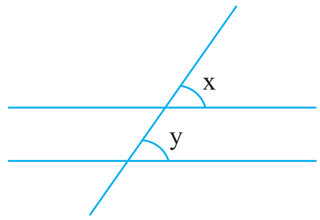
7. Fill up the blank spaces by writing the type of angle that each figure shown here represents.



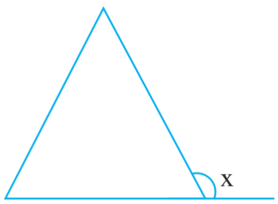
(a) ....



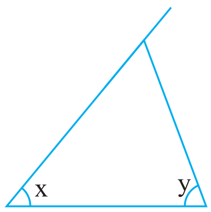
(b) ....



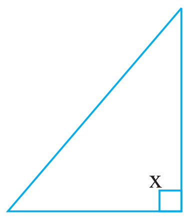
(c) ....



(d) ....

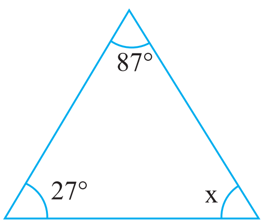


(e) ....

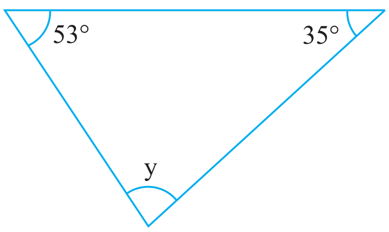


(f) ....

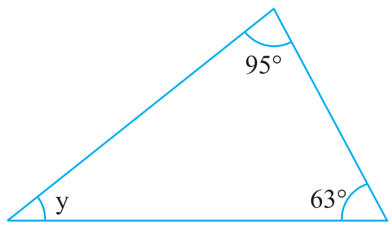
8. What are the two major differences between an equilateral triangle and an isosceles triangle ?
9. Find out the value of *x* and *y* in the following.



(a)



(b)

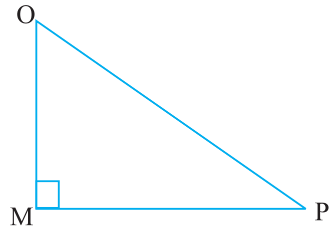


(c)

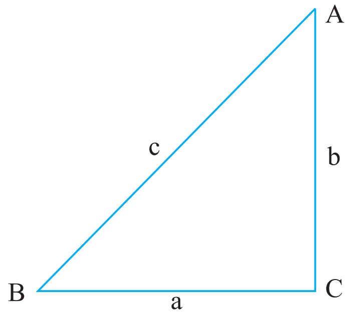
10. The length of two sides of a triangle is 6 cm and 8 cm. Between which two numbers can the length of third side fall ?

Right-angled Triangle

A right-angled triangle has one angle of 90°. The other two angles are acute angles, i.e. less than 90°. There cannot be two angles of 90° of each in any triangle. Look at the figure shown here.



In this figure, OM is the perpendicular, MP is the base and OP is the hypotenuse of $\triangle$OMP. $\angle$OMP is equal to 90°. The hypotenuse is the longest side of the right-angled triangles. The sides other than the hypotenuse are called **legs** of the right-angled triangle. We can all represent the sides of this triangle by small letters. Look at the figure shown here.

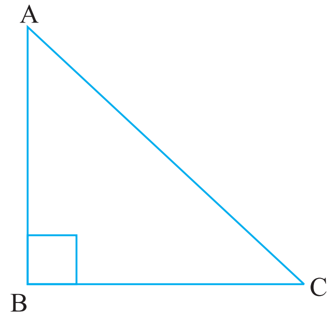


Pythagoras theorem

Pythagoras, a Greek philosopher of early sixth century B.C., found a very useful property of right-angled triangles. That property is called **Pythagorean theorem**. In ancient India, the Indian mathematician (Bandhayan) had also given a similar property of right-angled triangles.

According to Pythagoras, **the square on the hypotenuse is equal to the sum of the squares on legs**.

Let us consider a right-angled triangle ABC. So, ABC is right-angled at B. Look at the figure shown here.

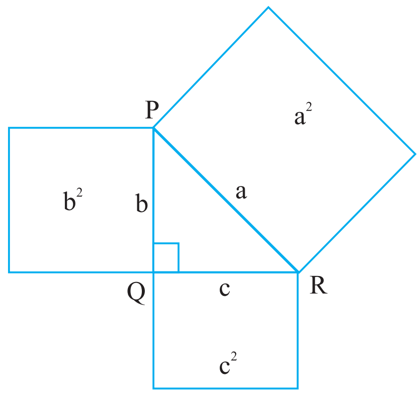


AC is the hypotenuse. $\overline{AB}$ and $\overline{BC}$ are the legs. Therefore, according to Pythagoras :

$$(AC)^2 = (AB)^2 + (BC)^2$$

$$\text{Or, } AC = \sqrt{AB^2 + BC^2}$$

Let us draw a triangle with one right angle. The hypotenuse and other two sides have been shown in Figure. The three squares have been drawn on three sides. You can measure the areas of these three squares. They have been marked *as*, a^2, b^2 and c^2 in the Figure shown here. You can confirm that $a^2 = b^2 + c^2$. Thus, 1 is a right-angled triangle.



Facts to Know

If the Pythagorean property is valid for a triangle, it must be a right-angled triangle.

Further, note that if $a^2 \neq b^2 + c^2$, then the triangle is not a right-angled triangle., This concept of building squares on the three sides of the right-angled triangle was, in fact, given by Baudhayan, an ancient Indian mathematician.

Pythagorean triplet

Any three positive integers or natural numbers are said to form a Pythagorean triplet if the square of one number is equal to the sum of squares of other two numbers.

Let us consider three numbers : 3, 4 and 5.

$$3^2 = 9$$
$$4^2 = 16$$
$$5^2 = 25$$
$$25 = 9 + 16$$

Facts to Know

If the sides of a triangle form the Pythagorean triplet, then that triangle is a right-angled triangle.

Thus, $25 = 9 + 16$

$(5)^2 = (3)^2 + (4)^2$

Or, $(3)^2 + (4)^2 = (5)^2$

So, 3, 4, 5 form a Pythagorean triplet.

Example 13 : **Which one of the following are Pythagorean triplets ?**

(a) 15, 10, 25

(b) 16, 12, 20

(c) 2.5, 6, 6.5

Solution : (a) We have three sides : 15, 10, 25

$$(15)^2 = 225$$
$$(10)^2 = 100$$
$$(25)^2 = 625$$
$$(10)^2 + (15)^2 = 100 + 225$$
$$= 325$$
$$325 \neq 625$$
$$(10)^2 + (15)^2 \neq (25)^2$$

Hence, these numbers do not form a Pythagorean triplet.

(b) We have three sides : 16, 12, 20

$$(16)^2 = 256$$
$$(12)^2 = 144$$
$$(20)^2 = 400$$
$$(12)^2 + (16)^2 = 144 + 256 = 400$$
$$\Rightarrow \quad (12)^2 + (16)^2 = 400 = (20)^2$$
$$\Rightarrow \quad (12)^2 + (16)^2 = (20)^2$$

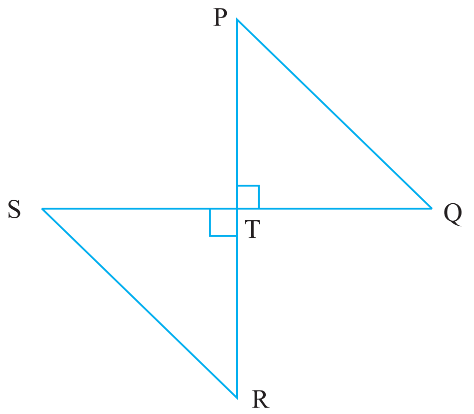
Hence, 12, 16 and 20 form the Pythagorean triplet.

(c) We have three sides : 2.5, 6, 6.5

$$(2.5)^2 = 6.25$$
$$(6)^2 = 36$$
$$(6.5)^2 = 42.25$$
$$(2.5)^2 + (6)^2 = 6.25 + 36$$
$$= 42.25$$
$$(2.5)^2 + (6)^2 = 42.25 = (6.5)^2$$
$$(2.5)^2 + (6)^2 = (6.5)^2$$

Hence, 2.5, 6 and 6.5 form the Pythagorean triplet.

Example 14 : **In the figure shown here, PR = 50 cm, SQ = 120 cm, T is the mid-point of PR and SQ. Further, SQ $\perp$ PR. Find out the length of SR and PQ.**



Solution : T is the mid-point of PR.

$$\Rightarrow \quad PT = TR$$

But $PR = 50$ cm

$$\Rightarrow \quad PT = TR = \frac{PR}{2} = 25 \text{ cm}$$

T is the mid-point of SQ.

$$\Rightarrow \quad ST = TQ$$

But $SQ = 120$ cm

$$\Rightarrow \quad ST = TQ = \frac{SQ}{2} = \frac{120}{2}$$

$= 60\text{ cm}$

$\angle PTQ = 90°$ (given)

Hence, $\triangle PTQ$ is a right-angled triangle.

Hence, we have:

$$PT^2 + TQ^2 = PQ^2 \quad \text{..............................(i)}$$

Here, $PT = 25\text{ cm}$

$TQ = 60\text{ cm}$

Putting these value in equation (i), we get

$$(25)^2 + (60)^2 = PQ^2$$

$\Rightarrow$ $PQ^2 = (25)^2 + (60)^2$

$= 625 + 3600$

$= 4225$

$\Rightarrow$ $PQ = \sqrt{4225}$

$= 65\text{ cm}$

Further, $\angle STR = 90°$ (given)

$\Rightarrow \triangle RTS$ is a right-angled triangle

$\Rightarrow$ $RT^2 + TS^2 = SR^2 \quad$ ....(ii)

$RT = TR = 25\text{ cm}$

$TS = ST = 60\text{ cm}$

Putting these values in equation (ii), we get

$$(25)^2 + (60)^2 = SR^2$$

$\Rightarrow$ $SR^2 = (25)^2 + (60)^2$

$= 625 + 3600 = 4225$

$\Rightarrow$ $SR = \sqrt{4225}$

$= 65\text{ cm}$

> The exterior angle of a triangle is equal to the sum of the interior opposite angle.

> The exterior angle of a triangle is always grater then either of the interior opposite angle.

Example 15 : **In a above figure the exterior $\angle$ PRS.**

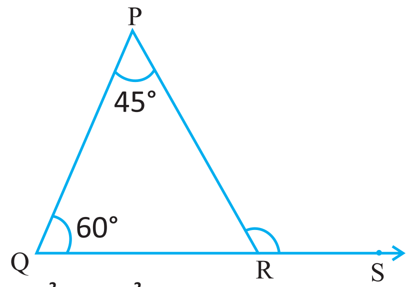
Solution :

$\Rightarrow$ $\angle PRS = (\angle QPR + \angle PQR)$

$\Rightarrow$ or, $\angle PRS = 45° + 60°$

$\Rightarrow$ or, $\angle PRS = 105°$

The exterior angle is equal to the sum of the interior opposite angle of a triangle.



Example 16 : **$\triangle ABC$ is an isosceles triangle, right-angled at C. Show that $AB^2 = 2AC^2$.**

Solution : In right-angled $\triangle ACB$, $\angle C = 90°$.

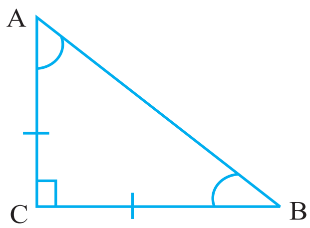
Further, the hypotenuse of a right-angled triangle is the longest side of it

So, it cannot be one of two equal sides

Hence, $AC = CB$

That is why it is also an isosceles triangle.

Now since $\triangle ACB$ is right-angled at C, we have :



$$AB^2 = AC^2 + CB^2 \quad \text{...(i)}$$

But $AC = CB$

Put $CB = AC$ in equation ....(i)

$\Rightarrow$ $AB^2 = AC^2 + AC^2 = 2AC^2$

Example 17 : **Find x, if the angle of triangle have measures (x + 40°), (2x + 20°) and 3x, Also, state which type of triangle this is.**

Solution :

$$x + 40 + 2x + 20° + 3x = 180° \quad \text{(angle sum property)}$$

$$\text{or, } 6x + 60° = 180°$$

$$\text{or, } 6x = 120°$$

$$\text{or, } x = 20°$$

$$\text{now, } x + 40° = 20° + 40° = 60°$$

$$2x + 20° = 2 \times 20° + 20°$$

$$= 60°$$

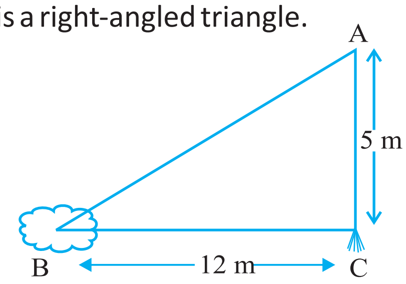
$$3x = 3 \times 20° = 60°$$

The three angles of the triangle is an "EQUILATERAL TRIANGLE

Example 18 : **A tree is broken at a height of 5 m from the ground. Its top touches the ground at a distance of 12 m from the base of the tree. Find the original height of the tree.**

Solution : The fallen tree is making an angle of 90° with the ground. So, ABC is a right-angled triangle.

BC = 12 m and AC = 5 m



$$\Rightarrow \quad AB^2 = BC^2 + AC^2$$

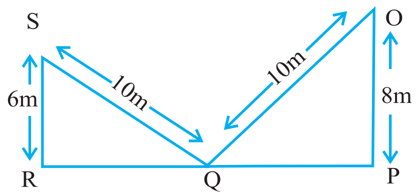
$$= (12)^2 + (5)^2$$

$$= 144 + 25$$

$$AB^2 = 169 \text{ m}$$

$$\Rightarrow \quad AB = \sqrt{169} = 13 \text{ m}$$

Example 19 : **A ladder is 10 m long. It reaches a window that is 8 m above the ground on one side of the road. Keeping its foot at the same point, the ladder is turned to the other side of the road to reach a window whose height is 6 m. What is the width of the road ?**



Solution : The figure is as follows :

OQ = SQ = length of ladder = 10 m

SR = 6 m and OP = 8 m

In $\triangle SRQ$, $\angle R = 90°$

$$\Rightarrow \quad SR^2 + RQ^2 = SQ^2 \quad \text{(Pythagorus Theorem)}$$

$$\Rightarrow \quad RQ^2 = SQ^2 - SR^2$$

$$= (10)^2 - (6)^2$$

$$100 - 36 = 64$$

$$\Rightarrow \quad RQ = \sqrt{64} = 8 \text{ m}$$

In $\triangle OPQ$, $\angle P = 90°$

$$\Rightarrow \quad OP^2 + PQ^2 = OQ^2$$

$$\Rightarrow \quad PQ^2 = OQ^2 - OP^2$$

$$= (10)^2 - (8)^2$$

$= 100 - 64$

$= 36$

$\Rightarrow \quad PQ = 6$ m

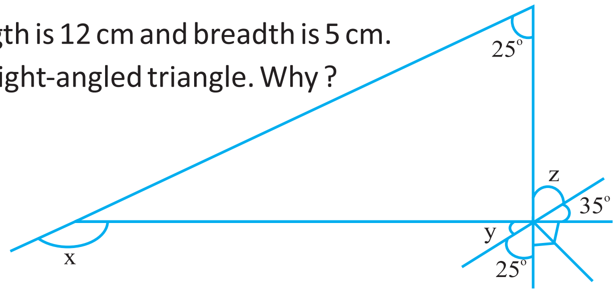
Hence, RP = length of the entire road

$= RQ + QP$

$= 8 + 6 = 14$ m

Exercise 10.2

1. A man goes 7 km east and then 24 km north. How far is he away from the initial point ?
2. In a right-angled triangle, one side is equal to 15 cm and the other equal to 20 cm. Find out length of the hypotenise side.
3. Find the perimeter of a rhombus whose diagonal measure 16 cm and 30 cm. Note that the diagonal of a rhombus intersect each other at right angles.
4. Find out the length of the diagonal of a rectangle whose length is 12 cm and breadth is 5 cm.
5. A triangle has sides of length 6 cm, 7.5 cm and 4.5 cm. It is a right-angled triangle. Why ?
6. Find out the value of x, y and z in the figure shown here.



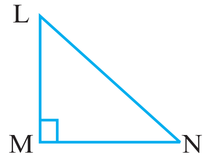
Points to Remember

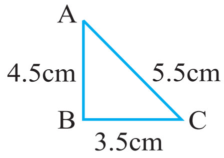
- A triangle is a polygon having three sides. It is a closed figure.
- The six elements of the triangle are its three sides and three angles.
- The sum of the angles of a triangle is 180° or two right angles (angles sum property).
- If we classify triangles on the basis of angles, we have three categories : acute-angled, right-angled and obtuse-angled triangles.
- If we classify triangles on the basis of side, we have three categories : equilateral, isosceles and scalene triangles.
- The basic features of triangles are height (perpendicular) base, median, altitude, interior, exterior, interior angle and exterior angles.
- The exterior angle of a triangle is equal to the sum of its opposite interior angles. (exterior angle property)
- The length of any two sides of a triangle, if added up, will always be more than the length of the third side. (side sum property)
- The sides opposite to the equal angles of a triangle are also equal. (equal side property)
- The angles opposite to the equal sides of a triangle are also equal. (equal angle property)
- A right-angles triangles has one angle of 90°, a hypotenuse, one base and one perpendicular.
- A right-angled triangle cannot have more than one angles of 90° measure.
- The square of the hypotenuse is equal to the sum of the squares of the other two sides or legs.. (Pythagorean theorem)
- If the Pythagorean triplet is valid for a triangle, it is a right-angled triangle.
- If the square of one side of a triangle equals the sum of squares of the other two sides, then the triangle is a right-angled triangle and also, the angle opposite to the longest side is a right angle. (converse of Pythagorean theorem)

EXERCISE

1. MULTIPLE CHOICE QUESTIONS (MCQs) :

Tick (✓) the correct options :





(a) The six elements of a triangle are its three sides and

(i) One angle (ii) Two angles

(iii) Three angles (iv) None of the these.

(b) The figure shown here, ∠M is equal to

(i) 22° (ii) 90°

(iii) 45° (iv) cannot be determined.

(c) A triangle ABC has been shown here. Which one of the following is true ?

(i) AB + BC = AC (ii) $AC^2 = AB^2 - Bc^2$

(iii) $AB^2 + BC^2 = AC^2$ (iv) ∠C = 90°

(d) In a right-angled triangle, the hypotenuse is the :

(i) Longest side (ii) Smallest side

(iii) Sum of the other two sides (iv) Difference of the other two sides

(e) Is the following set of number a Pythagorean triplet ?

4	5	7

(i) Yes (ii) No

(iii) Cannot be determined (iv) Need more data.

(f) The angle sum property of the triangle states that

(i) Three sides are equal (ii) Two sides are equal

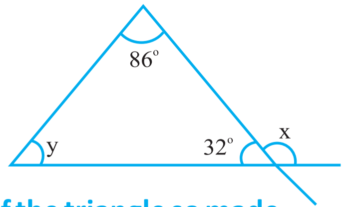
(iii) Sum of two angles is 180° (iv) None of these.

(g) The angles of an acute-angles triangle are :

(i) Less than 90° (ii) More than 90°

(iii) Equal to 90° (iv) None of these

2. Find the measure of x, y in the following figure.



3. Whether △PQR is possible in the following conditions ? Also tell about the type of the triangle so made.

(a) PQ = m cm and QR = $(m + 3)$ cm
RP = $(m + 2)$ cm, where $m > 1$ cm

(b) PQ = 9 cm, QR = 10 cm and RP = 11 cm

(c) ∠PQR = 28° 30′, ∠QRP = 90° and ∠RPQ = 61° 30′

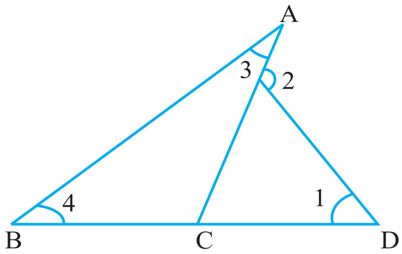
(d) ∠P = 90°, ∠Q = 90° and ∠R = 33° 10'

(e) ∠PQR = x, ∠QRP = $x + 80°$ and ∠RPQ = $x + 94°$

4. (a) State the angle sum property of triangle.
   (b) Prove that the exterior angle of a triangle is equal to the sum of its interior opposite angles.

5. One of the acute angles of a triangle (having one angle of 90°) is 47°. Find out the measure of the third angle. Is it an acute or obtuse angles.

6. **In the figure shown here,**



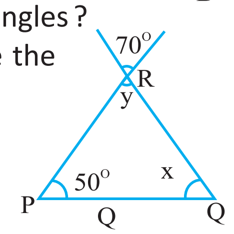
$\angle 1 = 43°$

$\angle 2 = 94°$

$\angle 3 = 32°$

Find out the measure of $\angle 4$.

7. (a) What is the difference between scalene and isosceles triangles ?
   (b) State Pythagorean theorem, draw a sketch to illustrate the concept of right-angled triangle given by Bandhayan.
   (c) Find the value of *x* and *y* in the figure shown here.

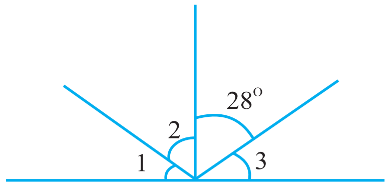


8. **Fill in the blanks.**
   (a) The angles of a triangle are in the ratio of 2 : 3 : 4. The angles are ______, ______ and _______.
   (b) If x and 107° are vertically opposite angles, then the value of x is ______.
   (c) In the figure shown here,

   $\angle 1 = 25°$

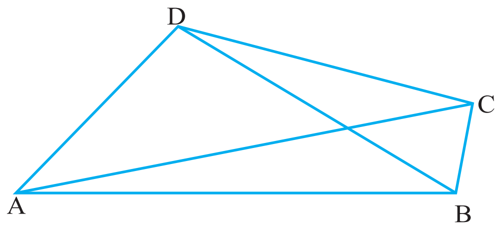
   $\angle 1 + \angle 2 = 90°$

   So, the sum of $\angle 1 + \angle 2 + \angle 3$ is equal to ___________.

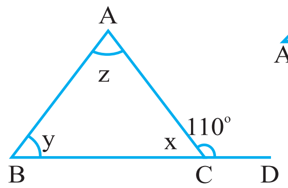


   (d) Three angles 60°, 60°, 60 are the angles of an _________ triangle.
   (e) In a right-angles triangle, the sum of the other two angles is _________.
   (f) The length of a rectangle is 40 m. Its diagonal is 41 m. So, its perimeter is _________ m.
   (g) In a right-angled triangle, the hypotenuse is the _______ side.
   (h) If a, b and c are the three sides of a triangle, a + b is ________ than c.

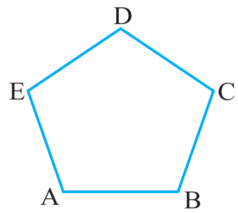
9. In the figure given here, prove that AB+BC + CD + AD > AC + BD



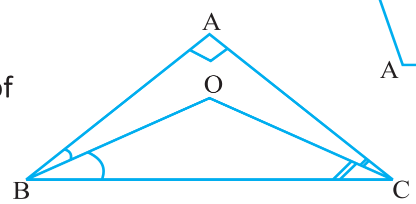
10. In the figure given here, y : z = 5 : 6. Further, $\angle ACD = 110°$. Find out the values of *x*, *y* and *z*.



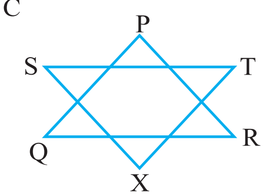
11. Prove that the sum of the interior angles of a regular pentagon is equal to 540°.



12. In a triangle ABC, right angled at A, the bisectors of $\angle B$ and $\angle C$ meet at O. Find the measure of $\angle BOC$.



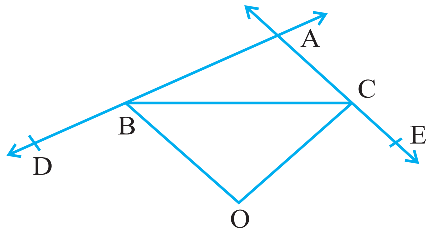
13. In the figure given here, find the sum of all angles that are the vertices of the figure.



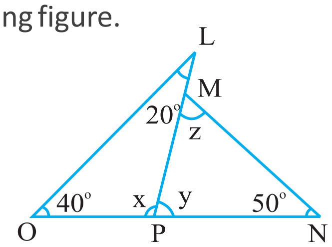
14. The length of two sides of a triangles is 5 cm and 11 cm, respectively. Find the maximum and minimum limits of the length of the third side of this triangles.

15. The sides AB and AC of $\triangle ABC$ have been produced up to D and E, respectively. The bisectors of exterior angles at B and C intersect each other at O.

Prove that $\angle BOC = 90° - \frac{\angle A}{2}$



16. Find out the value of *x*, *y* and *z* in the following figure.

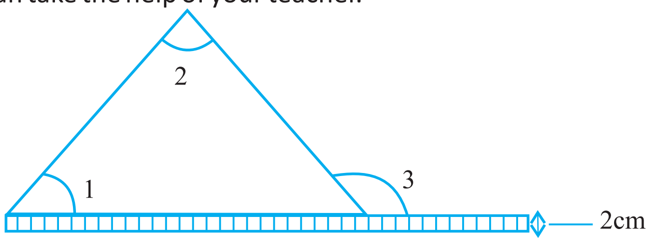


HOTS

Construct an isosceles triangle whose base is equal to 9 cm, and whose altitude from the opposite vertex to the base is 7 cm. [Hint: Altitude of an isosceles triangle divides the base into half.]

Lab Activity

Take a cardboard of size 1 foot by 1 foot. Draw the following figure on it with pencil, scale and compass. You can take the help of your teacher.



Now, cut out the figure from the cardboard sheet. You would get a thick base line, show as shaded part in figure. The solid triangles shape and its step must be cut neatly with the help of a cutter and scale. The base strip can be 2 cm thick.

Now, measure $\angle 1$, $\angle 2$, and $\angle 3$. What do you observe ?

Write down your observation here.

I am ________________(name)

My class is ____________(class)

I have observed about $\angle 1$, $\angle 2$ and $\angle 3$ that

....

....

....

Signature

11 Congruence

The object, which is same shape and same size are called congruent object such are carbon copy of each other or can say in all respect.

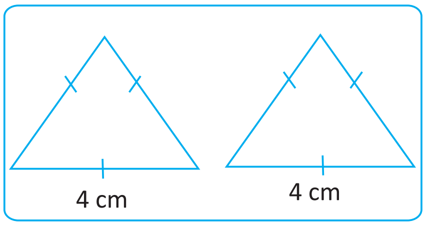
In geometry, two figure, identical in shape and size, are side to be congruent and this property of begin identical is called congruence its denoted by symbol "≅"

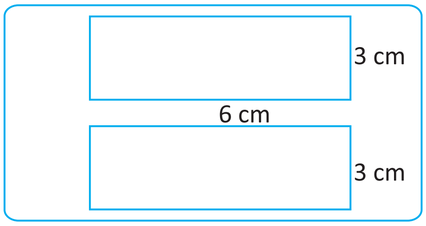
"To congrue" means to agree. In our everyday life, we come across many things which are the exact copies of some other things.

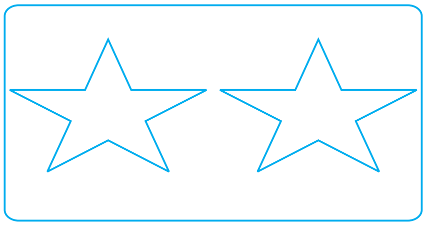
Look at the figures that follow.

In these figures, two objects of same size and shape have been shown. Their positions or orientation may become different but they are exact copies of each other (in all three figures shown here). These pairs are exactly the same in terms of size and shape. So, we can state that they are congruent to each other.

The same is the case with the figures used in geometry. Look at the following figure pairs. The geometrical figures







in each case are exact copies of each other.

How do we learn that these shapes are congruent to each other ? We can place one figure over the other to find this out. This method is called superimposition. However, this method can be applied only in case of objects like coin, blade and key. So, what should we do if want to check whether two triangles are congruent or not? We cannot superimpose one triangle over the other (in a practical, workable manner). So, we would have to learn about geometrical shapes and the rules for matching their shapes and size.

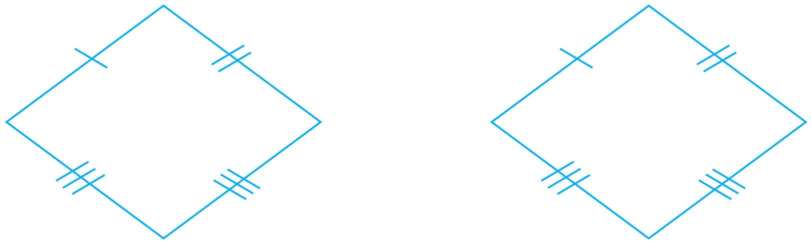
Congruence and its Representation

The property due to which two objects or geometrical shapes are exactly same to each other in terms of shape and size is known as **congruence.** If two objects are exactly the same in terms of shape and size, they are said to be **congruent**. More than two objects or geometrical figures can also be congruent to one another. The symbol of congruence is as follows:

≅ ↔ "Is congruent to"

In single-line diagrams, the method of showing congruence is different. The two corresponding sides of a geometrical figure are ticked with single lines, as has been shown here.

If the number of lines is more than one, you can use double-ticks and triple-ticks and so on. Look at these two congruent figures.



Note that only corresponding sides are to be shown as equal in each one of the two congruent figures.

Structure of This Lesson

In this lesson, we shall study the congruence of line segments, angles, triangles, quadrilaterals and circles, in this very order.

Facts to Know

- We use congruence in engineering and science. We compare objects of similar shape and design or make new objects similar to them.
- Two points are always congruent to each other. A point is a geomatric figure that occupies space in an X-Y plane.

Congruence of Line Segements

Two line segments are congruent if their length is equal. Look at the two line segments shown here.

Ideally, we should pick $\overline{AB}$ and place it over $\overline{CD}$. If A comes over C and B comes over D (and both line segments appear to be one line segment), we conclude that they are congruent to each other. However, a simple method is the measurement of each one of these line segments.

$\overline{AB}$ = 4.5 cm, $\overline{CD}$ = 4.5 cm

If the length is the same, it means that $\overline{AB}$ and $\overline{CD}$ are congruent to each other.

$$\therefore \overline{AB} \cong \overline{CD}.$$

Facts to Know

- Two geometrical figures are congruent if one of them can be turned and / or flipped and placed exactly on top of the other, this overlapping being exact and complete two figures with same shape and size are congruent to each other.

Important Properties (Line Segments)

Some more facts about the congruence of line segments are as follow:

1. Every line segment is congruent to itself.

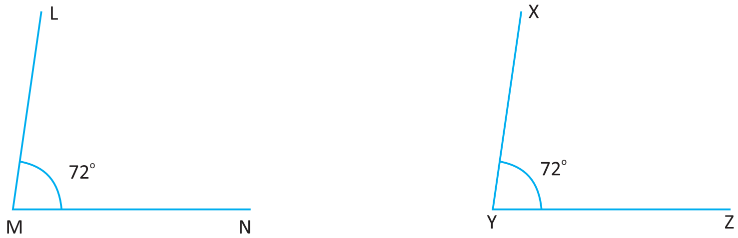
   Hence, $\overline{AB} \cong \overline{AB}$.

2. If $\overline{PQ}$ and $\overline{RS}$ are two line segments and $\overline{PQ} \cong \overline{RS}$, then $\overline{RS} \cong \overline{PQ}$.
3. If $\overline{PQ}$, $\overline{RS}$ and $\overline{ZK}$ are three line segments and $\overline{PQ} \cong \overline{RS}$ and $\overline{RS} \cong \overline{ZK}$, then $\overline{PQ} \cong \overline{ZK}$.

Congruence of Angles

Suppose that we have been given two angles — ∠LMN and ∠XYZ. Let us suppose that they have the same measure, i.e., 72°.

Look at these figures.



These two angles would be congruent if their measures are equal.

So, if $m\angle LMN = m\angle XYZ$

$\Rightarrow \angle LMN \cong \angle XYZ$

Note that the length of $\overline{LM}$, $\overline{MN}$, $\overline{XY}$ and $\overline{YZ}$ need not be equal. The equality of both angles is the only criterion for congruence (because the length of these four line segments can vary).

Important properties of Angles

1. Every angle is congruent to itself.

   Hence, we have $\angle M \cong \angle M$.

2. If ∠M and ∠N are two angles and $\angle M \cong \angle N$, then $\angle N \cong \angle M$.
3. If ∠M, ∠N and ∠R are three angles and $\angle M \cong \angle N$ and $\angle N \cong \angle R$, then $\angle M \cong \angle R$.

Facts to Know

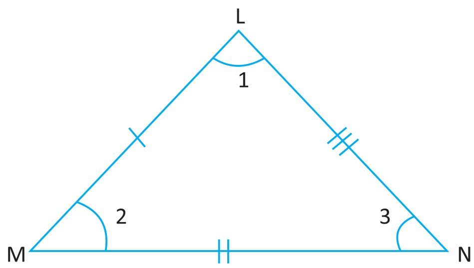
- You can make ∠PQR and ∠ABC and cut them from the sheet of paper. Superimpose ∠PQR over ∠ABC. If the angles are exactly equal, these two angles are congruent, else they are not so.

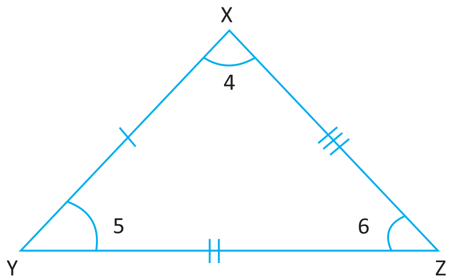
Congruence of Triangles

There are six parts in a triangle — three sides and three angles. If two triangles ΔLMN and ΔXYZ are to be congruent to each other, their corresponding sides and corresponding angles must be congruent to each other.

Let us take two triangles ΔLMN and ΔXYZ. Look at these figures.

If, in these two triangles, the following conditions are satisfied then we can state that they are congruent to each other.

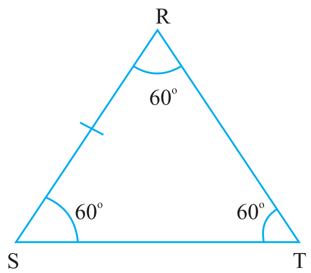


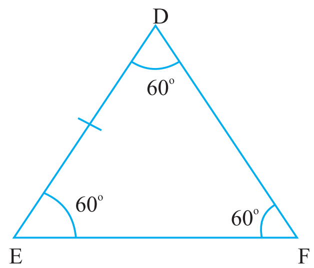


1.	$\overline{LM} \cong \overline{XY}$	2.	$\overline{MN} \cong \overline{YZ}$	3.	$\overline{LN} \cong \overline{XZ}$
4.	$m\angle 1 = m\angle 4$	5.	$m\angle 2 = m\angle 5$	6.	$m\angle 3 = m\angle 6$

Congruence of Equilateral Triangles

In an equilateral triangle, all sides are equal. Further, the measure of each angle of this triangle is 60°. Let us take up the case of two triangles — ΔRST and ΔDEF.





$m\angle S = m\angle E = 60° \Rightarrow \angle S \cong \angle E$ $m\angle R = m\angle D = 60° \Rightarrow \angle R \cong \angle D$

$m\angle T = m\angle F = 60° \Rightarrow \angle T \cong \angle F$

Further, all sides of an equilateral triangle are equal to one another.

For the ΔRST and ΔDEF, we have :

$RS \cong ST \cong TR \cong \overline{DE} \cong \overline{EF} \cong \overline{FD}$

Hence, $\Delta RST \cong \Delta DEF$

Hence, two equilateral triangles are congruent to each other if one side of one triangle is equal to one side of the other triangle.

Congruence of Quadrilaterals

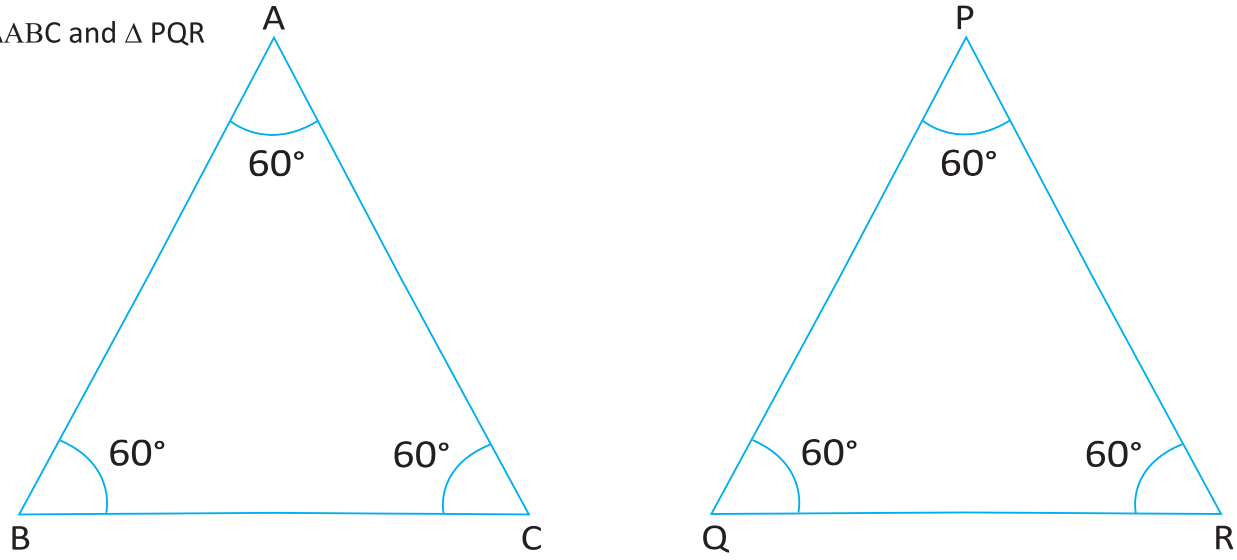
A quadrilateral is a polygon with four sides. So, square, rectangle, parallelogram, rhombus and trapezium fall under the category of quadrilateral. We shall discuss squares and rectangles in this section.

Congruence of Squares

A square is a quadrilateral in which all sides are equal and every angle is 90°. Two squares are congruent to each other if one side of a square is congruent to any one side of the other square. Let us take up two squares — WXZY and PQRS.

In an equilateral triangle, all sides are equal and each angle is of 60° consider two equilateral.

Triangles ΔABC and Δ PQR



Solution : In both the triangles, he measure of each angle is 60°. So $\angle A \cong \angle P$, $\angle B \cong \angle Q$ and $\angle C \cong \angle R$.

Again, all sides of an equilateral triangle are equal. So for the congruence of two equilateral triangles, it is sufficient to show that any one side of ΔABC is equal in length to any one side of ΔPQR.

Hence, we have : $\overline{AB} \cong \overline{PQ} \cong \overline{QR} \cong \overline{RP}$

$\Delta ABC \cong \Delta PQR$

By above method and the two equilateral triangles are said to be congruent it one side of one triangle is equal to one side of the other triangle.

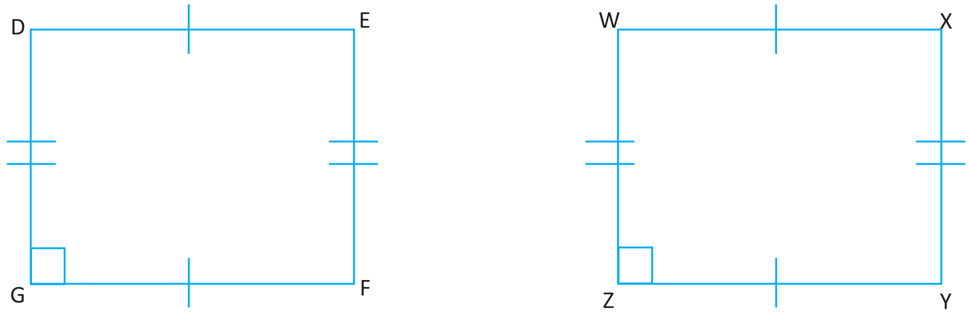
[So, $\Delta ABC \cong \Delta PQR$]

Congruence of Rectangles

A rectangle is a quadrilateral in which opposite sides are equal and each angle is equal to 90°.

Two rectangles are congruent to each other if they have equal length and breadth.

Let us take up two rectangles — DEFG and WXYZ. Look at the figures shown here.



We can easily prove that for these two rectangles:

$\angle G \cong \angle Z$, $\angle F \cong \angle Y$, $\angle E \cong \angle X$ and finally $\angle D \cong \angle W$

Now, we can prove that the length and breadth of one rectangle are equal to those of the other rectangle respectively.

Thus, we have:

Length		Breadth
GF ≅ ZY	;	FE ≅ YX
ED ≅ XW	;	DG ≅ WZ

Hence, two rectangles are congruent, if they have same length and breadth.

Facts to Know

- If you prove that length and breadth of two rectangles are equal, then you need not prove that their angles are also equal. The equality of length and breadth would automatically prove that the rectangles are congruent.

Congruence of Circles

If the radii of two circles are equal, they are congruent to each other.

Let us draw two circles with the radius 3.5 cm.

We have two circles, with centres O and O'. The radius of both circles is the same (3.5 cm). We can cut these circles and superimpose one over the other. But comparing the radii of these circles would be a more practical method.

In the case of these two circles, we have:

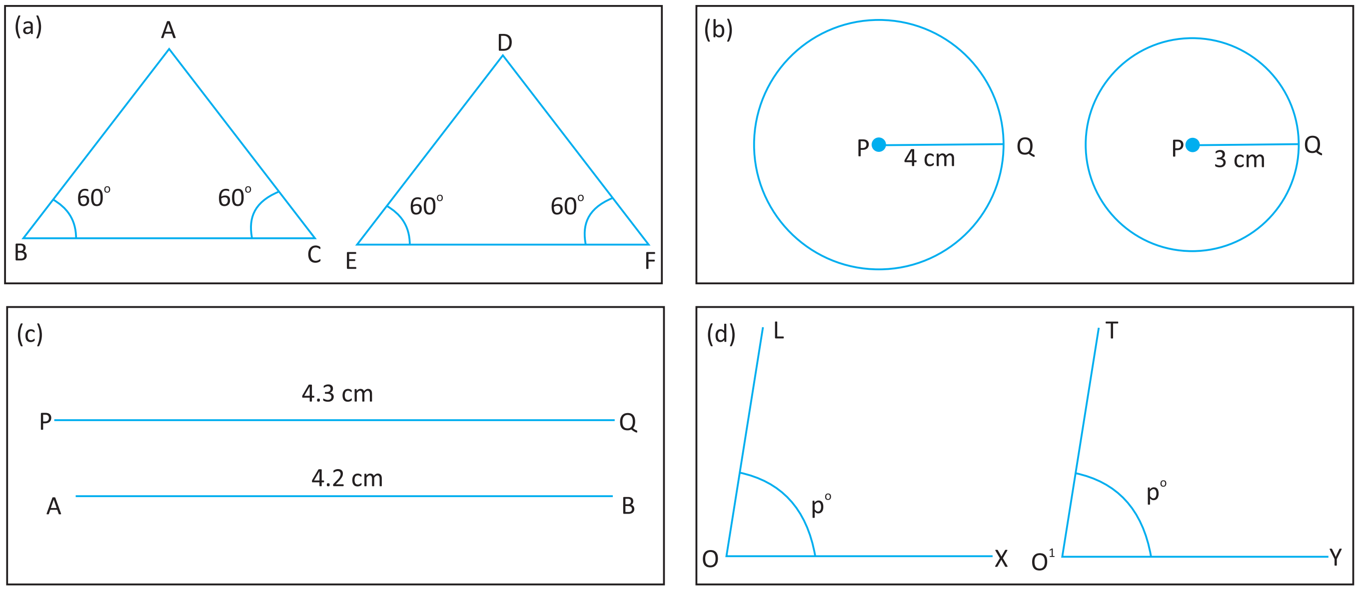
OA = OA’ = 3.5 cm

Hence, these two circles are congruent. Thus, we can state that:

Circle with centre O ≅ Circle with centre O'

OA = OA' = 3.5 cm

Example 1 : State whether the following pairs of figures are congruent or not:



Solution :

(a) We have to analyse the angles first.

In ΔABC, we have:

$\angle A + \angle B + \angle C = 180^\circ$

$\angle A + 60^\circ + 60^\circ = 180^\circ$

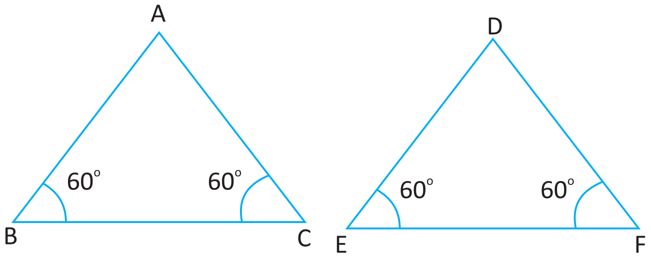
$\angle A = 180^\circ - 60^\circ - 60^\circ = 60^\circ$

In ΔDEF, we have:

$\angle D + \angle E + \angle F = 180^\circ$

$\Rightarrow \angle D + 60^\circ + 60^\circ = 180^\circ$

$\Rightarrow \angle D = 180^\circ - 60^\circ - 60^\circ = 60^\circ$



Hence, all angles of both triangles are 60° each.

Hence, they are equilateral triangles. Hence their sides are also equal.

In such a case, any one side of an equilateral triangle has to be shown to be equal to any one side of the other equilateral triangle.

So, $\overline{BC} = \overline{EF}$ (All sides of both triangles are equal)

$\Delta ABC \cong \Delta DEF$

(b) The circles have different radii.

$\overline{PQ} = 4$ cm

$\overline{PQ'} = 3$ cm

$\overline{PQ} \neq \overline{PQ'}$

Hence, the circles are not congruent to each other.

(c) $\overline{PQ} = 4.3$ cm

$\overline{AB} = 4.2$ cm

Clearly $\overline{PQ} \neq \overline{AB}$

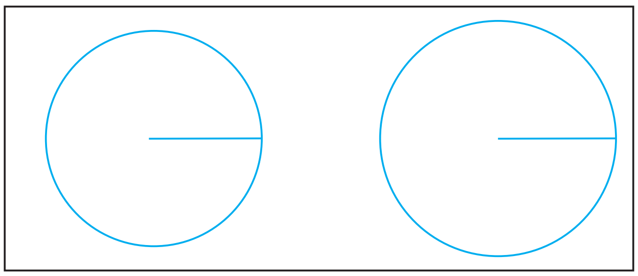
Hence, $\overline{PQ}$ and $\overline{AB}$ are not congruent to each other.

(d) $m\angle LOX = p^\circ$, $m\angle TO'Y = p^\circ$

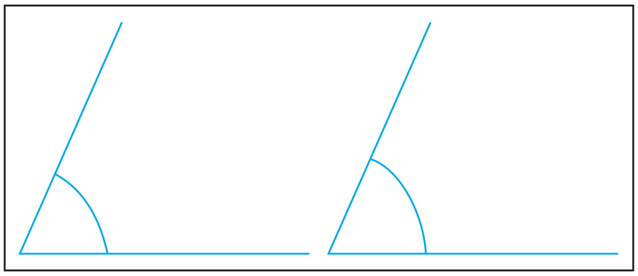
$\angle LOX \cong \angle TO'Y$

Hence two angles are congruent to each other.

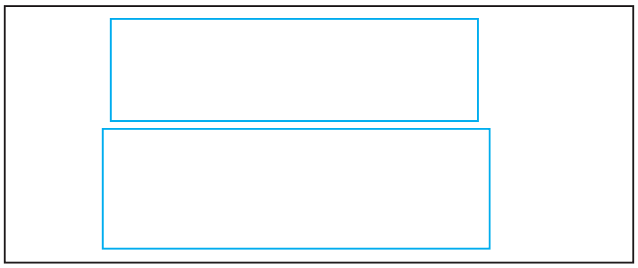
Example 2 : **Measure the following geometrical figures and tell whether the figures in each pair are equal to- each other or not:**



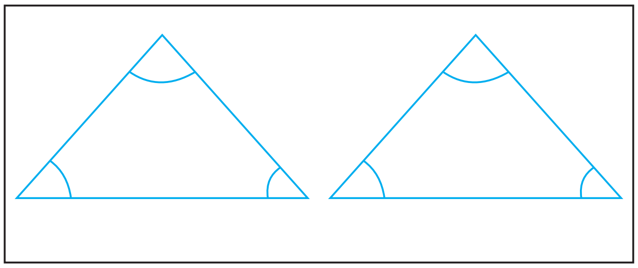
(a)



(b)



(c)



(d)

Solution : Here is a good exercise for you. We are giving the solutions here but you would have to measure the figures and find out what the real reasons behind these answers are.

(a) Figures are incongruent with each other.

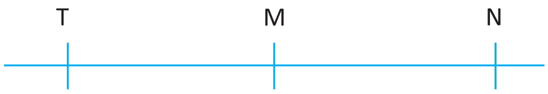
(b) Figures are incongruent with each other.

(c) Figures are incongruent with each other.

(d) Figures are congruent with each other.

Exercise 11.1

1. In the figures given below, the following observations were recorded:



TM ≅ XY
XY ≅ MN
Can you conclude that
MN ≅ TM ?

2. Is correct to state that any two right triangles are congruent to each other ? Give reasons for justifying your answer.

3. Complete the following statements:

(a) Two squares are congruent, if ________________.

(b) Two rectangles are congruent, if ________________.

(c) Two line segments are congruent, if ________________.

(d) Two circles are congruent, if ________________.

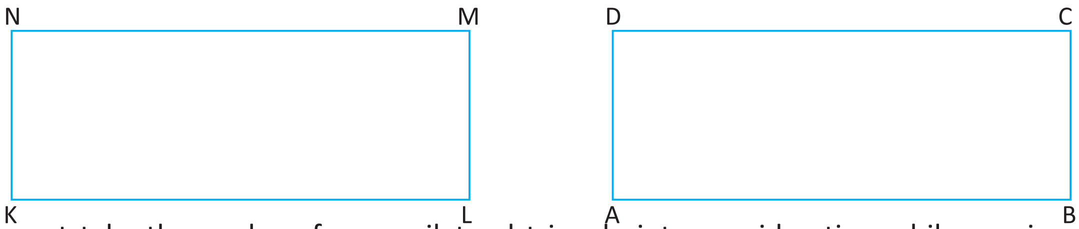
(e) Two angles are congruent, if ________________.

4. ABCD and PQRS are squares. If ABCD ≅ PQRS (in this order letters), then write the parts of PQRS that correspond to the following :

(a) ∠A (b) $\overline{BC}$ (c) ∠C (d) AB

5. Three angles are congruent to one another. If ∠1 + ∠2 + ∠3 = 180°, what are the measures of three angles?

6. These two figures are congruent to each other. What are the values of $\overline{KL}$, $\overline{MN}$, ∠L and ∠M. Use a ruler and protractor to do the measurements.



7. Why do we not take the angles of an equilateral triangle into consideration while proving that they are congruent to each other. Draw diagrams as well.

Facts to Know

- More then two line segments, circles, squares or rectangles can also be congruent. The criteria for congruency remain the same in all of these cases.

Congruence of Triangles

This section is the focus area of this chapter. That is why we are discussing triangle congruence separately.

We are aware of the fact that two triangles are congruent , if their corresponding sides and angles are congruent. So, all six equalities (three sides and three angles) must be valid for the two triangles that are being compared.

In some cases, however we may have less than six criteria. Then, how can we prove that the triangles are congruent? There are four other methods — besides the six-equality criterion – to prove that two triangles may be congruent. Let us discuss those four methods briefly.

Side Side Side (SSS) Congruence Criterion

It states :

If under a given correspondence, the three sides of one triangle, are equal to the three corresponding sides of another triangle, then the two triangles are congruent with each other.

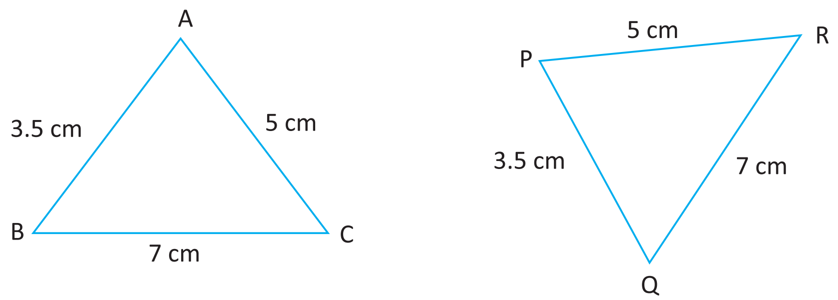
Look at the figures shown here:

In these two triangles, we have :

$AB \cong PQ = 3.5$ cm

$BC \cong QR = 7$ cm

$CA \cong RP = 5$ cm



Hence, $\Delta ABC \cong \Delta PQR$.

Note the order,

$A \leftrightarrow P$

$B \leftrightarrow Q$

$C \leftrightarrow R$

Δ ABC $\cong \Delta$ RQP.

The vertices, sides and angles of one triangle correspond to only particular vertices, sides and angles of another triangle. We cannot choose sides and angles on our own. Thus, congruence depends very much on the correspondence criterion.

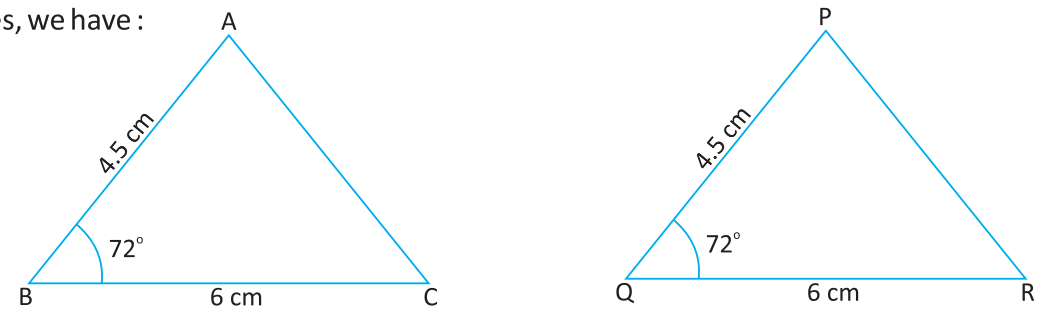
Side Angle Side (SAS) Congruence Criterion

It states :

If under a given correspondence, two sides and the included angle of one triangle are respectively equal to the two corresponding sides and the included angle of the other triangle, then these two triangles are congruent .

Look at the figures shown here:

In these two triangles, we have :



$AB \cong PQ = 4.5 cm$

$BC \cong QR = 6.0 cm$

$\angle ABC \cong \angle PQR = 72^\circ$

Hence $\Delta ABC \cong \Delta PQR$

So, two sides and their included angle of the first triangle, i.e., Δ ABC are congruent with the corresponding two sides and the included angle of the other triangle i.e., Δ PQR. Hence, these two triangles are congruent to each other.

Angle Side Angle (ASA) Congruence Criterion

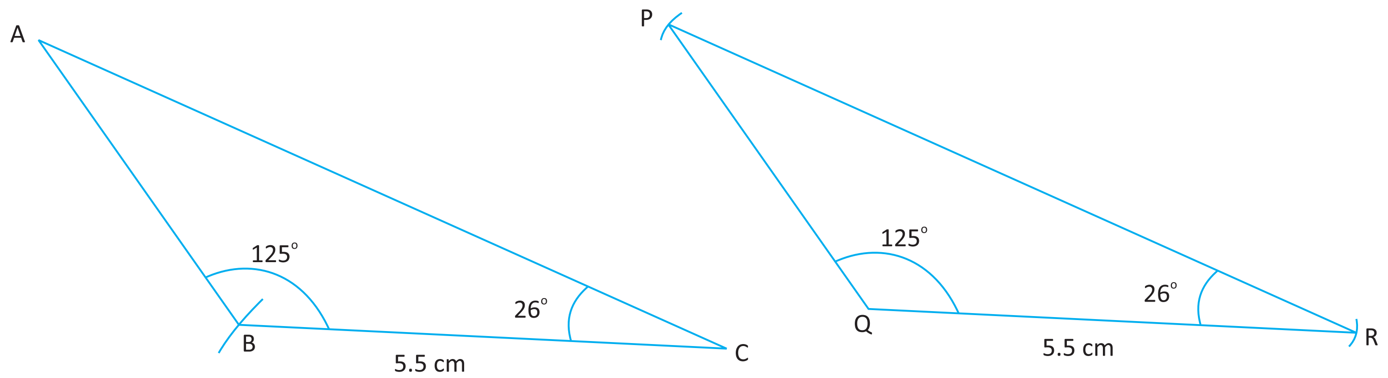
Facts to Know

- The SSA criterion of triangle congruence does not exist. The term SAS means - two sides and the angle included by them. So, SSA does not satisfy this condition.

It states :

If under a given correspondence, two angles and the included side of one triangle are equal to the two angles and the included side of another triangle, then those two triangles are congruent .

Look at the figures given here:



In these two triangles, we have :

$\angle ABC \cong \angle PQR = 125^\circ$

$BC \cong QR = 5.5 cm$

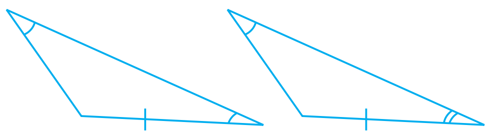
$\angle ACB \cong \angle PRQ = 26^\circ$

Hence, $\Delta ABC \cong \Delta PQR$

- AAS Congrunce Criterion : Two angles and the excluded side of one triangle are equal to the two angles and the excluded side of another triangle, them two triangle are congruent.

So, two angles and one included side of Δ ABC are congruent with the corresponding two angles and one included side of the Δ PQR. Hence, these two triangles are congruent to each other.

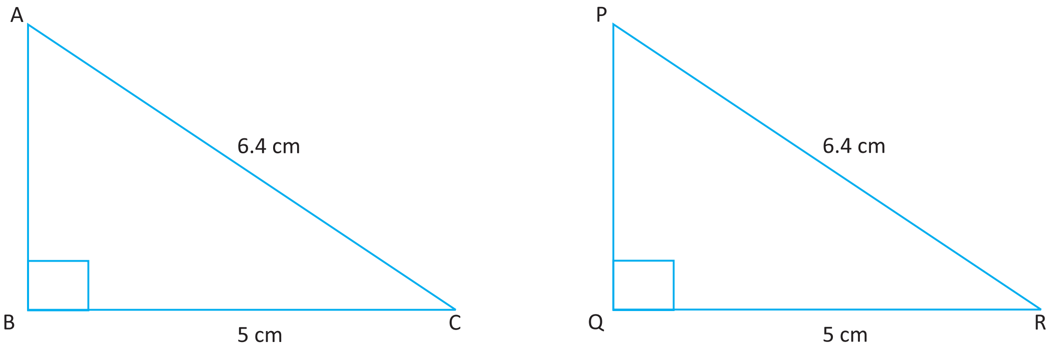
Right Angle Hypotenuse Side (RHS) Congruence Criterion



It states :

If under a given correspondence, the right angle hypotenuse and one side of a right-angled triangle are equal to the corresponding right angle, hypotenuse, and the one side of the other triangle, then these two triangles are congruent to each other.

Look at the figures given here.



In these two right– angled triangles, we have :

$\angle ABC \cong \angle PQR = 90°$

$AC \cong PR = 6.4$ cm [Hypotenuse criterion]

$BC \cong QR = 5$ cm.

So, the right angle, hypotenuse and base of Δ ABC are equal to the right angle, hypotenuse and base of Δ PQR, respectively. Hence, these two triangles are congruent to each other.

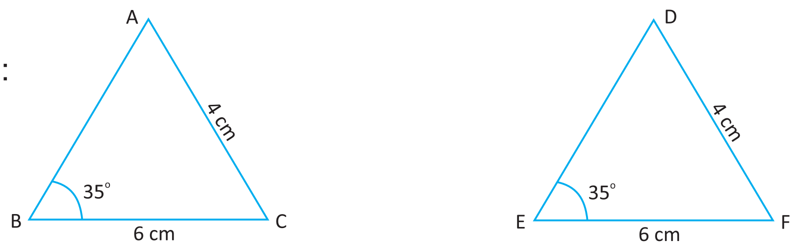
Facts to Know

- The AAA criterion for congruence of triangles does not exist! If three angles of a triangle are equal to the corresponding three angles of another triangle, then it is not necessary that these two triangles would be congruent to each other.

Example 3 : In DABC, BC = 6 cm, AC = 4 cm, $\angle B = 35°$

In ΔDEF, DF = 4 cm, EF = 6 cm, $\angle E = 35°$, check whether these two triangles are congruent or not.

Solution : First of all, draw the triangles. You can use a compass, protractor or a scale to do so.



In Δ ABC and Δ DEF, we have :

$\overline{BC} \cong \overline{EF} = 6$ cm

$\angle B \cong \angle E = 35°$

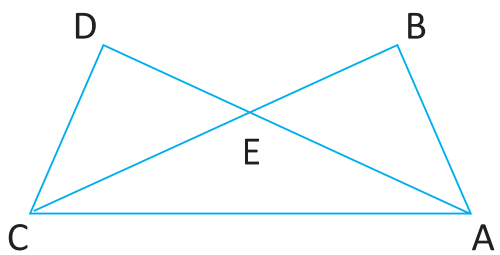
$\overline{AC} \cong \overline{DE} = 4$ cm

But $\angle B$ is not the included angle between those sides of the triangles that are being considered.

Hence, Δ ABC and Δ DEF are not congruent to each other.

Example 4 : In the figure shown here $\overline{AB} = \overline{CD}$ and $\overline{AD} = \overline{BC}$. Prove that :

$\Delta ADC \cong \Delta CBA$.

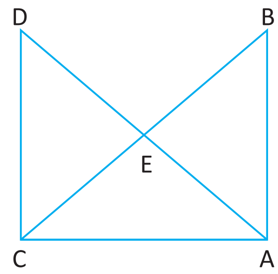


Solution :

Given : Two Triangles ΔADC and ΔCBA have been given, as shown in the figure.

$\overline{AB} = \overline{CD}$

$\overline{AD} = \overline{BC}$



To Prove $\Delta ADC \cong \Delta CBA$

In ΔADC and ΔCBA, we have :

$\overline{AB} = \overline{CD}$ (given)

$\overline{AD} = \overline{BC}$ (given)

$\overline{CA} = \overline{CA}$ (common)

Hence, $\Delta ADC \cong \Delta CBA$ (SSS Criterion of Congruence)

Example 5 : **Prove that the sides opposite to equal angles in a triangle are equal.**

Solution :

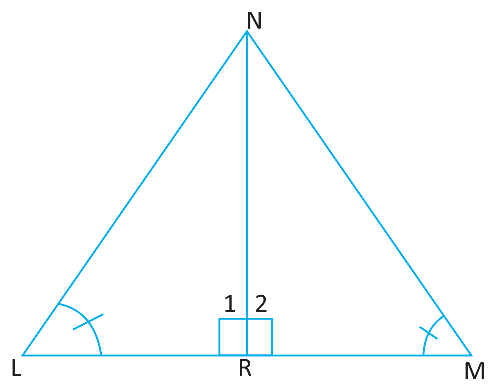
Given : ΔLMN in which

$\angle L = \angle M$

To Prove $\overline{NL} = \overline{NM}$

Construction

Draw $NR \perp LM$ from N.



Proof In ΔNLR and NMR, we have :

$\angle L = \angle M$ (given)

$NR = NR$ (common)

$\angle 1 = \angle 2$ [each equals 90° due to construction]

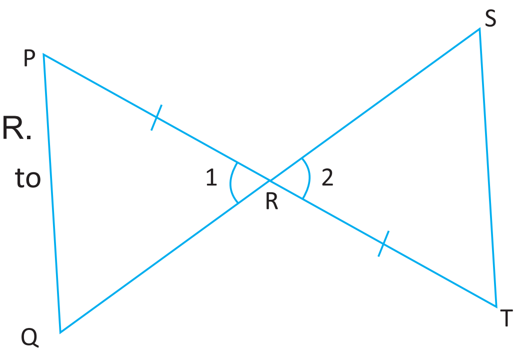
Hence, $\Delta NLR \cong \Delta NMR$ (ASA Criterion of Congruence)

Example 6 : **In the figure given here, what should be the minimum available condition or criterion that must be provided to you to make the ΔPQR and ΔSTR congruent to each other ?**

Solution : In ΔPQR and ΔSTR, two sides are equal :

$\overline{PR} = \overline{RT}$ (given)

Further, $\angle 1 = \angle 2$ (vertically opposite angles)



Hence, they are equal. $\angle 1$ is the included angle between PR and QR.

Hence, if QR is proved equal to $\overline{RS}$, then the SAS criterion can be applied to these two triangles . If that is so, the final conclusion can become :

$\overline{PR} = \overline{RT}$ [given]

$\overline{QR} = \overline{RS}$ [condition we are seeking]

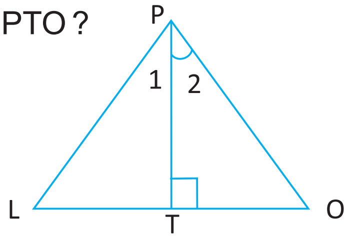
$\angle 1 = \angle 2$ [vertically opposite angles and included between the side pairs for congruence purpose]

Then , $\Delta PQR \cong \Delta STR$

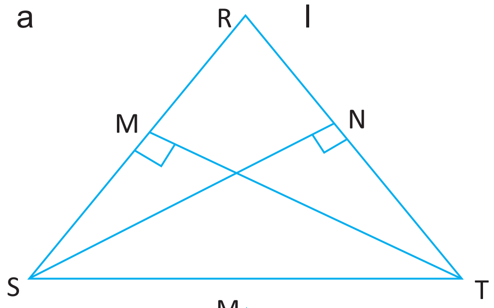
Hence, the minimum available condition or criterion needed for congruence is $\overline{QR} = \overline{RS}$.

Exercise 11.2

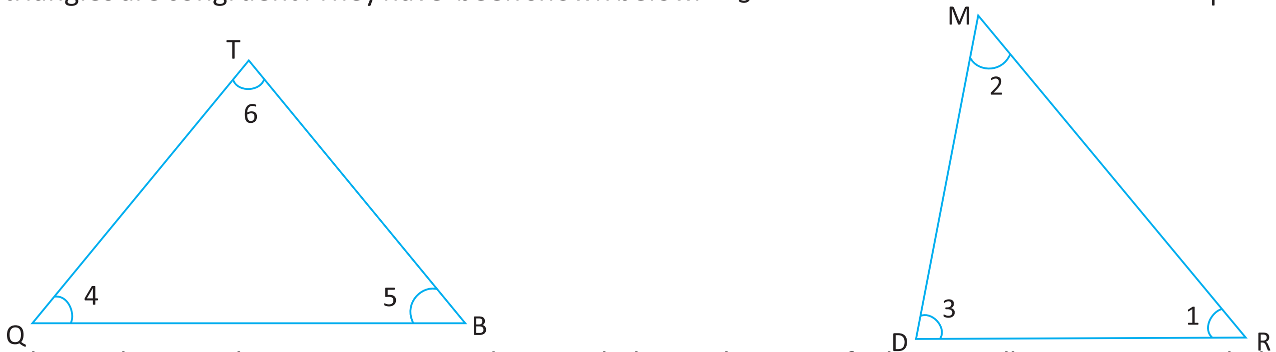
1. In the figure given here, PT bisects $\angle P$ and $PT \perp LO$. Is $\Delta PTL \cong \Delta PTO$?



2. In the figure given here, $TM \perp RS$ and $SN \perp RT$. Further it has been given that the perpendiculars are also equal. Is $\Delta SNT \cong \Delta TMS$?

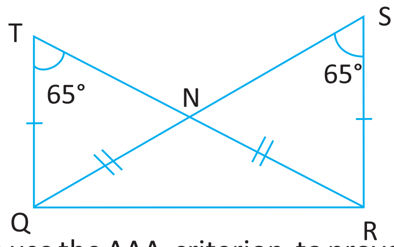


3. Two triangles are congruent. They have been shown below.



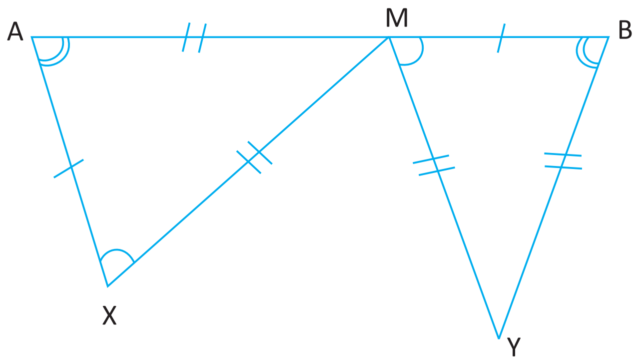
Write the six elements that are in correspondence with the six elements of other as well as congruent with them.

4. Which two triangles are congruent in this figure? Are there two such pairs?



5. Why can't we use the AAA criterion to prove that two triangles are congruent?

6. In this figure, which two triangles are congruent?



7. Write 'T' for true and 'F' for False for the following statements.

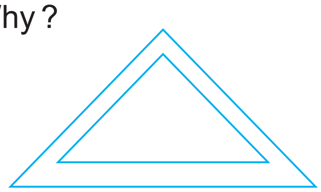
(a) Two triangles with congruent angles need not be congruent with each other.

(b) If $\Delta PQL \cong \Delta XMT$, it means that $PQ \cong XM$.

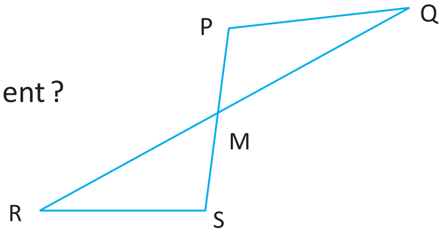
(c) In SSS criterion the congruence of angles is not needed.

(d) If SAS criterion is applicable to prove two triangles as congruent figures, then it means all the six elements have been proved to be congruent.

8. Are these triangles congruent? Why ?



9. In the given figure PQ || RS. Also, $\overline{PQ} = \overline{RS}$. Are these two triangles congruent ?



10. In the figure given here, AB = AC , ∠1= ∠2. Give reasons to show that

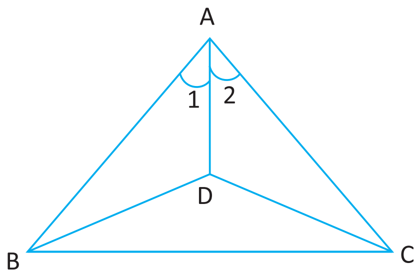
(a) $\Delta ABD \cong \Delta ACD$

(b) $\angle ABD = \angle ACD$

(c) $BD = CD$

(d) $\angle DBC = \angle DCB$

(e) ΔDBC is an isosceles triangle.



Points to Remember

- The property due to which two objects or geometrical shapes are exactly same to each other in terms of shape and size is known as congruence. Its symbol is ≅.
- Two line segments are congruent, if their length is equal .
- Two angles are congruent , if the measures of their angles are equal .
- Two triangles are congruent, if their three sides and three angles are equal in the order of correspondence.
- Two equilateral triangles are congruent if any of the sides of one triangle is equal to any one of the sides of theother one.
- Two squares are congruent, if any one side of one square is equal to any one side of the other one.
- Two rectangles are congruent to each other, if they have the same length and breadth.
- Two circles are congruent, if their radii are equal.
- The criterion of SSS and AAA taken together (six elements) is rarely seen in congruence of triangles. We have touse
- more practical criterion to prove their congruence. These four criteria are — SSS, SAS, ASA and RHS.
- In the SSS criterion , if three sides of a triangle are equal to three corresponding sides of another triangle, then these two triangles are congruent.
- In the SAS criterion, if two sides and the including angle of a triangle are equal to the corresponding two sides and the including angle of another triangle, then these two triangles are congruent.
- If the right angle, hypotenuse and one side of a right-anlgled triangle are equal to the right angle, hypotenuse and corresponding one side of another right–angled triangle, than these two triangles are congruent.

EXERCISE

1. **MULTIPLE CHOICE QUESTIONS (MCQs):**
   Tick (✓) the correct options :

(a) In which criterion of congruence, are two sides of triangle involved ?

(i) ASA ☐ (ii) RHS ☐ (iii) SAS ☐ (iv) Both (ii) and (iii) ☐

(b) If two circles are congruent , then their

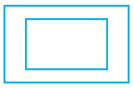
(i) radii are equal ☐ (ii) chords are equal ☐

(iii) diameters are equal ☐ (iv) Both (a) and (c) ☐

(c) ∠PQR and ∠ABC are congruent . If ∠PQR = 60°, then ½ ∠ABC = ?

(i) 60° ☐ (ii) 120° ☐ (iii) ☐ 30° (iv) None of these ☐

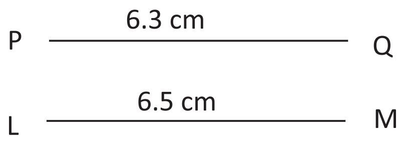
(d) Two rectangles have been shown here. Are they congruent to each other ?



(i) Yes ☐ (ii) No ☐

(iii) Need to measure them ☐ (iv) Need to get more data ☐

(e) Are these line segments congruent?



(i) Yes ☐

(ii) No ☐

(iii) Need to measure ☐

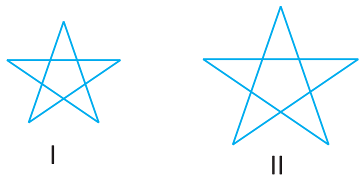
(iv) Need to get more data. ☐

(f) If two rectangles have equal area, they are congruent. This statements

(i) Is true ☐ (ii) Is false ☐

(iii) Requires more information ☐ (iv) None of these ☐

(g) Figures I and II shown here are not



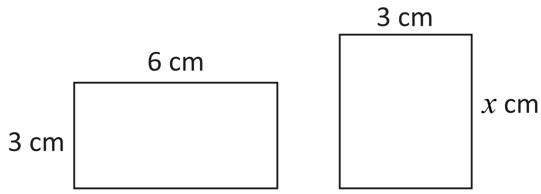
(i) Similar ☐

(ii) Congruent ☐

(iii) Equal ☐

(iv) Dissimilar ☐

(h) If these two rectangles are congruent to each other, then the value of x is



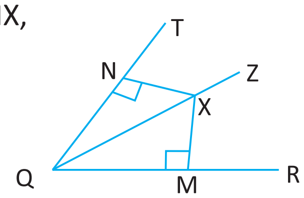
(i) 3 cm ☐

(ii) 4 cm ☐

(iii) 7 cm ☐

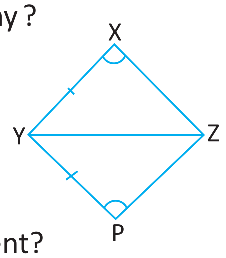
(iv) None of these . ☐

2. In the figure shown here, $\overline{XN} \perp \overline{QT}$ and $\overline{XM} \perp \overline{QR}$. If $\overline{NX} = \overline{MX}$, prove that $\Delta NXQ \cong \Delta MXQ$.



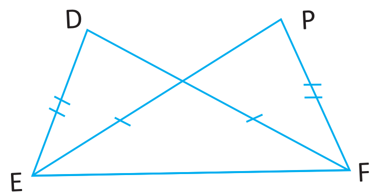
3. In Δ RXT, $\angle R = 60°$, $\angle X = 80°$ and RT = 5 cm.
In Δ DBM, $\angle B = 60°$, $\angle M = 80°$ and DB = 6 cm.
Are these two triangles congruent ? Why ?

4. Are these two triangles congruent to each other ? Why ?

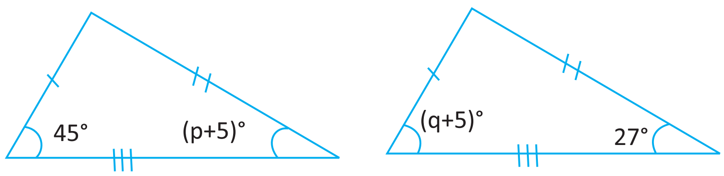


5. The areas of two squares are equal. Are they congruent?

6. Keeping in view the property of correspondence , find out which two triangles are congruent in this figure.



7. Find the values of p and q in these two triangles.

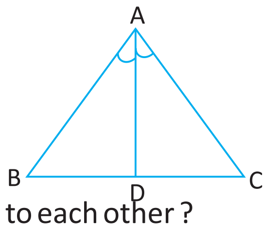


8. In an isosceles triangle ABC, AB = AC. The bisector of $\angle$A meets BC in D.

(a) Is $\Delta ABD \cong \Delta ACD$? How?

(b) Is D the mid-point of BC ? How ?

(c) What is the measure of $\angle$ADB ?



9. The areas of two circles are 154 sq. cm each. Are these two circles congruent to each other ?

10. Fill in the blanks:

(a) $\Delta ABC \cong DEF$. If BC = 4.5 cm, then EF = ...

(b) In Δ PQR, the side included between $\angle$P and $\angle$Q is ....

(c) Two squares are congruent, if their .... are equal to each other.

(d) The scalene triangles can be ....to each other.

(e) The .... condition of congruence does not exist.

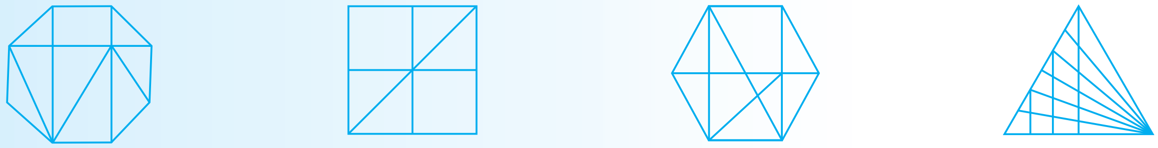
(f) In Δ ABC, m $\angle A = 40°$, m$\angle C = 70°$

If $\Delta ABC \cong \Delta DEF$, m$\angle$E= ....

(g) The .... condition of congruence is applied to right-angled triangles.

HOTS

Which of these contains the greatest number of triangles?



(a) (b) (c) (d)

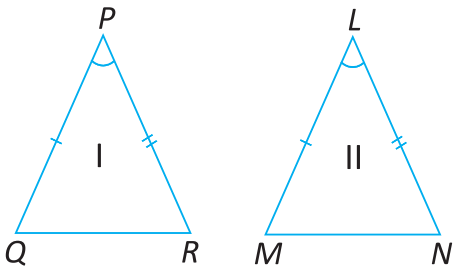
Lab Activity

Establishing Congruency of Triangles

Objective : To establish congruency of triangles by making paper models and superposing

Materials Required : Paper, pencil, ruler, and scissors

Draw two triangles, PQR and LMN Where $\overline{PQ} = \overline{LM}$, $\overline{PR} = \overline{LN}$, and $\angle P = \angle L$. Cut out these triangles.



Now we are ready to prove the following theorem.

'If two sides and the included angle of one triangles are respectively equal to two sides and the included angle of another triangle, the triangles are congruent'.

Procedure : Students may work individually or in pairs taking turns.

Step 1. Overlap $\overline{PQ}$ of triangle I with $\overline{LM}$ of triangle II. It will coincide exactly as they are equal.

Step 2. Overlap ∠P on to ∠L. They too will coincide as their measures are equal.

Step 3. Now, since $\overline{\text{PR}}$ and $\overline{\text{LN}}$ ar equal, they will coincide exactly and ∠R will coincide with ∠N.

Step 4. This means that $\overline{QR}$ and $\overline{MN}$ too will fall along each other.

Thus, triangle I coincides with triangle II.

Hence S.A.S. holds.

Record the Activity–

ΔPQR	ΔLMN	Equal/Not equal
∠P	∠L	
Side PQ	Side LM	
Side PR	Side MN	

Try out with different triangles having different sets of congruent parts.

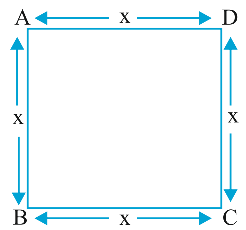
12 Perimeter and Area

In the previous class Thus, Perimeter is the measure around something we have already learnt about the perimeters of plane figures and areas of squares and rectangles. We learnt that perimeter signifies the distance around a closed figure, whereas area is the part of a plane or region occupied by the closed figure. Here, we will recapitulate what we have learnt last year and will try to learn more about perimeters and areas of some more plane figures.

Perimeter

Perimeter is the distance measured around a closed figure. The word perimeter is made of two words peri means **around** and **meter** means **measure**. It means perimeter is the measure around something. You will come across many everyday applications of the perimeter. For example, groundmen measure the perimeter of a cricket ground to fix the boundary. As perimeter signifies the distance around a figure, it is expressed in the different units like *m*, *cm*, *km* etc.

Perimeter of a Square



ABCD is a square of side measuring unit

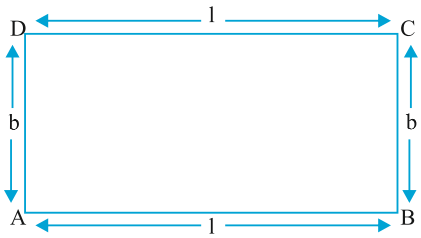
Perimeter (P) = AB + BC + CD + DA

= $x + x + x + x$

= $4x$

= 4 × side

Perimeter of a Rectangle



ABCD is a rectangle of length *l* units and breadth *b* units

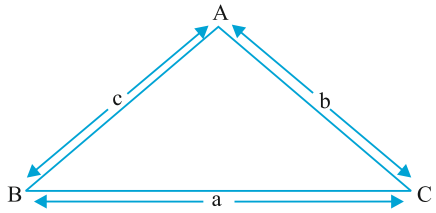
Perimeter (P) = AB + BC + CD + DA

= $l + b + l + b$

= $2(l + b)$

= 2 (length + breadth)

Perimeter of a Triangle



ABC is a triangle with sides measuring *a*, *zb* and *c* units.

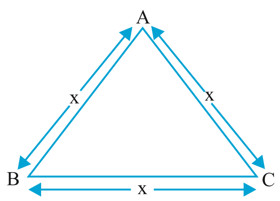
Perimeter (P) = AB + BC + CA

= $c + a + b$

= $a + b + c$

= sum of all sides

Perimeter of an equilateral triangle (all sides equal)

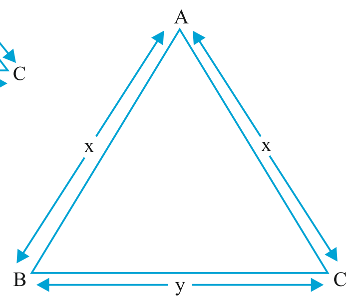


= AB + BC + CA

= $x + x + x$

= $3x$

Perimeter of an isosceles triangle (only two sides equal)



= AB + BC + CA

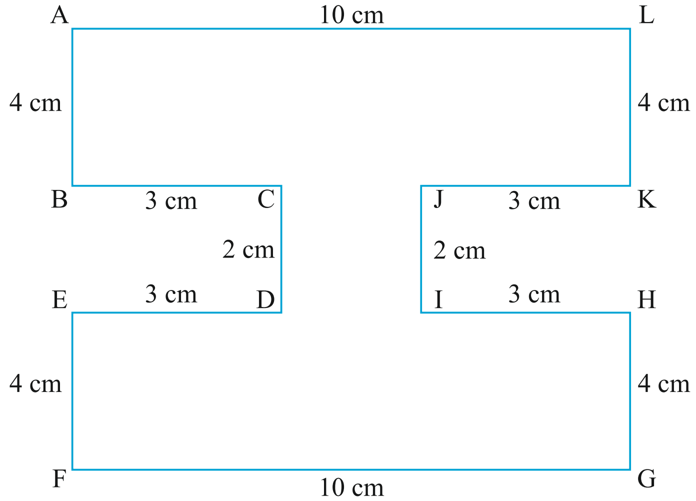
= $x + y + x$

= $2x + y$

Facts to Know

- The perimeter of any figure is the sum of all its sides.

Example 1 : Find the perimeter of the given figure.

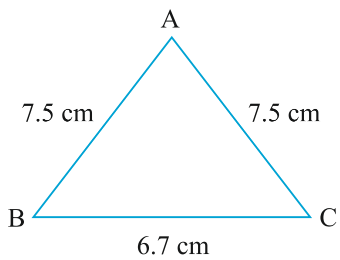


Solution : Perimeter = AB + BC + CD + DE + EF + FG + GH + HI + IJ + JK + KL + LA

= 4 cm + 3 cm + 2 cm + 3 cm + 4 cm + 10 cm + 4 cm + 3 cm + 2 cm + 3 cm + 4 cm + 10 cm

= 52 cm

Example 2 : Find the perimeter of an isosceles triangle that has its equal side 7.5 cm and third side 6.7 cm.



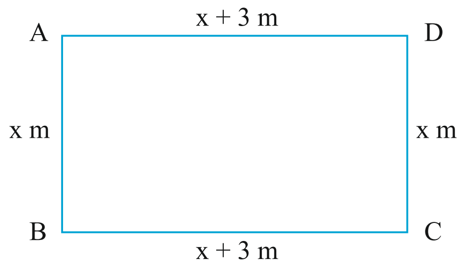
Solution : Perimeter of isosceles triangle

= AB + BC + CA

= 7.5 cm + 6.7 cm + 7.5 cm

= 21.7 cm.

Example 3 : Find the length and breadth of a rectangle, if its length is 3 m longer than its breadth and its perimeter is 86 m.



Solution : Let the breadth of the rectangle be x.

Length = $x + 3$ m

According to the question,

Perimeter of the rectangle = 2 (length + breadth)

$\Rightarrow$ 86 m = $2(x + 3 + x)$

$\Rightarrow$ $4x + 6 = 86$ m

$\Rightarrow$ $4x = (86 - 6)$ m

$\Rightarrow$ $x = \frac{80}{4} = 20$ m

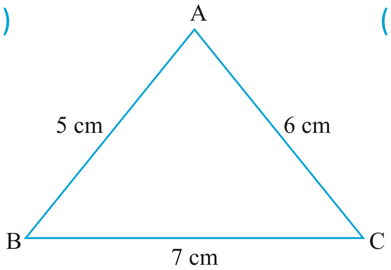
Hence, breadth = 20 m and length = 23 m.

Exercise 12.1

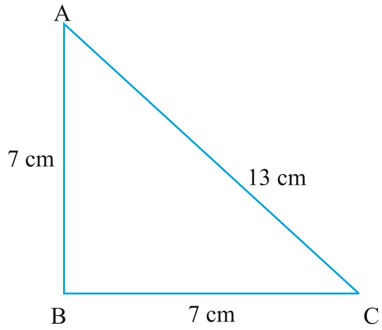
1. Find the perimeter of an equilateral triangle with its side measuring 7.2 m.
2. **Find the perimeter of the rectangles given that**
   (a) length = 7 cm and breadth = 9 cm. (b) length = 3.4 cm and breadth = 5.6 cm.

3. **Find the perimeter of the following triangles.**

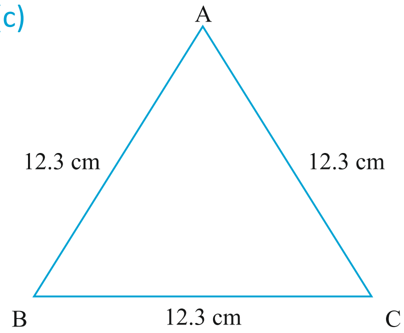
(a)



(b)

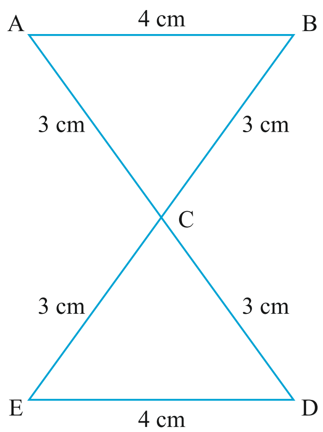


(c)

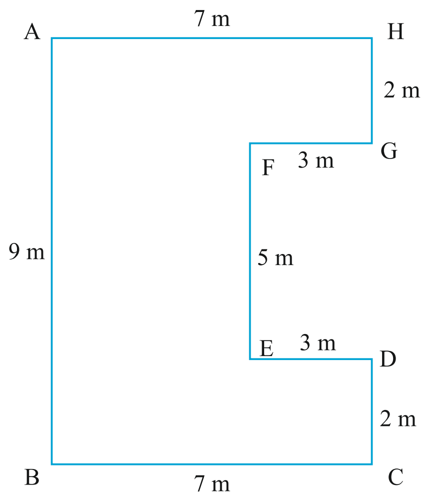


4. If the perimeter of an equilateral triangle is 111m, find the length of each side.
5. The length of equal sides of an isosceles triangle is 16.7 cm. Find the length of third side if its perimeter is 50 cm.
6. The length of a rectangular field is 3 times its breadth. If the perimeter of the field is 96 m, find its length and breadth.
7. The measure of two adjacent sides of a rectangle are in the ratio 4 : 3. If the perimeter of the rectangle is 686 cm, find its length and breadth.
8. Three sides of a triangle are in the ratio 2 : 3 : 4. Find the measurement of each side if its perimeter is 108 m.
9. If the perimeter of a regular pentagon is 105 cm, find the measure of its sides.
10. **Find the perimeter of the following figures :**

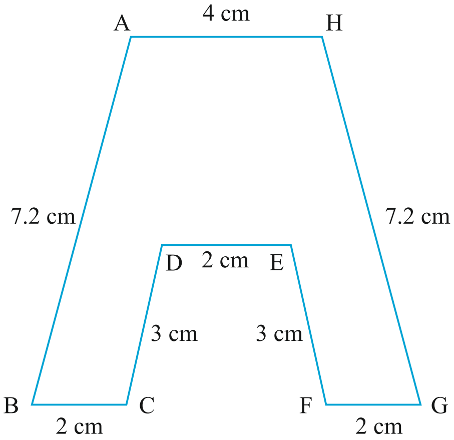
(a)



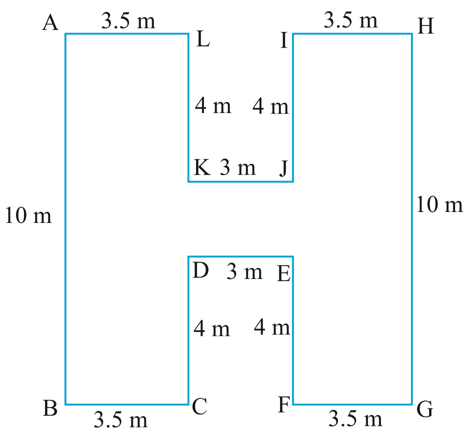
(b)



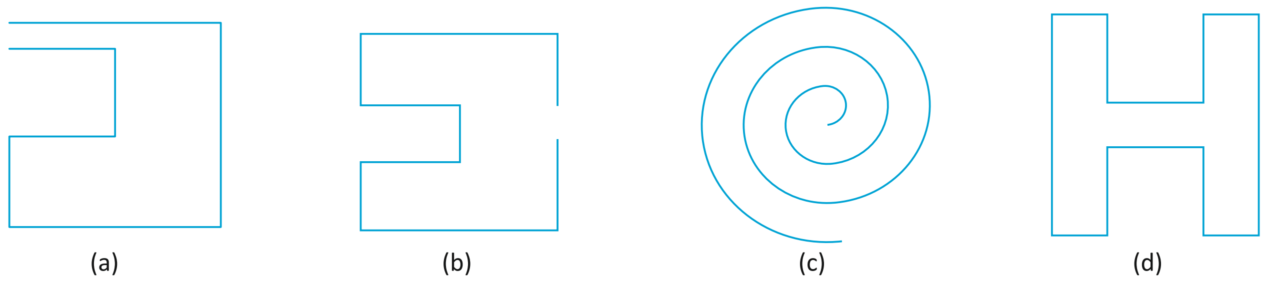
(c)



(d)



Concept of Area

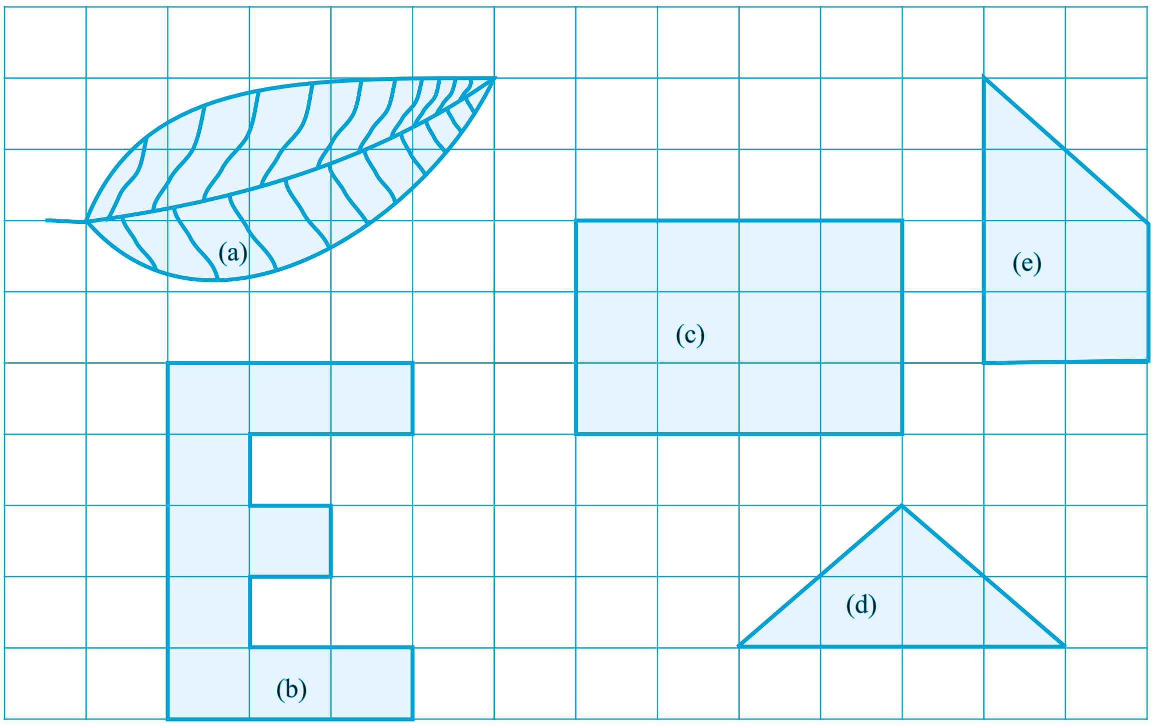


You are well versed with the concept of area. Do you think the above given figures represent area ?

The answer is obvious. Only figure (d) represents area as it is a closed figure. Rest of the figures are open ended. A closed figure occupies some amount of surface. The amount of the space within a closed figure is called its **area**.

To measure the area of a figure we generally find the numbers of square units contained in the figure. For example, a square that is 1 unit on all sides covers an area of 1 square units. If the unit of measurement is cm, then area is measured in square centimetre. Other area measuring units are like square decimetre, square metre, square kilometre etc. Generally we use square centimetre as standard unit of a square. It is briefly written as sq. cm or cm^2.

Look at the given figure :



Can you measure the area of the above given shapes ? The area of any figure can be measured by counting the number of square units the figure occupied. You can easily measure the area of figures (b) and (c) as the square units fit them evenly. Follow these steps to measure the area of (a), (d) and (e) :

- Count the full squares.
- Count the squares as one which are more than half.
- Neglect the squares which are less than half.
- Count the half squares as half units.

Area of a figure = No. of full squares + $\frac{1}{2}$ × No. of half squares + No. of squares which are less than half.

Let's find area of each figure separately.

Area of figure (a) = 3 + 7

= 10 sq. units

(∵ No. of full squares = 3 and squares more than half = 7)

Area of figure (b) = 10 sq. units

(∵ No. of full squares = 10)

Area of figure (c) = 12 sq. unit

(∵ No. of full squares = 12)

Area of figure (d) = $2 + \frac{1}{2}(4)$

= 4 sq. units

(∵ No. of full squares = 2 and half square = 4)

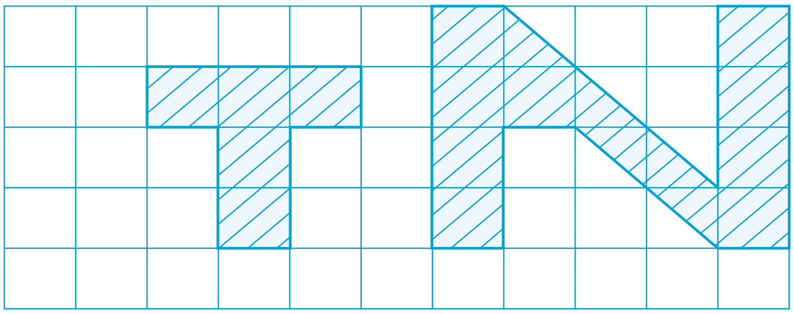
Area of figure (e) = $5 + \frac{1}{2}(2)$

= 6 sq. units

(∵ No. of full squares = 5 and half squares = 2)

You must be observing that it is difficult to measure the area enclosed by a figure if the square units don't fit into them evenly. Also, it is not always practical to measure the area of a particular figure by counting the number of units as it is not accurate and gives us only an estimated area. Therefore, we use generalised formula to find the area of a closed figure.

Example 4 : **Find the area of the given alphabets.**



Solution : Area of alphabet **T** = 5 sq. units

(∵ No. of full squares = 5)

Area of alphabet **N** = $9 + \frac{1}{2}(5)$

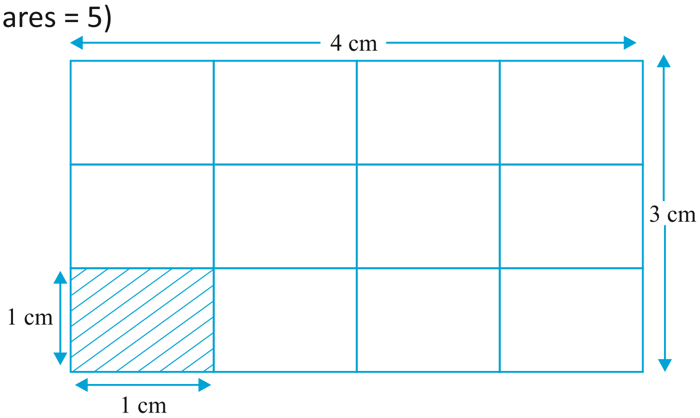
= $9 + 2\frac{1}{2}$

= $11\frac{1}{2}$ sq. units

(∵ No. of full squares = 9 and half squares = 5)

Area of a Rectangle

Let us take a rectangle with its length and breadth as 4 cm and 3 cm respectively.



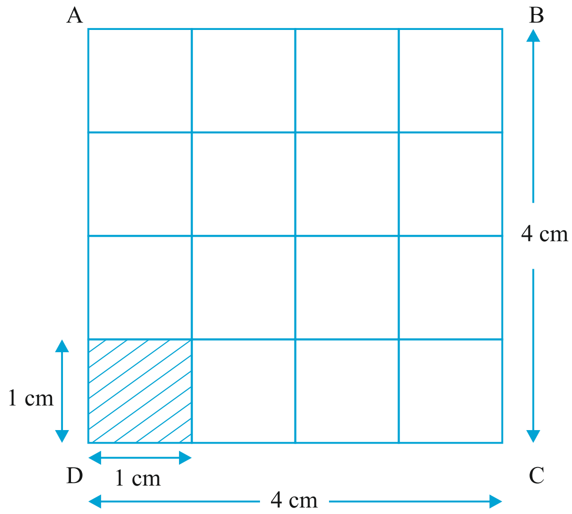
Divide the rectangle into unit squares such that each unit square is of 1 square centimetres. There are 12 such unit squares. We can say that the area of the rectangle is 12 sq. centimetres. Now, find the product of length and breadth of the rectangle and you will find that the product obtained is also 12 square centimetres. Thus, we conclude that,

Area of a rectangle = length × breadth

or, A = $l \times b$ where 'l' is the length and b is the breadth.

Area of a Square

Let us take a square of side 4 cm. Divide the square into unit squares such that each unit square is of 1 square centimetre (shown is figure by linings).



There are 16 such unit squares. We can say that the area of the square is 16 square centimetres. Now, find the area of this square by conventional method (i.e., side × side) and you will find that the area obtained is also 16 square centimetres.

Thus, we conclude that

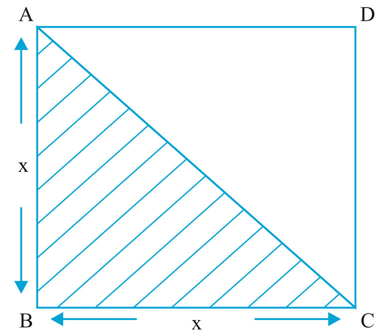
Area of a square = side × side

= side^2

= a^2, where a is the side of the square.

Area of a Right Isosceles Triangle

Let us take a square ABCD of side x units. Diagonal AC divides it into two congruent-isosceles triangles △ABC and △ADC, i.e. △ABC ≅ △ADC



Area of square ABCD = Area of △ABC + Area of △ADC

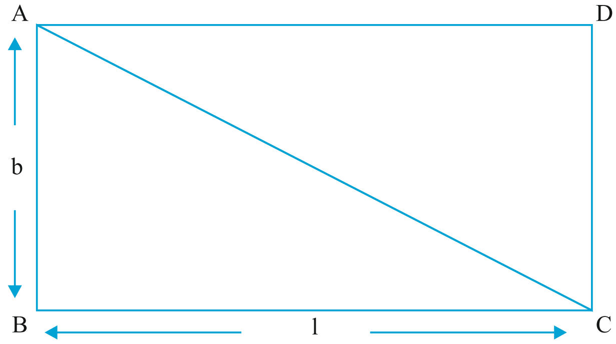
∴ x^2 = 2 Area of △ABC

∴ Area of △ABC $= \frac{1}{2}x^2$

Therefore, the area of right-isosceles triangle is half the square of its base.

Area of a Right-angled Triangle

Let us take a rectangle ABCD with its length and breadth as *l* units and *b* units respectively. Diagonal AC bisect it into two congruent right-angled triangle △ABC and △ADC, i.e., △ABC ≅ ADC



Area of given figure = Area of rectangle ABCD

⇒ Area of △ABC + Area of △ADC = length × breadth

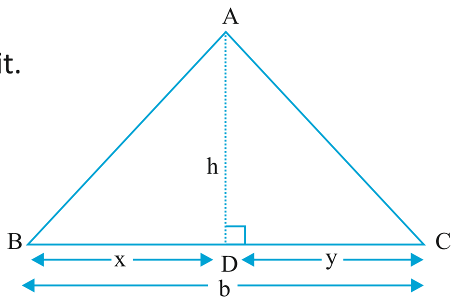
⇒ 2 × Area of △ABC = $l \times b$

⇒ Area of ABC = $\frac{1}{2} \times l \times b$

Therefore, the area of a right-angled of triangle is half the product of its base (*l*) and altitude (*b*).

Area of a Triangle

Let us take a triangle ABC with its base BC as *b* unit.



A perpendicular AD measuring *h* is drawn on line BC, such that it divides base BC into two parts BD and DC measuring *x* and y respectively. This perpendicular AD represents the altitude of △ABC.

Area of △ABC = Area of △ADB + Area of △ADC

$= \frac{1}{2} \times BD \times AD + \frac{1}{2} \times DC \times AD$

$= \frac{1}{2} \times AD \times (BD + DC)$

$= \frac{1}{2} \times h \times (x + y)$

$= \frac{1}{2} \times h \times b$ [∵ BC = BD + DC]

Facts to Know

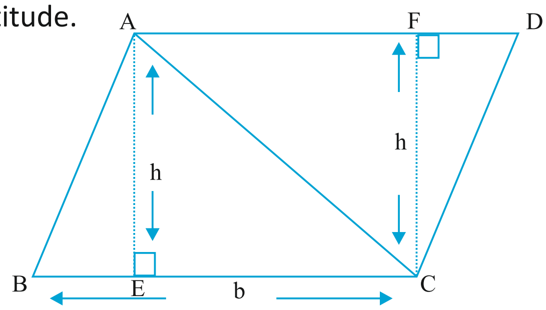
- All the congruent triangles are equal in area but the triangles equal in area need not be congruent.

or, **Area of △ABC** = $\frac{1}{2} \times b \times h$

Therefore, the area of a triangle is half the product of its base and altitude.

Area of a Parallelogram

Let us take a parallelogram ABCD with its base as *b* unit. Diagonal AC divide it into two triangle, △ABC and △CDA.



Draw AE ⊥ BC and CF ⊥ AD as shown in the figure.

For a parallelogram ABCD.

BC || AD, so AE = CF (say h unit)

And BC = AD = b unit

Area of parallelogram ABCD = Area of $\triangle$ABC + Area of $\triangle$CDA

$$= \frac{1}{2} \times BC \times AE + \frac{1}{2} \times AD \times CF$$

$$= \frac{1}{2} \times b \times h + \frac{1}{2} \times b \times h$$

$$= b \times h \; [\because BC = AD \text{ and } AE = CF]$$

or, **Area of a parallelogram** = $b \times h$

Therefore, the area of a parallelogram is the product of its base and altitude.

Example 5 : **Find the area of the following :**

(a) A rectangle with its sides measuring 16 m and 8 m.

(b) A square with its side measuring 19 m.

Solution : (a) Area of a rectangle = length × breadth

= 16 × 8

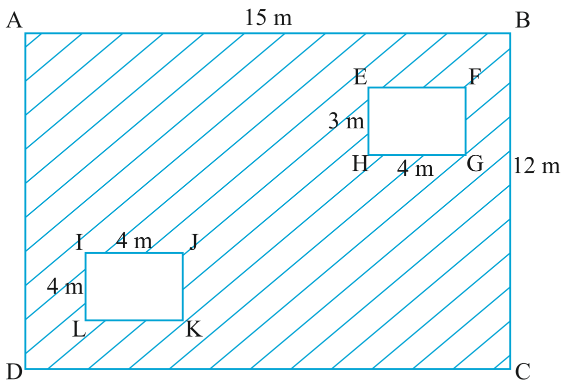
= 128 m^2

(b) Area of a square = $side^2$

= 19 × 19

= 361 m^2

Example 6 :



Find the area of shaded portion.

Solution : Area of rectangle ABCD = length × breadth

= 15 × 12

= 180 m^2

Area of rectangle EFGH = length × breadth

= 4 × 3

= 12 m^2

Area of square IJKL = $side^2$

= 4 × 4

= 16 m^2

Area of shaded portion = Area of rectangle ABCD – (Area of rectangle EFGH + Area of square IJKL)

= 180 – (12 + 16)

= 180 – 28

= 152 m^2

Example 7 : **Find the area of a right angled isosceles triangle which has its equal sides measuring 16 m.**

Solution : Area of right angled isosceles triangle $\frac{1}{2}$ = × (equal side)^2

$= \frac{1}{2} \times 16 \times 16$

$= 128\ m^2$

Example 8 : **Find the area of a rectangular field if its perimeter is 210 m and its length and breadth are in the ratio 4 : 3.**

Solution : Let length and breadth of the rectangle be 4x and 3x respectively.

Perimeter of a rectangle = 2 (length × breadth)

$\Rightarrow$ 210 m = 2 (4x + 3x)

$\Rightarrow$ 7x = 105 m

$\Rightarrow$ x = 15 m

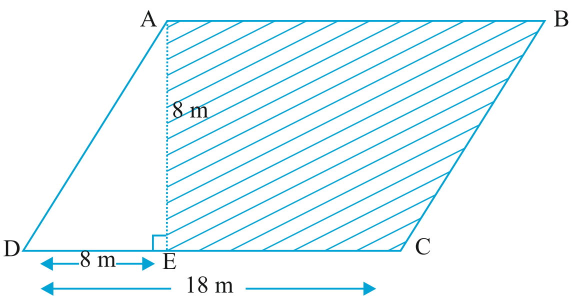
Length = 4x = 60 m and breadth = 3x = 45 m

Area of a rectangle = length × breadth

= 60 × 45

= 2700 m^2

Example 9 :



If ABCD is a parallelogram, find the area of the shaded portion.

Solution : Area of parallelogram ABCD = base × altitude

= 18 × 8

= 144 m^2

Area of $\triangle$AED $= \frac{1}{2}$ × base × altitude

$= \frac{1}{2} \times 8 \times 8$

$= 32\ m^2$

Area of shaded portion ABCD = Area of parallelogram ABCD – Area of ΔAED

= 144 – 32 = 112 m^2

Example 10 : **The base and the altitude of a triangle are in the ratio 3 : 4 and its area is 1350 m^2. Find its base and altitude.**

Solution : Let the base and the altitude of the triangle be 3x and 4x respectively.

Area of a triangle $= \frac{1}{2}$ × base × altitude

$\Rightarrow$ $1350 = \frac{1}{2} \times 3x \times 4x$

$\Rightarrow$ $6x^2 = 1350$

$$\Rightarrow \quad x^2 = \frac{1350}{6} m^2$$

$$\Rightarrow \quad x^2 = 225 m^2$$

$$\Rightarrow \quad x = 15 \text{ m}$$

Base = $3x$ = 45 m and altitude = $4x$ = 60 m

Exercise 12.2

1. Find the area of the following rectangles :

(a) Length = 15 m ; Breadth = 8 m

(b) Length = 7.5 m ; Breadth = 6 m

(c) Length = 12 m ; Breadth = 7.5 m

(d) Length = 14.5 m ; Breadth= 4 m

2. Find the area of a square ABCD, given that :

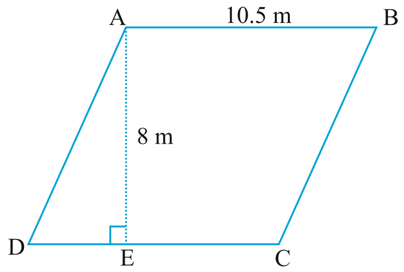
(a) BC = 7.5 m (b) AB = 13 m (c) CD = 7.2 m (d) DA =12 m

3. Find the area of △ ABC, right angled at B and AB and BC measuring 16 cm and 10 cm respectively.

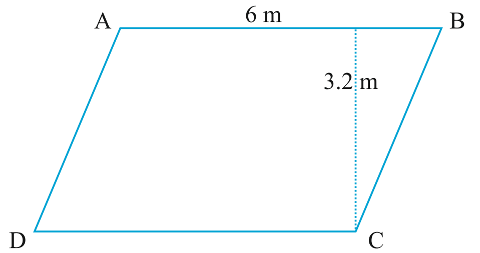
4. The cost of ploughing a rectangular field at the rate of ₹4 per m^2 is ₹ 1280. If the length of the field is 20 m, find its width.

5. Find the area of the following parallelograms.

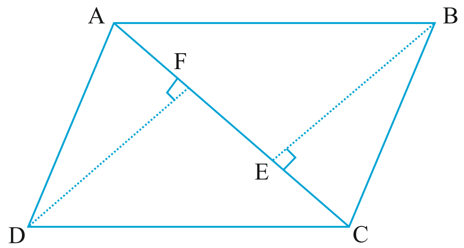
(a)



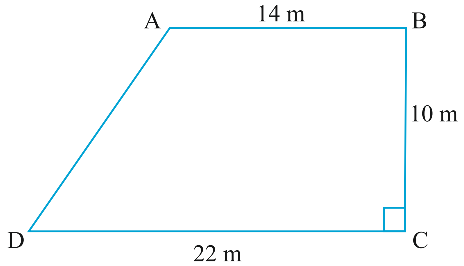
(b)



6. Find the area of a parallelogram ABCD, if AC = 36 m and BE = DF = 12 m.

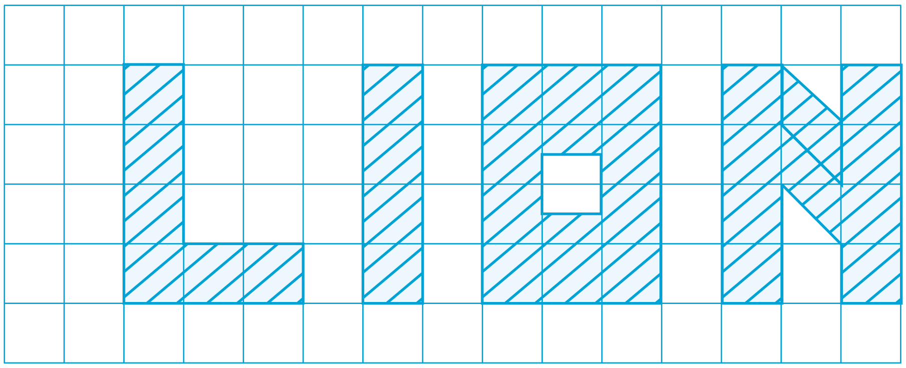


7. Find the area of the quadrilateral ABCD as shown in the figure.

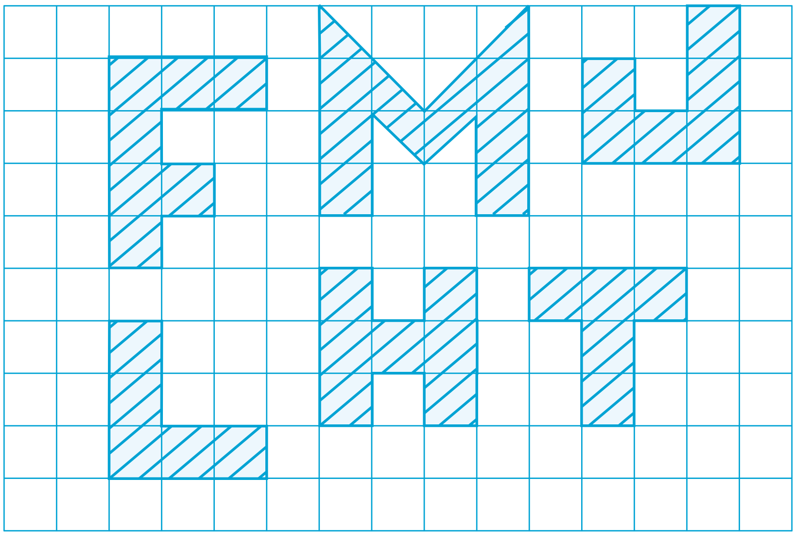


8. The base and the altitude of a parallelogram are in the ratio 3 : 2. Find the measures of its base and altitude if the area of the parallelogram is 3750 m^2.

9. Following figure have been drawn on squared paper. Find the area of shaded word (LION). Take each square as 1 cm^2.



10. Find the area of the shaded alphabets drawn on squared paper. (Take each square as 1 cm^2)

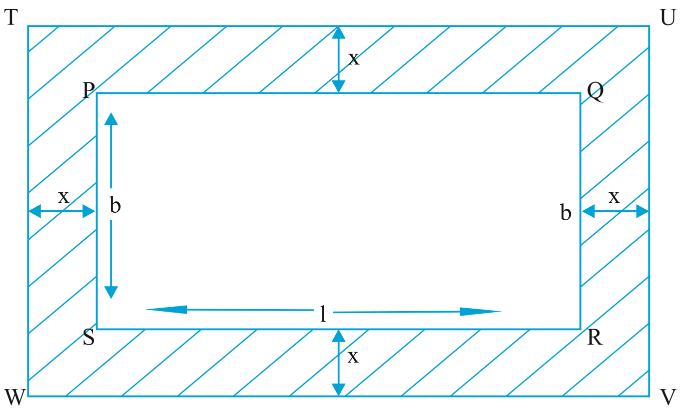


11. The length and breadth of a rectangular field are 64 m and 16 m respectively. If it is exchanged with a square field of the same area, find the side of the square field.

12. The cost of ploughing a triangular field at the rate of ₹7 per m^2 is ₹6300. If the base of the field is 25 m, find its altitude.

AREA BETWEEN TWO RECTANGLES

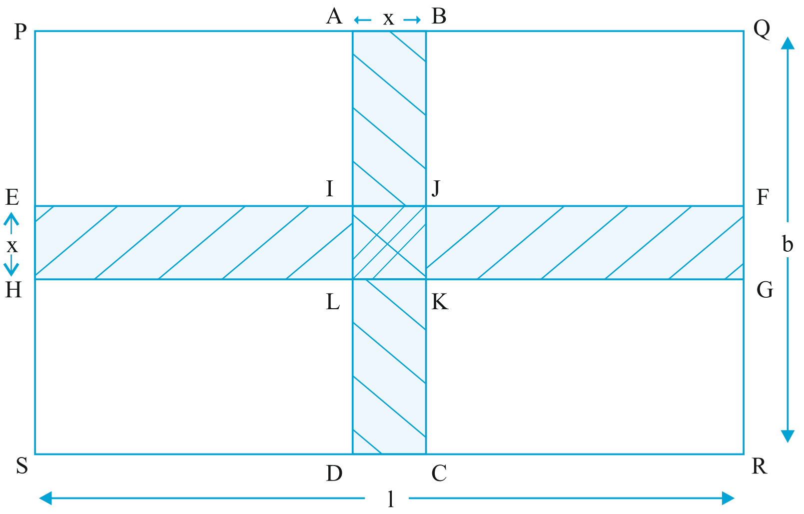
Let us take a rectangular field PQRS with length *l* units and breadth *b* units. A path of *x* units width runs around the field as shown below :



Here, length of the outer rectangle TUVW is TU $= x + l + x = l + 2x$
Breadth of the outer rectangle TUVW is UV $= x + b + x = b + 2x$
Area of outer rectangle $= (l + 2x) \times (b + 2x)$
Area of inner rectangle $= l \times b$
Area of path = Area of outer rectangle – Area of inner rectangle.
$= [(l + 2x)(b + 2x) - l \times b]$ sq. units

Area of Cross Roads

Let us take a rectangular field PQRS with length l units and breadth b units. Two paths of width x units run across in the middle of a garden, one parallel to the length and the other parallel to the breadth as show in the figure.

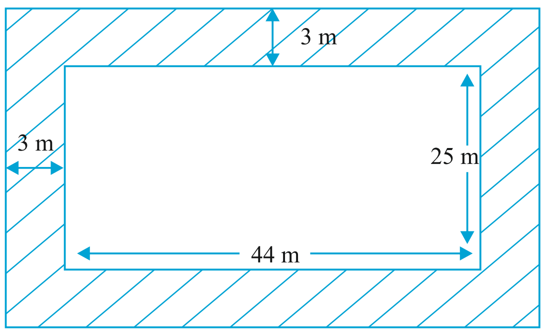


Area of path EFGH parallel to length $= l \times x$ sq. units
Area of path ABCD parallel to breadth $= b \times x$ sq. units

Here, the total area of the path can not be equal to the sum of the areas of paths along its length and breadth because the area of junction IJKL is added twice.

Hence,
Area of path = Area of path along the length + Area of the path along breadth – Area of junction
= Area of EFGH + Area of ABCD – Area of IJKL
$= (l \times x) + (b \times x) - (x \times x)$ sq. units

Example 11 : A rectangular plot measuring 44 m × 25 m is surrounded externally by a 3 m wide path. Find the area of the path.



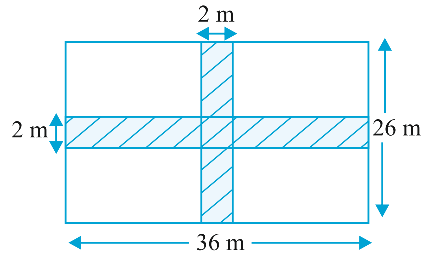
Solution : Area of inner rectangular plot
$= 44 \times 25$
$= 1100 \text{ m}^2$
Length of outer rectangle
$= 44 + 3 + 3$
$= 50$ m
Breadth of outer rectangle $= 25 + 3 + 3$
$= 31$ m
Area of outer rectangular plot = length × breadth
$= 50 \times 31$
$= 1550 \text{ m}^2$
Area of path = Area of outer rectangle – Area of inner rectangle
$= 1550 - 1100$
$= 450 \text{ m}^2$

Example 12 : **A rectangular lawn is 36 m × 26 m. It has two roads, each 2 m wide running in the middle of it one parallel to the length and other parallel to the breadth. Find the area of the roads.**

Solution : Area of path parallel to length

$= 36 \times 2$

$= 72\ m^2$



Area of path parallel to breadth

$= 26 \times 2$

$= 52\ m^2$

Total area of path

= Area of path along the length + Area of path along the breadth – Area of Junction.

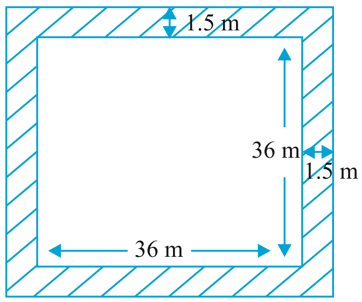
$= 72 + 52 - 4$

$= 120\ m^2$

Example 13 : **A path 1.5 m wide is running around a square field whose side is 36 m. Determine the area of the path.**

Solution : Area of inner square $= 36^2$

$= 1296\ m^2$



Side of the outer square

$= 36 + 1.5 + 1.5 = 39$ m

Area of outer square $= 39^2$

$= 1521\ m^2$

Area of path = Area of outer square – Area of inner square

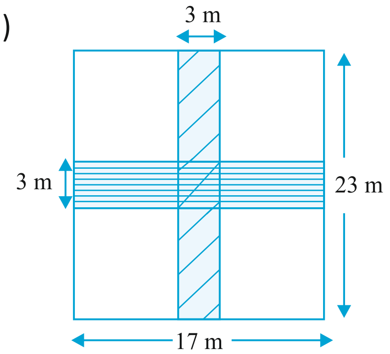
$= 1521 - 1296$

$= 225\ m^2$

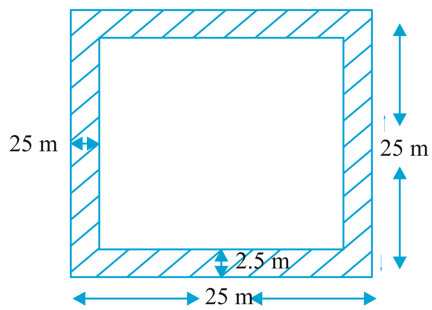
Exercise 12.3

1. Find the area of the path of the following.

(a)



(b)



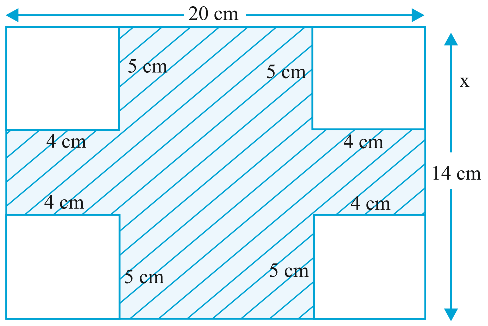
2. A garden is 44 m long and 40 m broad. A path 2.5 m wide is to be added outside around it. Find the area of the path.

3. A rectangular field measuring 36 m × 24 m is it be surrounded externally by a path which is 2.5 m wide. Calculate the cost of constructing this path at the rate of ₹ 4.5 per m^2.

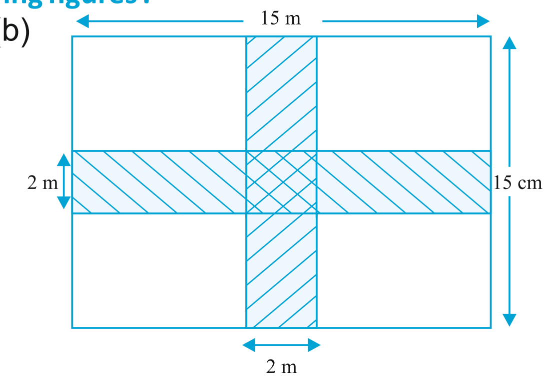
4. Two paths of 5 m wide running perpendicular to each other in the centre of a rectangular path 80 m long and 70 m wide. Find the area of the remaining park.

5. A 2 m wide flower bed is to be constructed around a football ground of dimension 80 m by 60 m. Find the cost of gardening the beds at the rate of ₹ 8 m^2.
6. Two crossroads, each of width 4 m, run at right angles through the centre of a rectangular park of length 60 m and width 45 m and parallel to its sides. Find the cost of constructing the roads at the rate of ₹ 225 per m^2.
7. A sheet of paper measures 36 cm by 24 cm. A strip of 2.5 cm wide is cut from it all round. Find the cost of colouring the remaining sheet at the rate of ₹ 2.5 per cm^2.
8. A rectangular plot is 50 m long and 45 m broad. A path 4 m wide is to be built all around it along its border. Find the area of the path.
9. **Find the area of the shaded region in each of the following figures :**

(a)

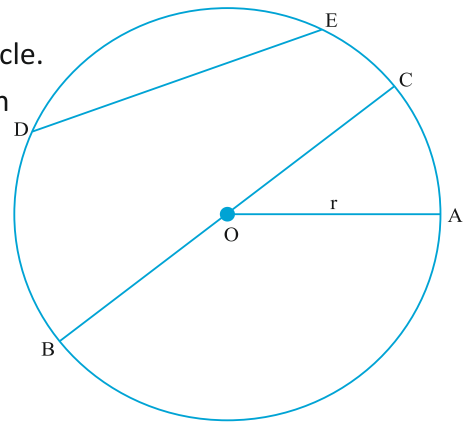


(b)



CIRCUMFERENCE OF A CIRCLE

We are well versed with the word 'circle'. Let's recall the basics of circle once again. A **circle** is described as a closed figure in a plane in which all the points on its boundary are equidistant from a fixed point. This fixed point is called the **centre** of a circle. It is generally denoted by O.

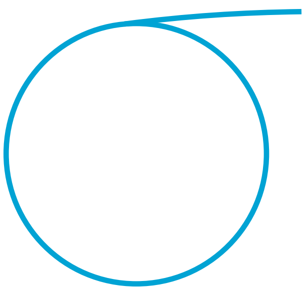


Radius is the line segment joining the centre of a circle to any point on the circle. It is denoted by 'r'. **Diameter** of the circle is the line segment passing through the centre of the circle with its end points lying on the circle. It is generally denoted by 'd'. **Chord** of a circle is a line segment joining any two points on a circle. In the given figure is the centre, OA is the radius and BC is the diameter of the circle. DE and BC are the chords of the given circle. In a circle, its diameter is always twice of its radius.

$$d = 2r$$

$$\text{Diameter} = 2 \times \text{radius}$$

The perimeter of a circle is called its **circumference**. It is the distance around a circular region. It is denoted by 'c'.



Let's find the circumference of circular hard-board without using the formula. Put the circular hard-board on a table and put a string around it completely. Now spread this thread as straight line and measure its length using a scale. This measurement is measure from the circumference of the hard-board.

Have a look at the given table. Draw circles of radii 2.5 cm, 3 cm and 3.5 cm and find their circumference by using string. Fill the rest of blanks.

Radius	Diameter	Circumference	Ratio of circumference to diameter
2.5 cm	5 cm	15.714 cm	
3 cm	6 cm		
3.5 cm	7 cm	22 cm	3.14

You will observe that

$$\frac{\text{Circumference}}{\text{Diameter}} = \frac{c}{d} = constant$$

This ratio is a constant and is denoted by π (pi).

Its approximate value is $\frac{22}{7}$ or 3.14.

Now, $\frac{c}{d}$ = constant

or, $\frac{c}{d} = \pi$

or, $c = \pi d$

or, $c = 2\pi r$ ($\because$ Diameter = 2 × radius)

Thus, **circumference** = 2 × π × radius

Example 14 : **The radius of a circular pipe is 28 cm. What length of tape is required to wrap it twice around the pipe?**

Solution : Radius of the pipe (r) = 28 cm

Circumference of the pipe $= 2\pi r$

$= 2 \times \frac{22}{7} \times 28$

= 176 cm

So, length of the tape needed to wrap twice around the pipe = 2 × 176 = 352 cm

Example 15 : **Julie goes for morning walk daily. If she takes 7 rounds of a circular park of radius 42 m, how much distance does he actually cover ?**

Solution : Radius of the circular park (r) = 42 m

Circumference of the park $= 2\pi r$

$= 2 \times \frac{22}{7} \times 42$

= 264 m

So, total distance covered by Julie = 7 × 264 = 1848 m

Example 16 : **Circumference of a circular boundary is 770 m. How much distance a kid will cover if he has to reach at the centre from the boundary ?**

Solution : Circumference of circular boundary = 770 m

$\Rightarrow 2\pi r = 770$

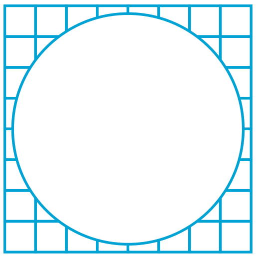
$\Rightarrow \quad r = \dfrac{770 \times 7}{2 \times 22}$

$\Rightarrow \quad r = 122.5$ m

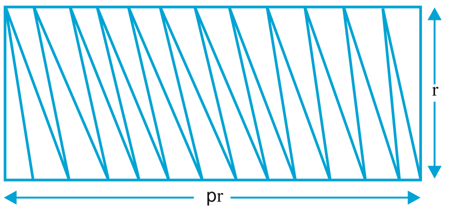
So, the kid will cover 122.5 m to reach at the centre.

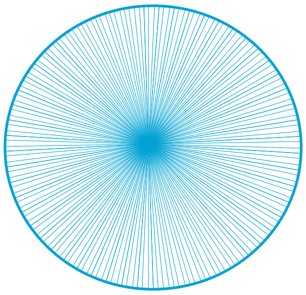
AREA OF A CIRCLE

There are more than one method to find the area of a circle. We can find its area by drawing it on a graph paper and counting the numbers of enclosed squares. We only get a rough estimate of the area of circle as its edges are not straight.



Let's see another method of finding the area of a circle. Draw a circle of any measurement and divide it in maximum numbers of sectors and arrange these as shown. It represents almost like a rectangle with length measuring πr and breadth measuring r.





Area of the circle = Area of rectangle thus formed

= length × breadth

= $\pi r \times r = \pi r^2$

Thus, the area of the circle = πr^2 unit ^{2}

Example 17 : Find the area of a circular cricket ground whose diameter is 56 m.

Solution : Diameter of the cricket ground (d) = 56 m

So, radius (r) $= \dfrac{56}{2} = 28$ m

Area of a circle (A) $= \pi\ r^2$

$\Rightarrow \quad = \pi \times (28)^2$

$\Rightarrow \quad = \dfrac{22}{7} \times 28 \times 28 = 2464\ m^2$

Thus, area of the cricket ground = $2464\ m^2$

Example 18 : The circumference of a circle is 88 cm. Find its area.

Solution : Circumference = 88 cm

$\Rightarrow \quad 2\pi r = 88$

$\Rightarrow \quad r = \dfrac{88}{2\pi}$

$\Rightarrow \quad r = \dfrac{88 \times 7}{2 \times 22}$

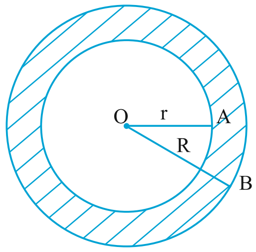
$\Rightarrow \quad r = 14$ cm

Area of circle (A) = πr^2

$= \frac{22}{7} \times 14 \times 14 = 616 \text{ m}^2$

Area between two Concentric Circles

Circles with the same centre are called **concentric circles**. Observe the given figure. Both circles, inner circle with radius OA = r and outer circle with radius OB = R, have the same centre as O. So, these circles are concentric.



Area of shaded region = Area of outer circle – Area of inner circle

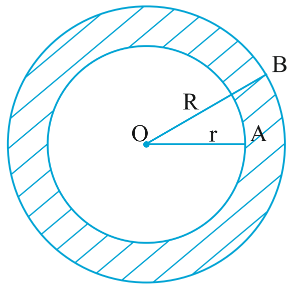
$= \pi R^2 - \pi r^2$

$= \pi (R^2 - r^2)$

$= \pi (R + r)(R - r)$

or, $A = \pi (R + r)(R - r)$

Example 19 : **The given figure shows two circles with the same centre. The diameter of the larger circle is 36 m and the radius of the smaller circle is 12 m. Find the area of the shaded portion.**



Solution : Diameter of the larger circle = 36 m

Radius of the larger circle (R) = $\frac{36}{2}$ = 18 m

Area of the larger circle $= \pi R^2 = \pi \times 18^2$

Radius of smaller circle (r) = 12 m

Area of the smaller circle $= \pi \times 12^2$

Area of the shaded portion = Area of larger circle – Area of smaller circle

$= \pi \times 18^2 - \pi \times 12^2$

$= \pi (18^2 - 12^2)$

$= \frac{22}{7} \times (18 + 12)(18 - 12)$

$= \frac{22}{7} \times 30 \times 6$

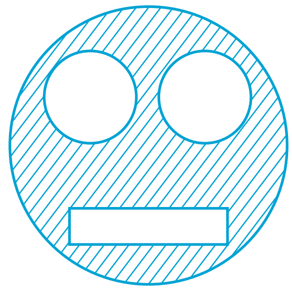
$= 565.714 \text{ m}^2$

Thus, the area of the shaded portion = 565.714 m^2

Exercise 12.4

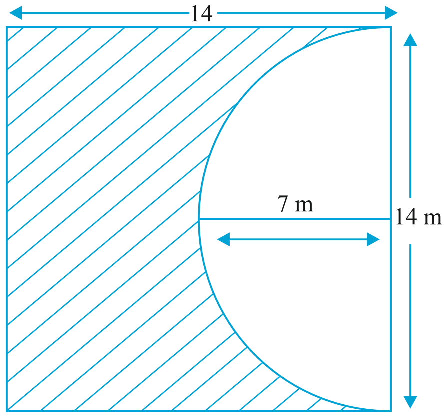
1. **Find the circumference of a circle whose radius is :**
   (a) 28 cm (b) 3.5 cm (c) 10.5 m (d) 4.2 cm
2. **Find the circumference of a circle whose diameter is :**
   (a) 7 m (b) 4.2 cm (c) 84 m (d) 14 m
3. If the circumference of a circular sheet is 308 m, find its area.
4. The perimeter of a square-shaped wire is 88 cm. If it is shaped into a circle, find its area.

5. The radius of the given figure is 35 m. Two circles of radii 3.5 m and 2.8 m, and rectangle of length 7 m and breadth 4 m are removed. Find the area of the shaded portion.

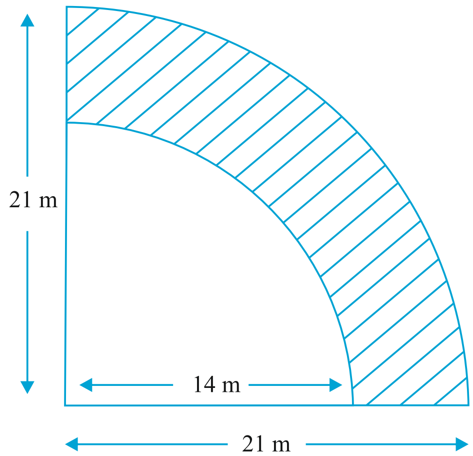


6. A circle of radius 10.5 cm is cut out from a square piece of an iron sheet of side 23 cm. What is the area of the left over iron sheet?
7. Two cows are tied with a rope to a pole in the middle of a grass field. The radius of the ropes are 18 m and 11m respectively. Find the difference in their area covered while grazing the grass.
8. The diameter of the flower bed is 56 cm. if this has to be surrounded by a path 7 cm wide, what is the area of this path?
9. Find the area of a circular ring whose inner and outer radii are 22 cm and 13 cm.
10. **Find the area of the shaded portion :**

(a)



(b)



Points to Remember

- Perimeter is the distance measured around a closed figure.
- Perimeter of a square = 4 × side, Perimeter of a rectangle = 2 (length + breadth).
- Perimeter of scalene triangle = a + b + c, isosceles triangle = 2a + c (equal sides measure a), equilateral triangle = 3a (all sides measure a)
- Area is the amount of space within a closed figure.
- Area of rectangle = length × breadth, Area of square = side^2, Area of parallelogram = base × altitude.
- Area of triangle = $\frac{1}{2}$ × base × altitude
- Area of path around the rectangular field = Area of outer rectangle – Area of inner rectangle.
- Area of crossroads = Area of path along length + Area of path along breadth – Area of junction.
- The ratio of circumference and diameter of any circle is always constant. It is known as π (pie). Its value is $\frac{22}{7}$ = 3.14 (approx)
- Circumference of circle = π × diameter.
- Area of circle = πr^2.
- Area between two concentric circles with radii Rand r = $\pi (R^2 - r^2) = \pi (R + r) (R - r)$

EXERCISE

1. MULTIPLE CHOICE QUESTIONS (MCQs):

Tick (✓) the correct options.

(a) Perimeter of an isosceles triangle is

(i) a + b + c ☐ (ii) abc ☐ (iii) a + a + b ☐ (iv) 2ab ☐

(b) If the side of a square becomes thrice, then its area becomes

(i) 6 times ☐ (ii) 9 times ☐ (iii) 3 times ☐ (iv) 27 times ☐

(c) If the length of a rectangle is multiplied by 4 and its breadth is divided by 2, the area increases by

(i) 2 times ☐ (ii) 4 times ☐ (iii) 6 times ☐ (iv) no change ☐

(d) Area of parallelogram is

(i) × side × side ☐ (ii) side^2 ☐ (iii) 2 (l + b) ☐ (iv) base × altitude ☐

(e) The area of a square field is 196 m^2, its side is

(i) 13 m ☐ (ii) 14 m ☐ (iii) 56 m ☐ (iv) $49m^2$ ☐

(f) The perimeter of an isosceles triangle with equal sides 14 m and other side 13 m is

(i) 40 m ☐ (ii) 41 m ☐ (iii) 182 m ☐ (iv) 27 m ☐

(g) The ratio of the radii of two circles is 3 : 4. The ratio of their perimeter is

(i) 9 : 16 ☐ (ii) 16 : 9 ☐ (iii) 4 : 3 ☐ (iv) 3 : 4 ☐

(h) The diameter of a circular lawn whose circumference is 154 m is

(i) 14 m ☐ (ii) 28 m ☐ (iii) 49 m ☐ (iv) 50 m ☐

2. The length of a rectangular field is 2.5 times its breadth. If the perimeter of this field is 119 m, find its length and breadth.

3. Three sides a triangle are in the ratio 3 : 2 : 2. Find the measurement of each side if its perimeter is 84 m.

4. The cost of putting an embankment around a square shaped pond at the rate of ₹ 15/ m is ₹ 3360. Find the measure of its side.

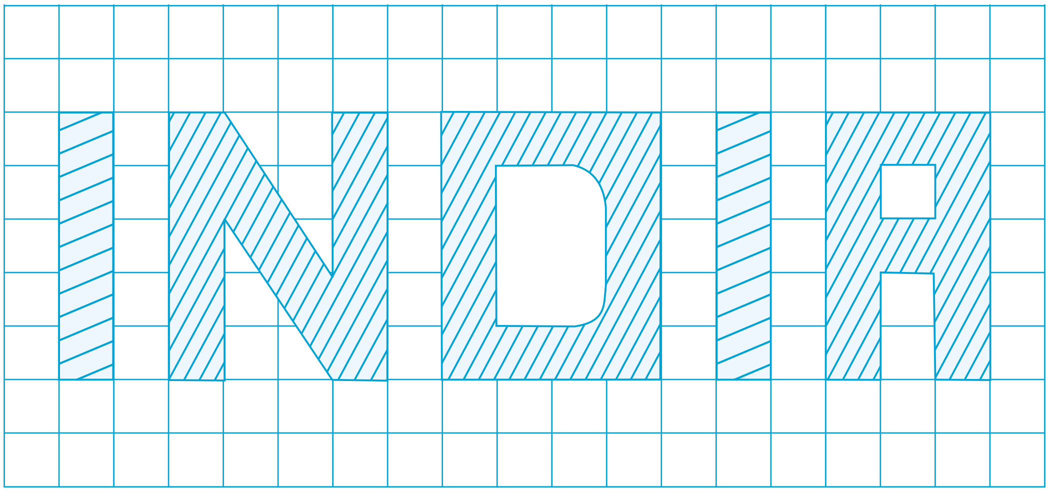
5. Rahul and Shweta are health conscious. Rahul on morning walk takes 5 rounds of a rectangular park measuring 80 m long and 75 m wide. Shweta goes for evening walk and complete 4 rounds of a square shaped garden with side 80 m. Who covers more distance and how much ?

6. Find the area of a rectangle field if its perimeter is 450 m and its length and breadth are in the ratio 5 : 4.

7. The length and breadth of a rectangle are in the ratio 5 : 2. If the area of this parallelogram is 490 m^2 find its perimeter.

8. The side of a square-shaded field is 72 m. Its perimeter is exchanged with a rectangular field of length 48 m. Find the area of this rectangular field.

9. Find the area of the shaded alphabets drawn on squared paper. (Take each square as 1 cm^2)



10. A sheet of paper measures 24 cm by 18 cm. A strip of 2 cm is cut from it all around. Find the cost of colouring the remaining sheet at the rate of ₹ 4 per cm^2.

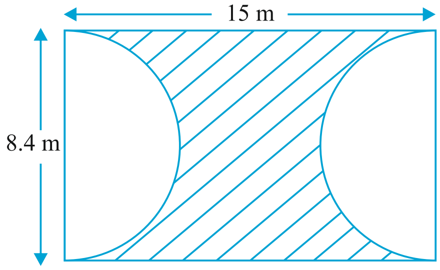
11. A rectangular park is 80 m × 60 m. It has two footpaths, each 3 m wide running in the middle of it - one parallel to the length and other parallel to the breadth. Find the area of the foot paths.

12. Two paths each of width 5 m, are running perpendicular to each other in the centre of a rectangular park measuring75 m by 55 m. Find the area of the remaining park.

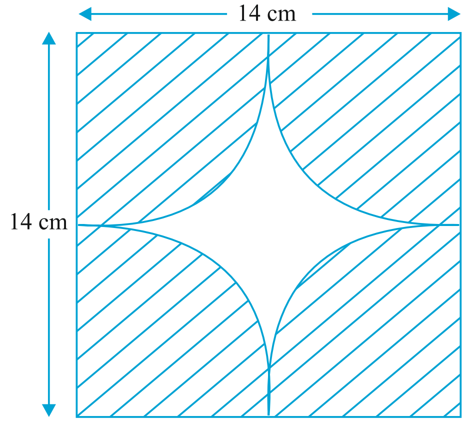
13. An athlete complete 8 rounds of circular field of radius 105 m. Find the total distance covered by him.

14. Find the area of the shaded portion :

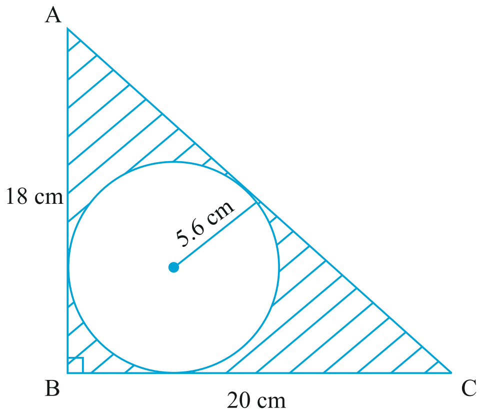
(a)



(b)

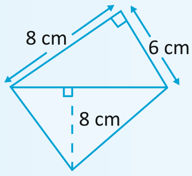


(c)

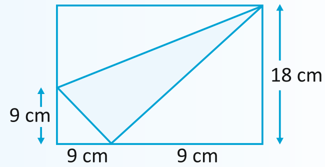


HOTS

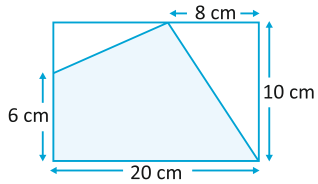
Find the area of the shaded portions in each of the following:



(a)



(b)



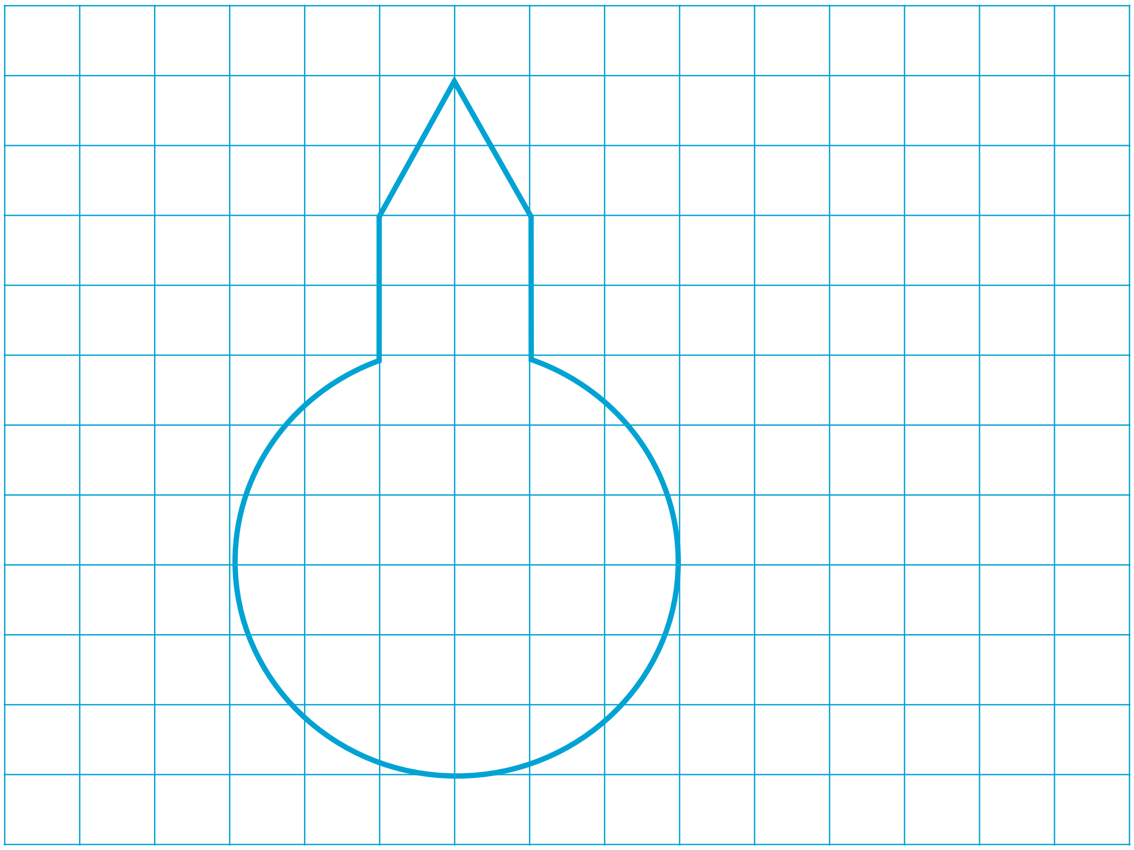
(c)

Lab Activity

Objective : To determine the area of closed figures by counting the numbers of squares on a graph sheet.

Materials Required : Graph sheet, pencil.

Procedure : Step 1. Take a graph sheet and make a closed figure of your choice. A figure is drawn for your convenience.



Step 2. Count the numbers of full squares. Here it is 22.

Step 3. Count the squares as one which are more than half. These are 14.

Step 4. Neglect the squares which are less than half.

Step 5. Count the half squares as half units. Here there are none.

Step 6. Add the counted numbers of step 2, step 3 and step 5.

Step 7. Area of the given figure = 22 + 14 + 0

= 35 square units.

13 Construction Geometry

In the previous class, we learnt the use of ruler, protractor, divider, compasses and set square to draw some geometrical figures. Here we shall take some other geometrical constructions, using ruler and compasses only. Let's first recapitulate what we learnt in class VI and then take up the construction of a triangle in details.

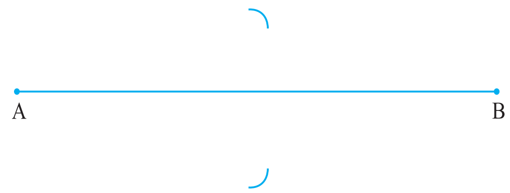
To Bisect a Given Line Segment

Bisecting a line segment means to divide a given line segment into two equal parts. Let's bisect a line segment AB measuring 6 cm at a point O.

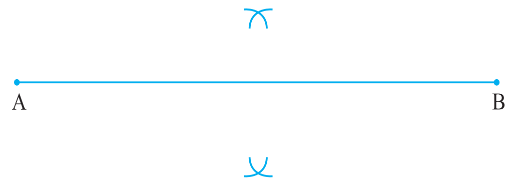
Steps of construction:

Step 1 : Using a scale draw a line segment AB = 6 cm.

Step 2 : Using compasses, with A as centre and radius more than half of AB, draw two arcs above and below line segment AB.

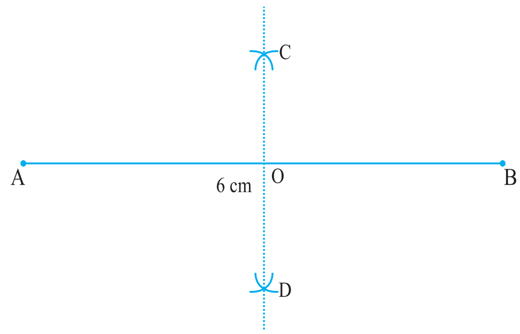


Step 3 : With B as centre and taking the same radius, draw two arcs above and below line segment AB, cutting the earlier drawn arcs at point C and D.



Step 4 : Join C and D cutting line segment AB at point O. Thus, CD intersects AB at point O dividing it into two equal parts.

Thus, AO = OB = 3 cm.

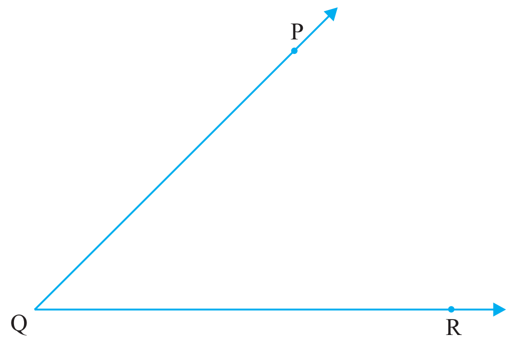


To Bisect Given Angle

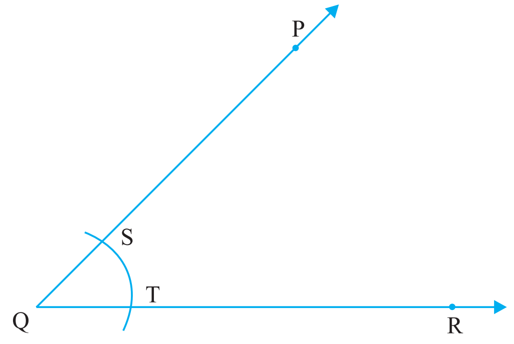
Bisecting an angle means to divide a given angle into two equal angles. Let's bisect ∠PQR.

Steps of construction:

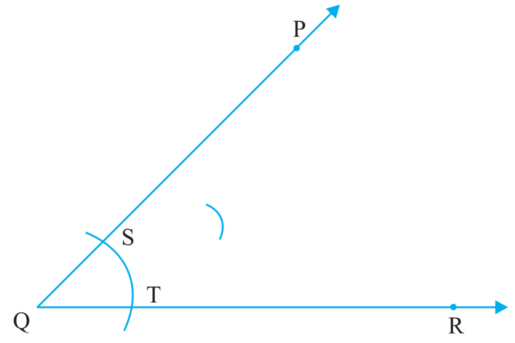
Step 1 : Draw ∠PQR of any measurement.



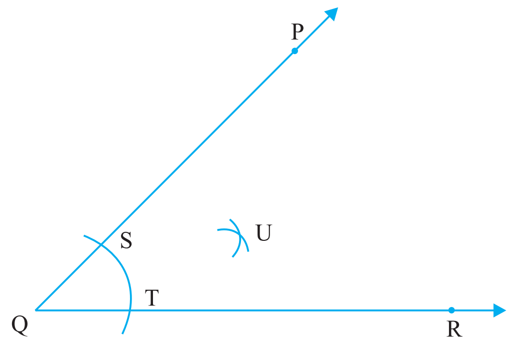
Step 2 : Using compasses with vertex Q as centre and a reasonable radius, draw an arc cutting PQ and QR at points S and T respectively.



Step 3 : With S as centre and radius more than half of ST, draw an arc in the interior part of ∠PQR.

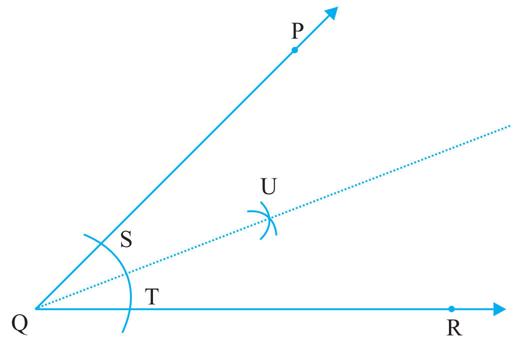


Step 4 : With T as centre and the same radius, draw another arc, cutting the earlier drawn arc at point U.



Step 5 : Join QU. $\overrightarrow{QU}$ is the required bisector of ∠PQR.

Thus, ∠PQU = ∠RQU

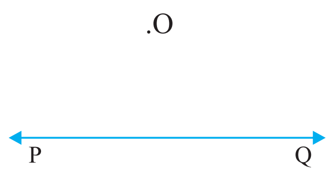


To Draw a Line Perpendicular to a Given Line from a Point not Lying on it

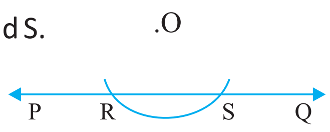
Let $\overleftrightarrow{PQ}$ be the given line and O be the given point outside it. Let's draw a perpendicular on $\overleftrightarrow{PQ}$ from O.

Steps of construction :

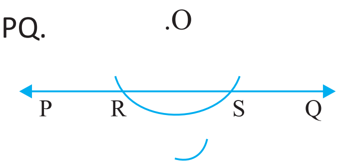
Step1 : Draw line $\overleftrightarrow{PQ}$ and denote point O, from where perpendicular is to be drawn.



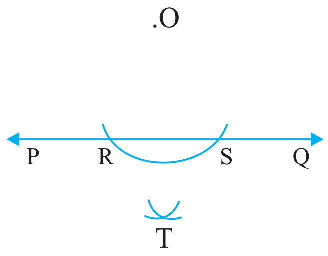
Step 2 : With O as centre and taking reasonable radius, draw an arc cutting $\overleftrightarrow{PQ}$ at R and S.



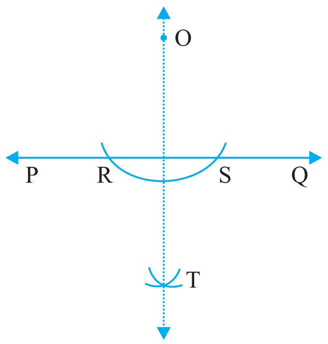
Step 3 : With R as centre and taking a radius more than half of RS, draw an arc below PQ.



Step 4 : With S as centre and the same radius, draw another arc cutting the earlier drawn arc at T.



Step 5 : Draw a line through O and T.



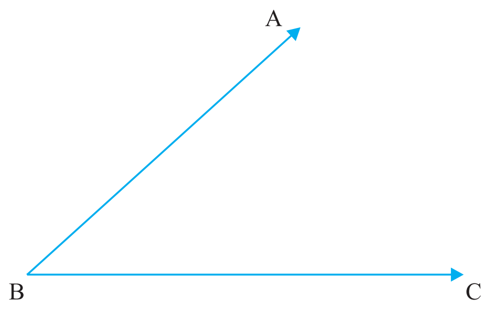
Thus, $\overleftrightarrow{OT}$ is the required perpendicular drawn from point O to $\overleftrightarrow{PQ}$.

To Draw an Angle Equal to a Given Angle

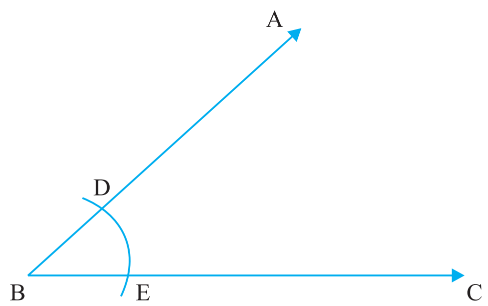
Let ∠ABC is the given angle and we have to construct another angle ∠RPQ whose measure is equal to that of ∠ABC.

Steps of construction :

Step 1 : Draw the given ∠ABC

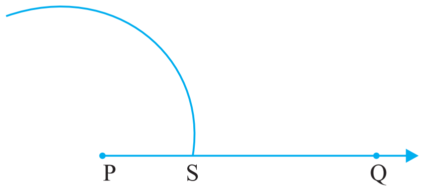


Step 2 : With the vertex B as centre and taking a reasonable radius cut AB and BC at point D and E respectively.

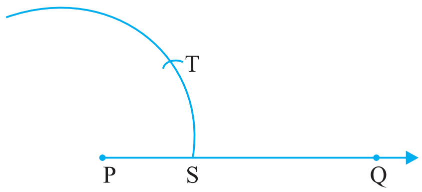


Step 3 : Draw a ray PQ.

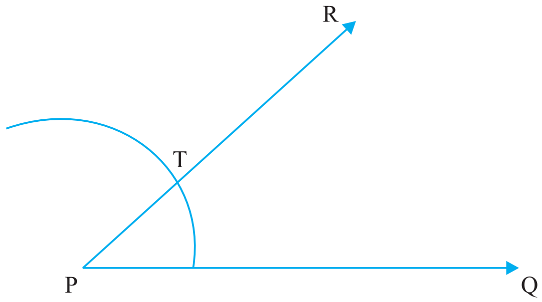
Step 4 : Keeping the same radius as step 2, draw an arc from P cutting PQ at S.



Step 5 : With S as centre and DE (from step 2) as radius, draw another arc to cut the earlier drawn arc at T.



Step 6 : Join P and T and extend it to form $\overrightarrow{PR}$.



Thus, ∠RPQ is the required angle.

Construction of Parallel Lines

Two lines in the same plane are parallel if they do not meet however far they are produced in either direction. Let's recap some important facts related to parallel lines :

(i) Parallel lines never meet. These are equidistant everywhere.

(ii) When a transversal intersect parallel lines, then its corresponding angles and alternate interior angles are equal and vice-versa.

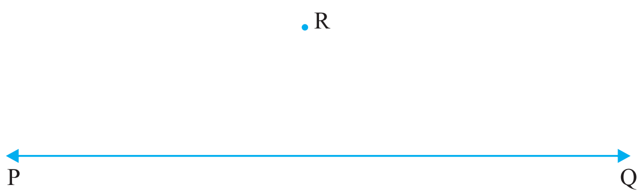
These facts are of great use while constructing parallel lines.

Construction of a Line Parallel to a Given Line from a Point Outside it

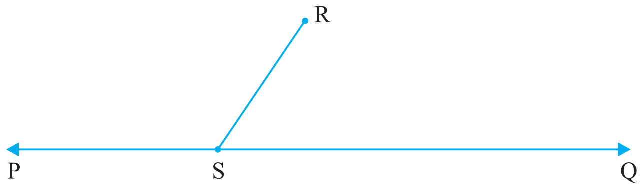
Let PQ is the line and R is the point outside, from where a parallel line is to pass through it.

Steps of construction :

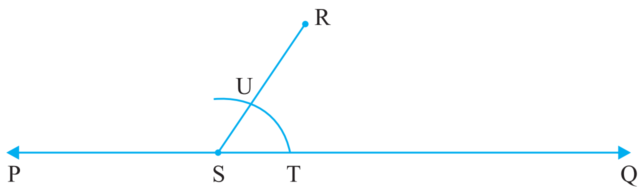
Step 1 : Draw a line PQ and denote the point R outside line PQ.



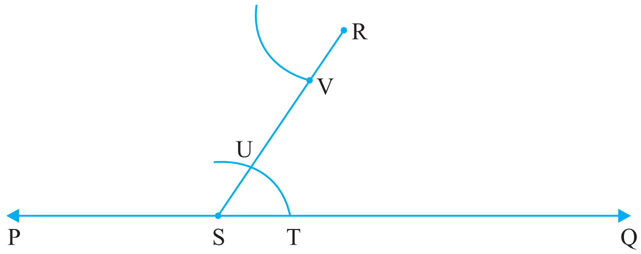
Step 2 : Take any point S on PQ and join RS.



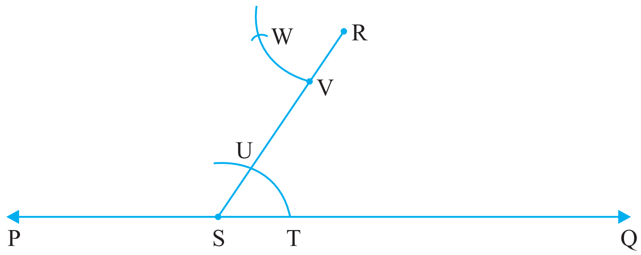
Step 3 : From point S, taking a reasonable radius, draw an arc cutting $\overrightarrow{SQ}$ and $\overline{RS}$ at points T and U respectively.



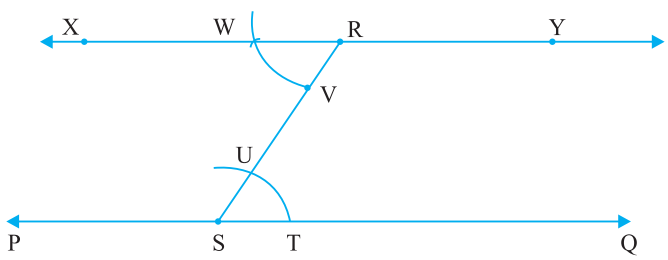
Step 4 : With R as centre and keeping the same radius as in step 3, draw an arc on the opposite side of RS to cut at RS at V.



Step 5 : With V as centre and the radius equal to TU, draw an arc cutting the earlier drawn arc in step 4 at W.



Step 6 : Join RW and produce it in both directions to meet at points X and Y.



Thus, $\overleftrightarrow{XY}$ is the required parallel line to $\overleftrightarrow{PQ}$ and passing through R.

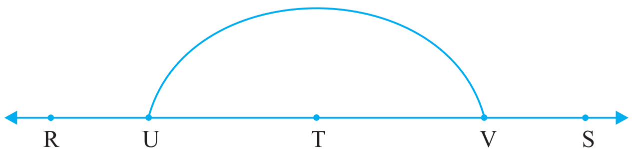
Construction of a Line Parallel to a Given Line at a Given Distance

Let RS is the line and $\overleftrightarrow{AB}$ is to be drawn parallel it at a distance of 3.5 cm from RS.

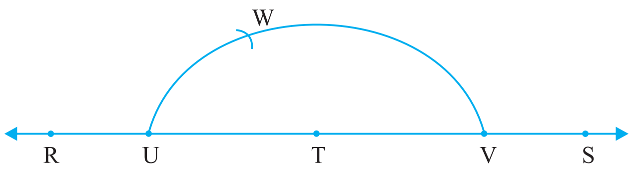
Steps of construction :

Step 1 : Draw a line RS.

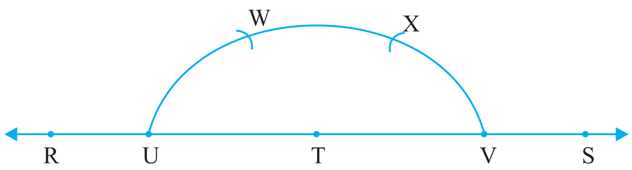
Step 2 : Mark a point T on RS. With T as centre draw an arc cutting RS at point U and V.



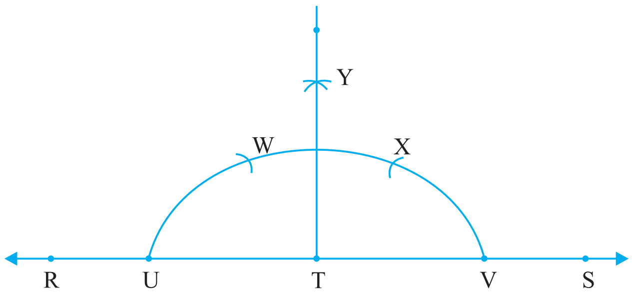
Step 3 : Taking U as centre and with the radius same as step 2, draw an arc intersecting $\overset{\frown}{UV}$ at W.



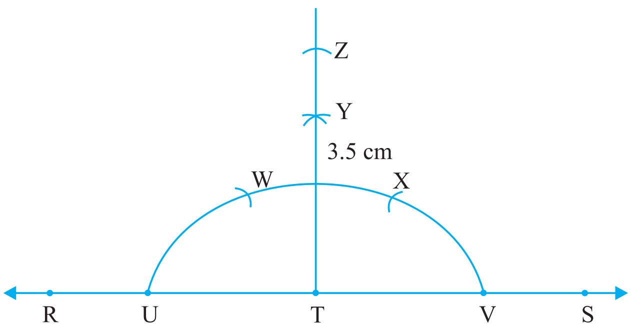
Step 4 : Taking W as centre and with the same radius, draw another arc interesting $\overset{\frown}{UV}$ at X.



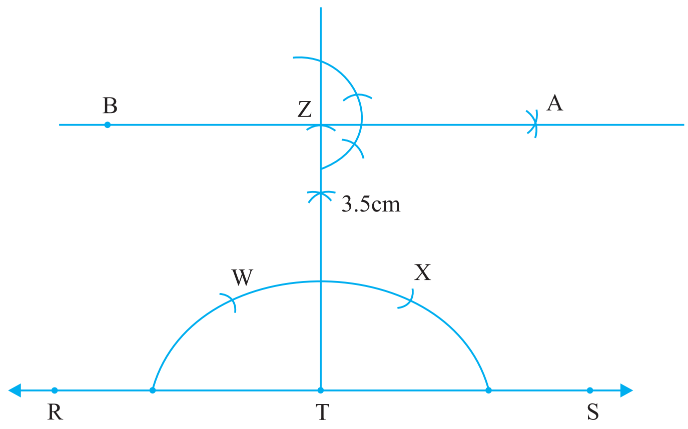
Step 5 : With W and X as centres and with the same radius, draw arcs such that they intersect each other at point Y. Join YT such that $\angle YTV = \angle YTU = 90°$.



Step 6 : Now mark a point Z on perpendicular YT such that ZT = 3.5 cm.



Step 7 : Again construct a right angle at Z as we did earlier.



Step 8 : Since ∠AZT = ∠ZTS = 90° (corresponding angles)

So, AB is parallel to RS at a distance of 3.5 cm away from RS.

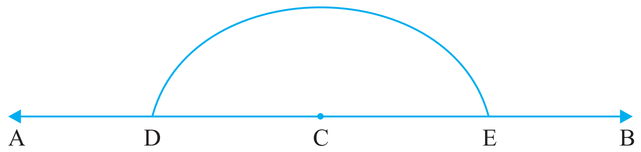
Example 1 : Draw a line IJ parallel to AB at a distance of 4 cm. Do write the steps of construction.

Solution :

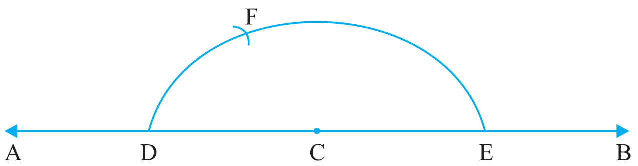
Step of construction :

Step 1 : Draw a line AB using a scale.

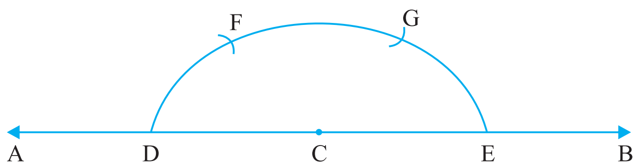
Step 2 : Mark a point C on AB. With C as centre draw an arc cutting AB at points D and E.



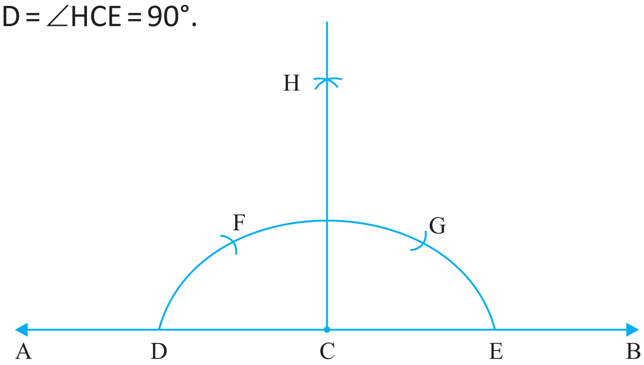
Step 3 : Taking D as centre and with the same radius as step 2, draw an arc intersecting $\overset{\frown}{DE}$ at F.



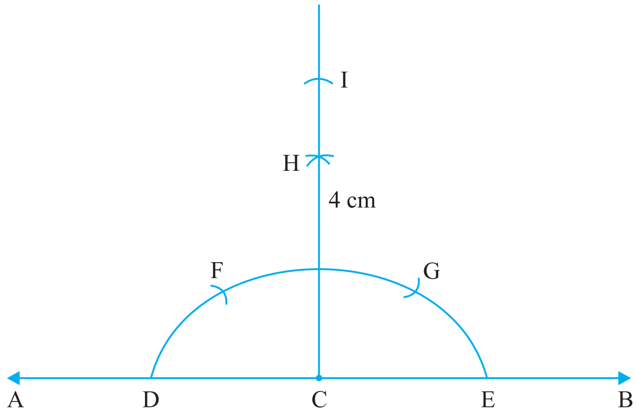
Step 4 : Taking F as centre and with the same radius, draw another arc intersecting $\overset{\frown}{DE}$ at G.



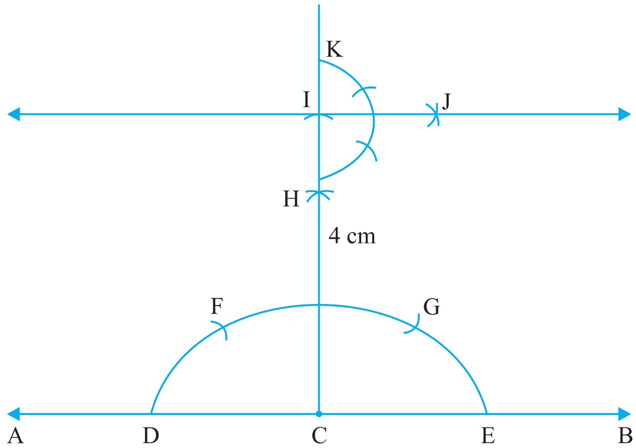
Step 5 : With F and G as centre and with the same radius, draw arcs such that they intersect each other at point H. Joint HC such that ∠HCD = ∠HCE = 90°.



Step 6 : Now mark a point I on perpendicular HC such that IC = 4 cm.



Step 7 : Again construct a right angle at I as we did earlier.



Step 8 : Since ∠JIC = ∠BCI = 90°. (Corresponding angles)
So, IJ is parallel to AB at a distance of 4 cm away from AB.

Exercise 13.1

1. Draw a line segment measuring 6.5 cm and bisect it. Do not forget to write the steps of construction.
2. **Draw the following angles using protractor and bisect it using compasses :**
   (a) 84° (b) 64° (c) 120° (d) 78°
3. Draw a line perpendicular to a line segment AB measuring 4.7 cm from a point not lying on it.
4. **Draw the following angles using protractor. Also construct equal angles of each using compass :**
   (a) 72° (b) 85° (c) 115° (d) 47°

5. Draw a line AB parallel to a given line XY at a distance of 3.7 cm.
6. Draw any line AB. Mark any point C at a distance of 4.7 cm from it. Through C, draw a line PQ parallel to AB using ruler and compasses.
7. Take any three non-collinear points P, Q and R, and draw DPQR. Through each vertex, draw a line parallel to the opposite side using ruler and compasses.

Construction of Triangles

Let's recall the components of a triangle. It has six parts— three sides and three angles. A triangle can only be constructed if three parts, including at least one side, are given. Before starting construction of triangle one should make a rough sketch of it mentioning its measurements.

To Construct a Triangle When Three Sides are Given (S.S.S. Triangle)

One must be very cautious before starting the construction of a triangle with three given sides. One should first check whether the construction of a triangle is possible or not. For this purpose, we should add two smaller sides and check whether the sum is greater than the third side or not. If yes, then only the construction of triangle is possible, otherwise not.

Facts to Know

- The sum of two sides of a given triangle should be greater than the third side.

Example 2 : **Is it possible to construct a triangle with sides 6 cm, 8 cm and 13 cm ?**

Solution : For a triangle to be constructed, the sum of two smaller sides should be greater than third side.

Here, 6 cm + 8 cm > 13 cm

$\Rightarrow$ 14 cm > 13 cm

Hence, it is possible to construct a triangle.

Example 3 : **Construct a triangles ABC which has sides AB = 4.5 cm, BC = 4 cm and CA = 6 cm.**

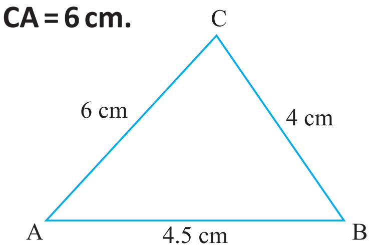
Solution : First check the possibility of the triangle to be formed.

Here, 4.5 cm + 4 cm > 6 cm

$\Rightarrow$ 8 .5 cm > 6 cm

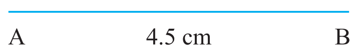
Hence, it is possible to construct a triangle.

Draw a rough sketch of the triangle with its measurement.

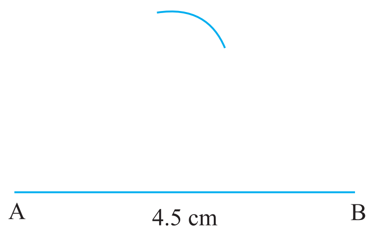


Steps of construction :

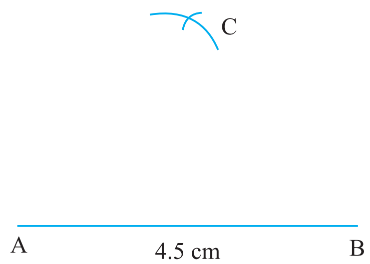
Step 1 : Draw a line segment AB = 4.5 cm.



Step 2 : Taking A as centre and radius 6 cm, draw an arc measuring 6 cm.

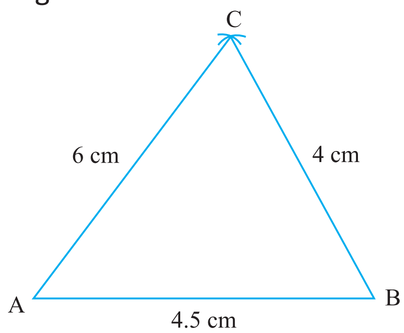


Step 3 : Now taking B as the centre and radius measuring 4 cm , draw another arc cutting the first arc at point C.



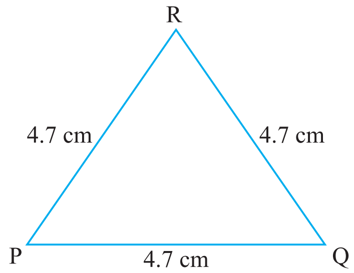
Step 4 : Join AC and BC.

Hence, ΔABC is the required triangle.



Example 4 : Draw an equilateral triangle PQR measuring sides 4.7 cm.

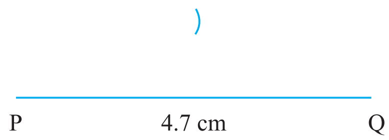
Solution : As the given triangle is equilateral, the sum of its two sides will always be greater than third side. So, it is possible to construct the triangle. Now, draw a rough sketch of the triangle with its measurement.



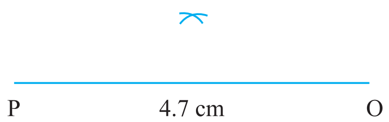
Steps of construction :

Step 1 : Draw a line segment PQ = 4 .7 cm.

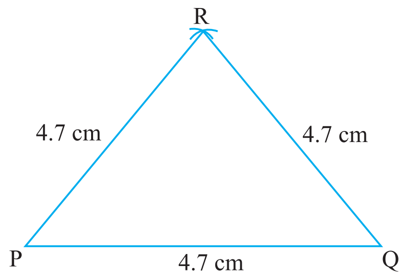
Step 2 : Taking P as the centre and radius measuring 4.7 cm, draw an arc.



Step 3 : Taking Q as the centre and same radius draw another arc cutting the previous arc at point R.



Step 4 : Join PR and QR.

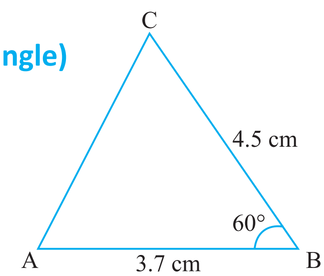


Thus, ΔPQR is the required equilateral triangle.

To Construct a Triangle When Two Sides and the Including Angles are Given (S.A.S . Triangle)

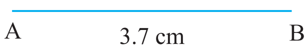
Example 5 : Construct a ΔABC, given that AB = 3.7 cm, BC = 4.5 cm and B = ∠60°.

Solution : Draw a rough sketch of the triangle with its measurement.

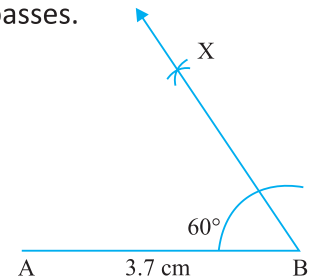


Steps of construction :

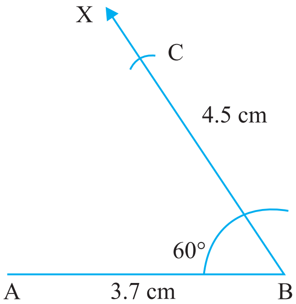
Step 1 : Draw a line segment AB = 3.7 cm.



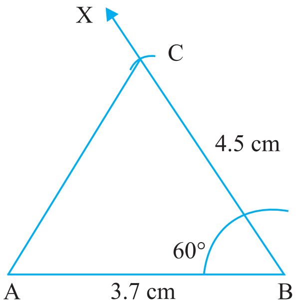
Step 2 : At B, draw ∠ABX = 60 ° using compasses.



Step 3 : Taking B as centre and radius measuring 4.5 cm, draw an arc cutting BX at C.



Step 4 : Join AC

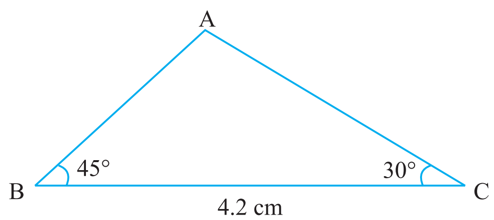


Thus, Δ ABC is the required triangle.

To Construct a Triangle When Two Angles and Included Sides are Given (A. S. A. Triangle)

Example 6 : Construct a triangle ABC, given that BC = 4.2 cm, ∠B = 45° and ∠C = 30°.

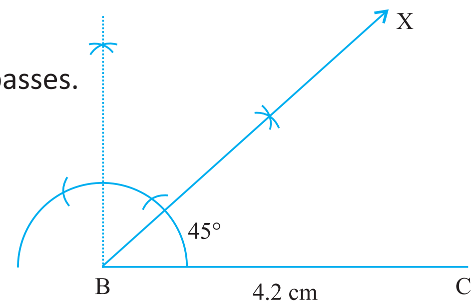
Solution : Draw a rough sketch of the triangle with its measurement.



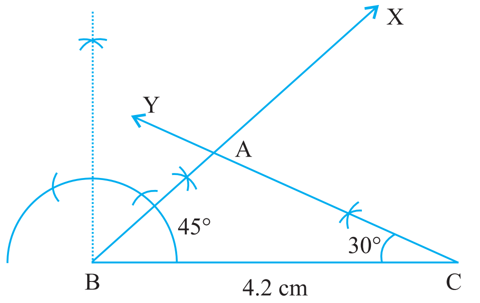
Steps of construction :

Step 1 : Draw a line segment BC = 4.2 cm.

Step 2 : At B, draw ∠CBX = 45° using compasses.



Step 3 : At C, draw ∠BCY = 30° using a compasses. Thus, BX and CY intersect each other at A.



Thus, ΔABC is the required triangle.

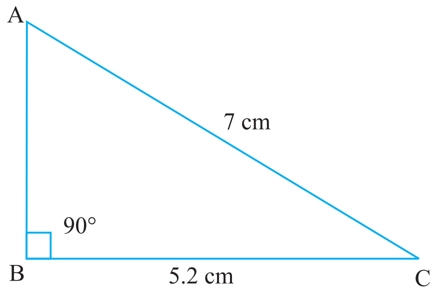
Facts to Know

- Sum of two angles of a triangle should be less than 180°

To Construct a Right-Angled Triangle Whose Two Sides are Given

Example 7 : Construct a right-angled ΔABC in which BC = 5.2 cm, ∠B = 90° and hypotenuse CA = 7 cm.

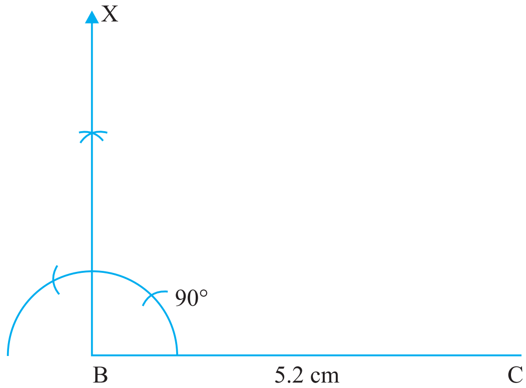
Solution : Draw a rough sketch of the triangle with its measurement.



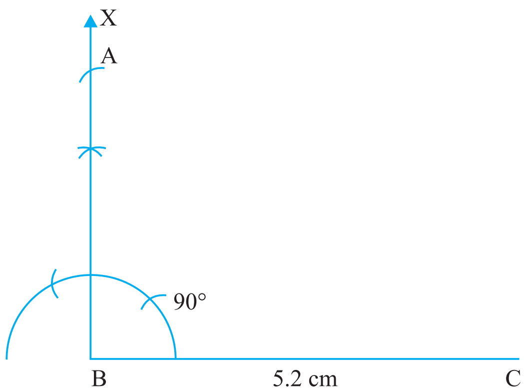
Steps of construction :

Step 1 : Draw a line segment BC = 5.2 cm

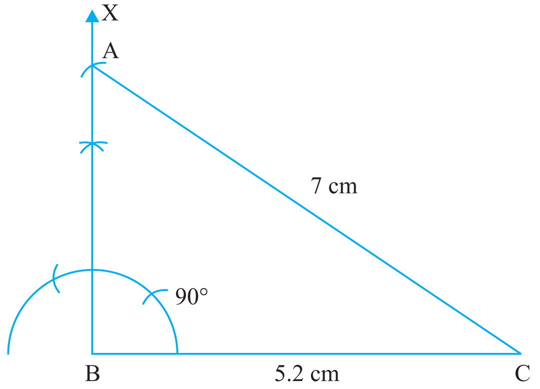
Step 2 : At B, draw ∠XBC = 90° using compasses.



Step 3 : With C as centre and radius measuring 7 cm, draw an arc cutting BX at point A.



Step 4 : Join AC



Thus, ∆ABC is the required right-angled triangle.

Exercise 13.2

1. Construct a ∆ ABC in which AB = 4.2 cm, BC = 5.6 cm and ∠B = 60°. Through each vertex of the triangle, draw a line parallel to the opposite side.
2. **Is it possible to construct a triangle with the following sides?**
   (a) 3 cm, 5 cm, 7 cm (b) 4.6 cm, 5.4 cm, 10 cm
   (c) 3.6 cm, 6.3 cm, 10 cm (d) 6.5 cm, 4.5 cm, 3.2 cm
3. **Construct the following triangles :**
   (a) An isosceles triangle with equal sides measuring 4.4 cm and other side measuring 3.7 cm.
   (b) An equilateral triangle with side 6.5 cm.
   (c) ∆ ABC ; AB = 3.6 cm, BC = 4.5 cm and CA = 5.4 cm.
   (d) ∆ PQR ; PQ = 6 cm, PS = 6.5 cm and QR = 7 cm
4. Construct a right-angled ∆PQR in which PQ = 5.2 cm, ∠Q = 90° and hypotenuse PR = 7cm.
5. Construct a ∆ABC right angled at B and sides AB and BC measuring 4 cm and 5 cm respectively.
6. Construct an isosceles triangle with base 7.2 cm and base angles 45°.
7. Construct a right-angled triangle PQR right angled at R, in which PQ = 4.8 cm and QR = 3.7 cm. Using protractor measure other two angles also.
8. The perimeter of a triangle whose sides are in ratio 2 : 3 : 4 is 18 cm. Construct the triangle.
9. Construct a ∆ABC if AB = 3.6 cm, ∠A = 105° and ∠B = 45°.
10. Construct a right-angled triangle whose hypotenuse is 7.5 cm and one of the legs is 4.2 cm long.

Points to Remember

- Bisecting a line segment signifies the division of a line segment into two equal parts.
- Bisecting an angle signifies the division of a line segment into two equal parts.
- Two lines are parallel if they do not meet, however far they are produced in either direction.
- Construction of parallel lines is possible if its alternate angles are equal.
- Constructions of a right angled triangle is possible when its hypotenuse and a side is given.
- Sum of two smaller sides of a triangle is greater than the third side.
- Sum of two angles of a triangle cannot be more than 180°.
- Before the construction of actual triangle, make a rough sketch of the triangle and denote the measurements on it.

EXERCISE

1. **MULTIPLE CHOICE QUESTIONS (MCQs):**

Tick (✓) the correct options :

(a) The number of triangle/s that can be constructed with angles in the ratio 3 : 4 : 5 is:

(i) 1 ☐ (ii) 2 ☐ (iii) 3 ☐ (iv) infinite ☐

(b) Sum of two angles of the given triangle should not be more than :

(i) 60° ☐ (ii) 90° ☐ (iii) 180° ☐ (iv) 360° ☐

(c) The case in which the construction of the triangle in not possible :

(i) 3 cm, 4 cm, 5 cm ☐ (ii) 4 cm, 6 cm, 9 cm ☐

(iii) 2.5 cm, 8.5 cm, 3.5 cm ☐ (iv) 12 cm, 16 cm, 20 cm ☐

(d) A line segment measuring 10.5 cm is bisected equally into three parts. The measure of each segment will be :

(i) 3.5 cm ☐ (ii) 5.25 cm ☐ (iii) 21 cm ☐ (iv) 31.5 cm ☐

(e) Construction of a triangle is not possible when :

(i) $\angle A = 90°, \angle B = 45°, \angle C = 45°$ ☐ (ii) $\angle A = 60°, \angle B = 60°, \angle C = 60°$ ☐

(iii) $\angle A = 120°, \angle B = 45°, \angle C = 15°$ ☐ (iv) $\angle A = 47°, \angle B = 60°, \angle C = 85°$ ☐

(f) With the help of ruler and compasses the angles which cannot be constructed is :

(i) $67\frac{1}{2}^{\circ}$ ☐ (ii) $37\frac{1}{2}^{\circ}$ ☐ (iii) 55° ☐ (iv) 105° ☐

(g) Construction of a right-angled triangle is possible when :

(i) one side is given. ☐ (ii) hypotenuse is given. ☐

(iii) hypotenuse and a side is given. ☐ (iv) one angle is given. ☐

(h) We can draw $\angle ABC = 37.50°$ and its bisector using :

(i) compasses ☐ (ii) protractor ☐

(iii) both compasses and protractor ☐ (iv) none of these ☐

(i) When two lines are perpendicular to the same line, then the lines are :

(i) parallel to each other. ☐ (ii) perpendicular to each other. ☐

(iii) intersecting each other. ☐ (iv) coinciding with each other. ☐

(j) Numbers of perpendiculars can be drawn from a point on the line :

(i) 1 ☐ (ii) 2 ☐ (iii) 4 ☐ (iv) infinite ☐

2. **Construct the following line segments and bisect it :**

(i) 7.2 cm (ii) 6.7 cm (iii) 4.5 cm

3. Draw $\angle PQR = 45°$ such that PQ = 4.2 cm and QR = 5 cm. Through P, draw a line parallel to PQ, and through R draw a line parallel to PQ, intersecting each other at S. Measure PS and SR.

4. **Construct the following angles using compasses and bisect it :**

(i) 45° (ii) 75° (iii) 120° (iv) 135°

5. **Draw the following angles using protractor and construct same angles of each using compasses :**

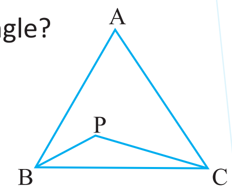
(i) 50° (ii) 75° (iii) 110° (iv) 25°

6. Draw a line PQ parallel to a given line RS at a distance of 5 cm.

7. Construct a triangle ABC in which AB = 5 cm, BC = 6.5 cm and CA = 7.2 cm. Through each vertex of the triangle, draw a line parallel to opposite side.
8. Construct an isosceles triangle ABC in which the lengths of each of its equal sides is 5.5 cm and angle between them is 60°.
9. Construct a right-angled triangle whose hypotenuse is 7.5 cm long and one of the legs is 5.5 cm.
10. Construct an isosceles right-angled triangle PQR where $\angle PQR = 90°$ and QR = 4.8 cm.
11. **Check out the possibility of a triangle to be constructed with the following measurements :**
    (i) ΔABC ; $\angle B = 105°$, $\angle C = 80°$, BC = 5 cm
    (ii) ΔPQR ; PQ = 7 cm, QR = 4 cm, PR = 3 cm
    (iii) ΔLMN ; $\angle L = 50°$, $\angle M = 70°$, $\angle N = 60°$
    (iv) ΔABC ; $\angle ABC = 75°$, AB = 4 cm, BC = 6 cm
12. Construct an isosceles ΔABC whose perimeter is 16 cm and its unequal side is 5 cm.
13. Construct a ΔPQR if $\angle PQR = 30°$, $\angle PRQ = 45°$ and QR = 6.7 cm.

HOTS

1. The three angles of a triangle are in the ratio 1 : 36 : 8. What is the measure of each angle?
2. BP and CP are angle bisectors. If $\angle A = 76°$, find the measure of $\angle BPC$.



3. Susmita's front door is 40 cm wide and 42 cm tall. She purchased a circular table is 96 cm in diameter. Will the table pass through the front door? Explain your answer.

Lab Activity

Objective : To verify that a triangle can only be constructed if the sum of lengths of two smaller sides is greater than the third side.

Materials Required : White sheet of paper, ruler, pencil, eraser, match box.

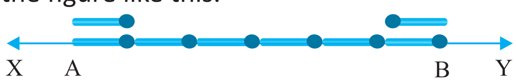
Procedure :

Step 1: Draw a straight line XY on the white sheet.

Step 2: Mark a point A on it.

Step 3: Take six matchsticks and glue them along the line starting from 'A'. Mark the end of sixth matchstick as point B an line XY.

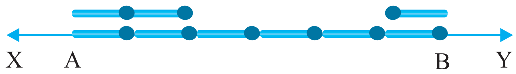
Step 4: (i) Take 2 matchsticks and try to make a triangle by taking 1 matchstick from A and other from B. We get the figure like this.



Here, 1 + 1 = 2 < 6 (Number of matchsticks in AB)

So, triangle is not formed.

(ii) Take 3 matchsticks and try to form triangle. We get the figure like this.

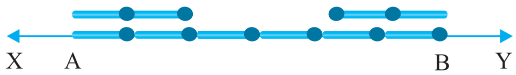


Here, $2 + 1 = 3 < 6$

So, triangle is not formed.

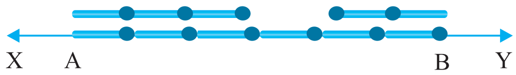
(iii) Take four matchsticks and try to form a triangle, we get the figure like this .

Here, $2 + 2 = 4 < 6$



So, triangle is not formed.

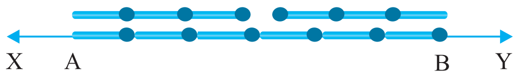
(iv) Take five matchsticks and try to form a triangle, we get the figure like this.



Here, $3 + 2 = 5 < 6$

So, triangle is not formed.

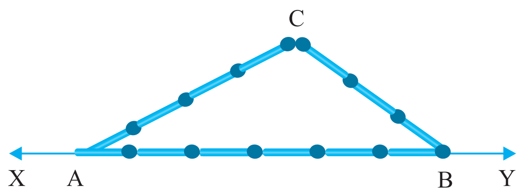
(v) Take six matchsticks and try to form a triangle, we get the figure like this.



Here, $3 + 3 = 6 = 6$

So, we get a straight line, not a triangle.

(vi) Take seven matchsticks and try to form a triangle, we get the figure like this.



Here, $4 + 3 = 7 > 6$

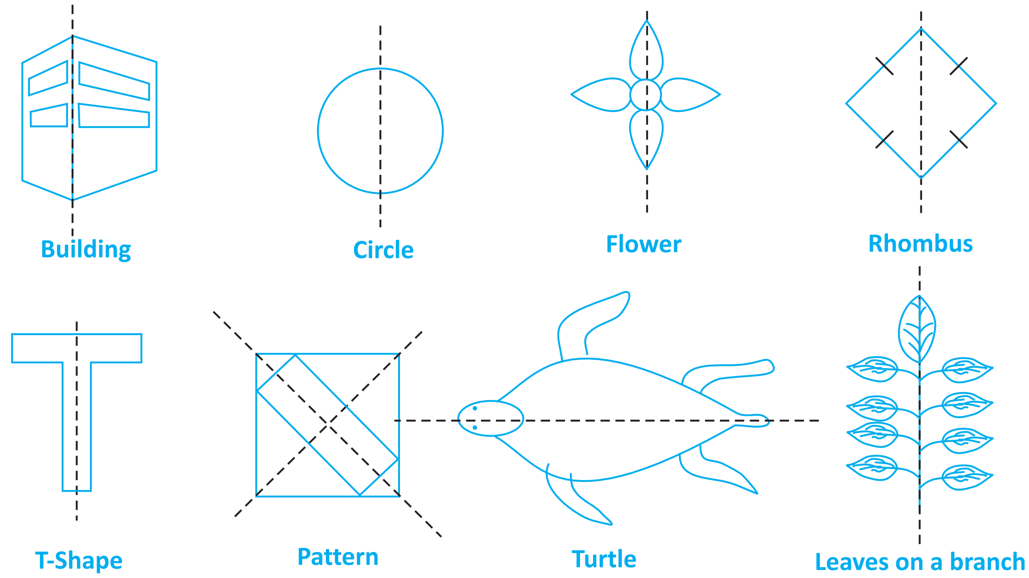
So, a triangle can be formed.

Hence, it is verified that a triangle is formed only if the sum of its two sides is greater than the third side.

14 Symmetry

Look at the following figures or designs :

Don't you think these figures or designs have something special about them ? They are uniform in terms of shape.



Dotted lines divide then into two equal halves or two congruent figures. Note that if a geometrical figure is exactly similar to another figure in terms of shape and size, the two figures are congruent to each other. In the figures shown here, the dotted lines are dividing the bigger figure into two exactly equal (congruent) halves. Note that if these figures are folded along their dotted lines, the two halves so obtained will be exactly equal to each other. Such figures are said to have line symmetry.

Line Symmetry

We observed the examples of line symmetry in the previous section. We had stated that if the figure is folded along a particular line (axis), it divides the entire figure into two exactly equal halves. This is the concept of line symmetry. We shall define it as follows.

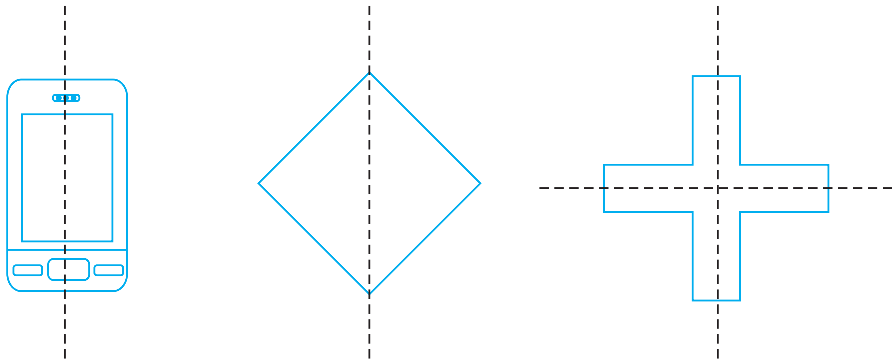
"A figure has line symmetry if there is a line about which the figure may be folded so that its two half parts (formed due to such folding) coincide with each other and are congruent to each other in terms of shape and size." "The line or axis of symmetry is the line around which the figure shows symmetry."

These are the figures which show line symmetry around at least one line or axis of symmetry. They can be folded along that line and we can see two parts, which would be convenient to each other. "A figure that shows line symmetry about at least one line or axis is called symmetry figure."

Facts to Know

- The line of symmetry and axis of symmetry are the same.
- The line of symmetry can be horizontal, vertical or slant.

Look at the following pictures. They all show symmetry along at least one line of symmetry: mobile, kite, red cross and triangle :

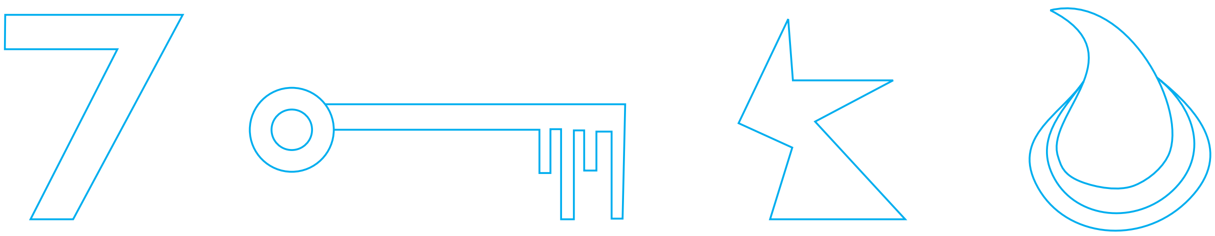


Mobile Kite Red Cross Triangle

- A figure that is symmetrical about a straight line is a mirror image of its own self.

Asymmetrical Figures

We do not always come across nice and symmetrical figures in our everyday life. Look at the following figures.

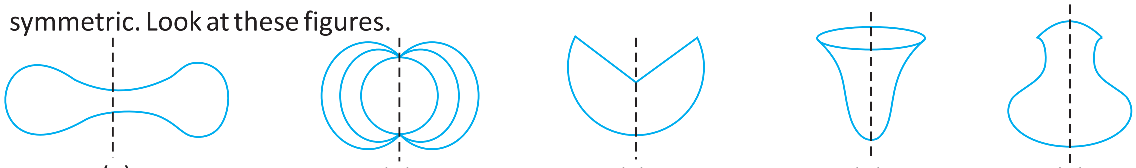


Seven Key Figure Design

In these figures, if the lines of symmetry are inserted and figures are folded along those lines of symmetry, the two parts of obtained would not coincide with each other. In other words, the two parts would not be congruent to each other. "A figure that displays this property is called Asymmetric figure. It means the figure does not have a single straight line about which it is symmetric. It is easily identified as unshapely, odd and asymmetric.

Facts to Know

- Figures with straight lines are not the only ones that can be symmetric. Even curved figures can also be symmetric. Look at these figures.



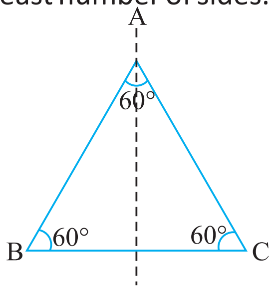
(a) (b) (c) (d) (e)

Facts to Know

- If only one-half part of a symmetric figure has been given, we can make the other half part easily.
- The number of lines of symmetry of a parellelogram is zero.

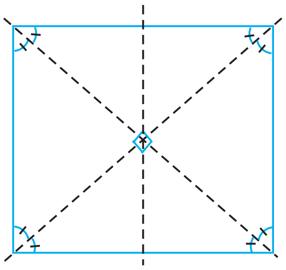
Lines of Symmetry for Regular Polygons

A polygon is a closed figure it has several line segments. A triangle is a polygon that has the least number of sides. A polygon is regular if all its sides are of equal length and all its angles are equal.



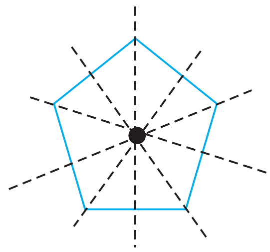
Look at the figure here. ΔABC is an equilateral triangle. Each one of its angles is 60°. So, it is a regular polygon of three sides (because all of its sides are of equal length). Its line of symmetry has been shown by the dotted line.

Now, look at the following figure. It shows a square, which is a polygon with four sides. It is a regular square (because the length of its sides is equal). Further, all of its angles are 90° each. Its diagonal are the perpendicular bisections of each other.



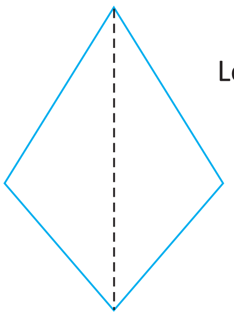
Its four lines of symmetry have been shown in the figure. They also include its two diagonals.

Now, refer to the next figure. It is a regular pentagon, with all five sides equal. It has five angles. It has five lines of symmetry.

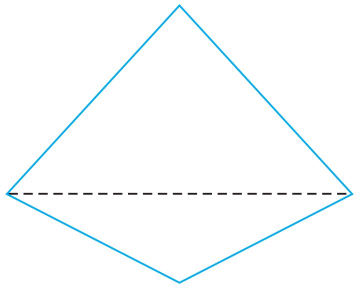


Thus, we conduce that the number of lines of symmetry of a regular polygon is equal to the total number of its sides.

In each one of these cases, we can conclude that if a mirror is prosed along a line if symmetry of any other portion will be its refection. So, the concept of symmetry is related to mirror reflection. A mirror line helps us understand and imagine the concept of line of symmetry.



Look at this figure, in this figure, the dotted line is a mirror line.

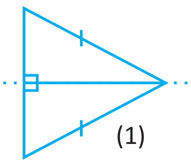


Here, the dotted line is NOT a mirror line.

Some Figures and their Lines of Symmetry

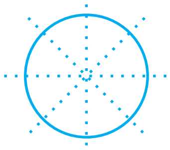
The number of lines of symmetry have been given in parentheses.

Isosceles triangle



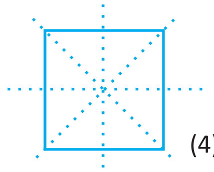
(1)

Circle



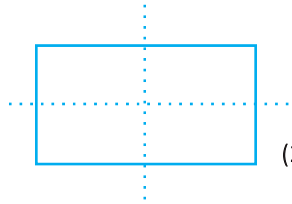
(Infinite)

Square



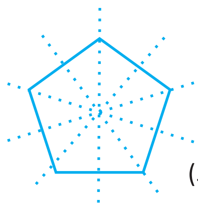
(4)

Rectangle



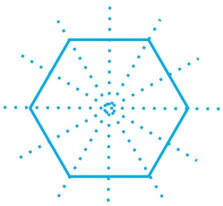
(2)

Pentagon



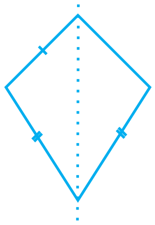
(5)

Hexagon



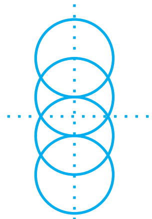
(6)

Kite



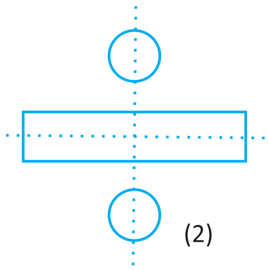
(1)

Chain of rings



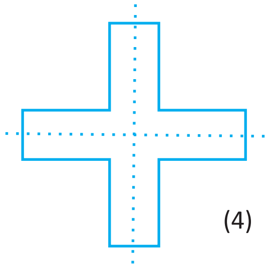
(2)

Division symbol



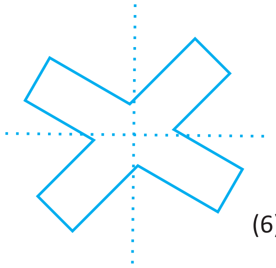
(2)

Addition symbol



(4)

Multiplication symbol

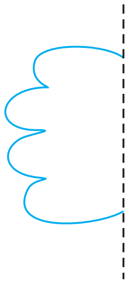


(6)

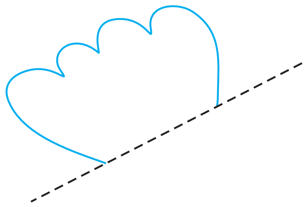
Parallelogram

(0)

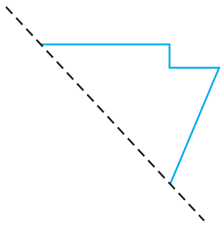
Example 1 : Draw the lines of symmetry in each one of the following figures.



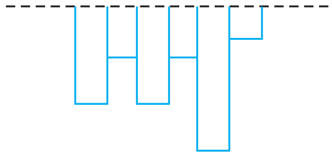
(a)



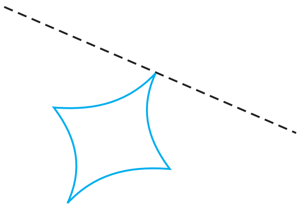
(b)



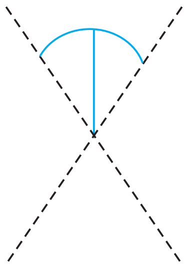
(c)



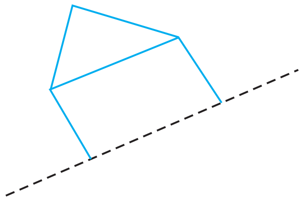
(d)



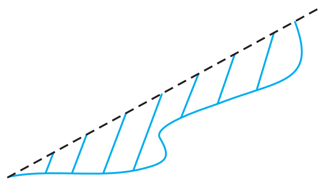
(e)



(f)

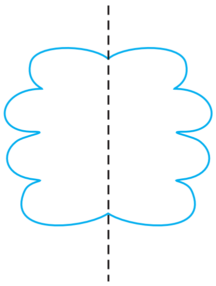


(g)

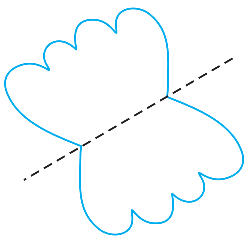


(h)

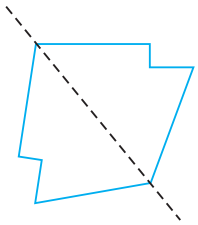
Solution :



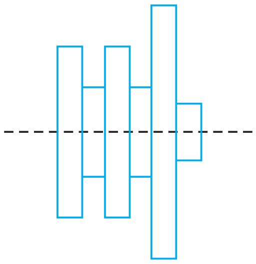
(a)



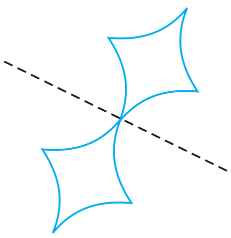
(b)



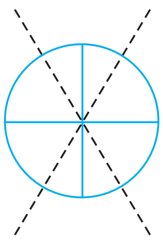
(c)



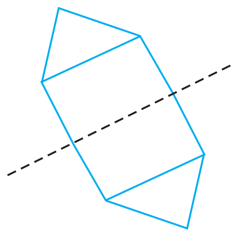
(d)



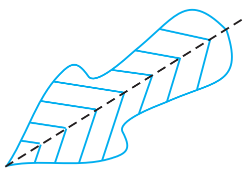
(e)



(f)

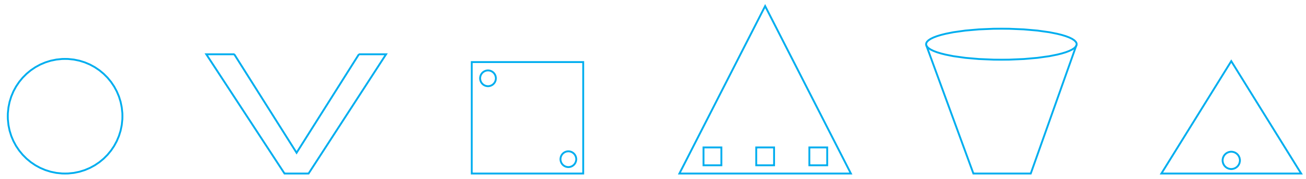


(g)



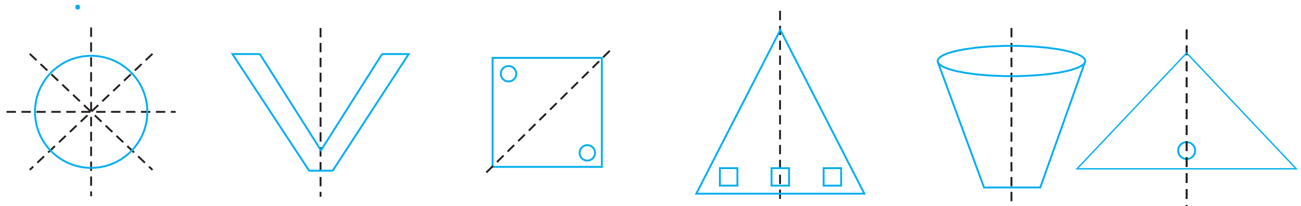
(h)

Example 2 : **Draw the lines of symmetry in each one of the following figures.**



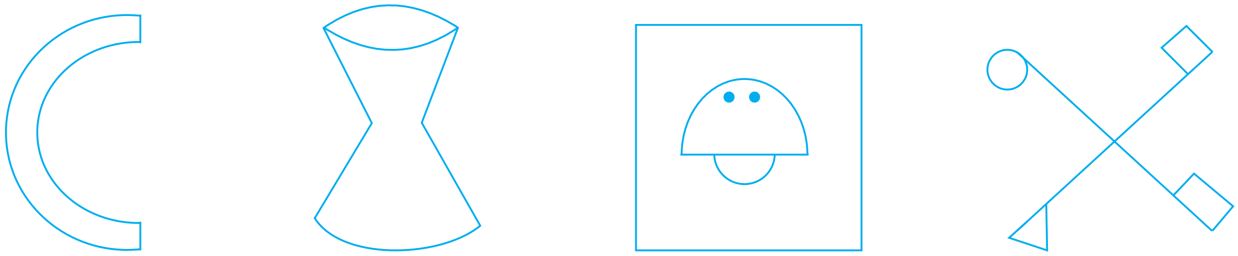
(a) (b) (c) (d) (e) (f)

Solution :

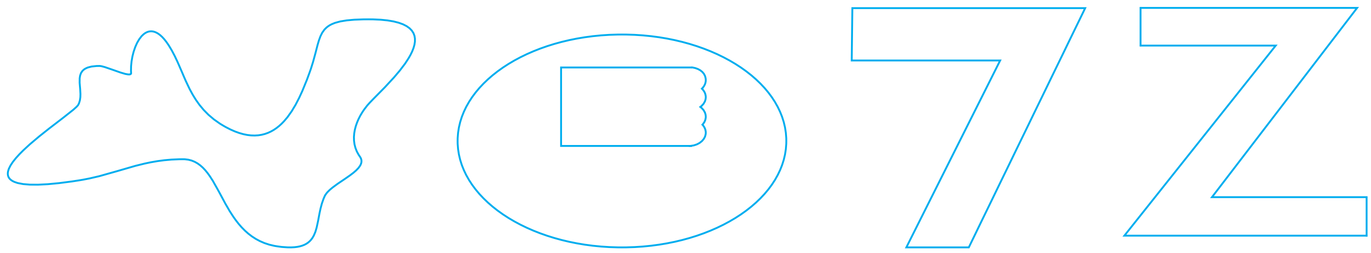


(a) (b) (c) (d) (e) (f)

Example 3 : **Which ones of the following figures are symmetrical ?**



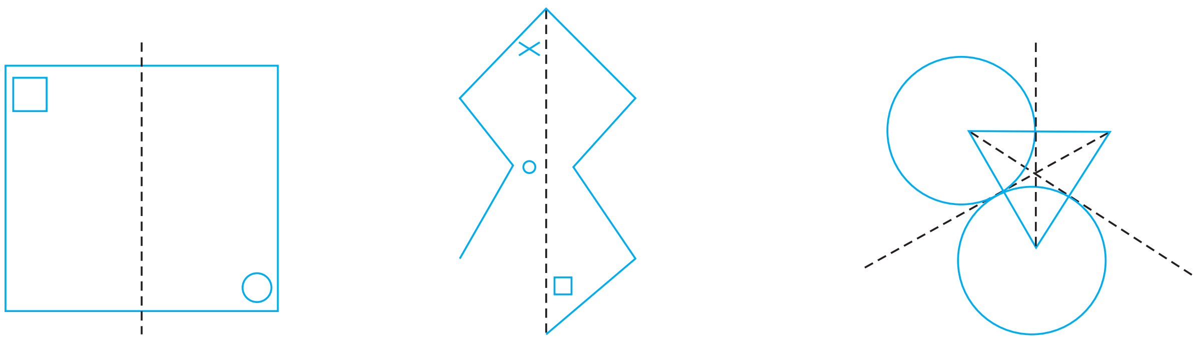
(a) (b) (c) (d)



(e) (f) (g) (h)

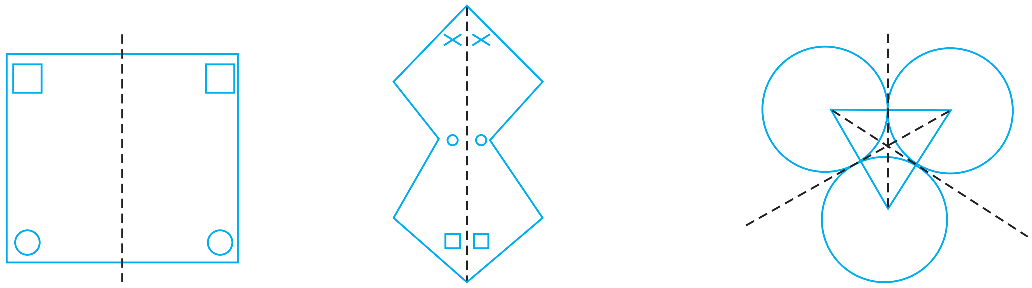
Solution : Except figures (d), (e), (f), (g) and (h), all others are symmetrical ones about at least one axis. So, figures (a), (b), and (c) are symmetrical.

Example 4 : **The objects missing in the following figures are to be provided to give symmetry to them. Complete these figures.**



(a) (b) (c)

Solution :



(a) (b) (c)

Example 5 : How many lines of symmetry are there for each one of the following figures ?

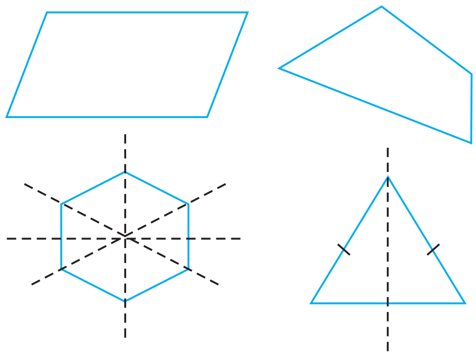
(a) Parallelogram (b) Quadrilateral (c) Regular hexagon (d) Isosceles triangle

Solution : (a) Parallelograms have not even a single line of symmetry.

(b) Quadrilateral do not have a single line of symmetry. They are irregular in most cases. Note that rectangles, squares and rhombuses are special cases of quadrilaterals. So, they have lines of symmetry.

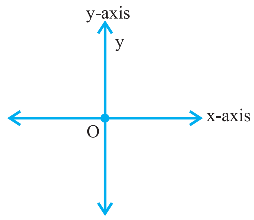
(c) Regular hexagon has six lines of symmetry.

(d) An isosceles triangle has one line of symmetry.



Facts to Know

- The point A has two coordinates. The coordinate along *x*-axis is known as **abscissa**. The coordinate along *y*-axis is known as **ordinate**. They lie in the *xy* plane.



Exercise 14.1

1. **How many lines of symmetry are there for each one of the following figures ? Draw sketches, too :**

(a) Rectangle (b) Circle (c) Square

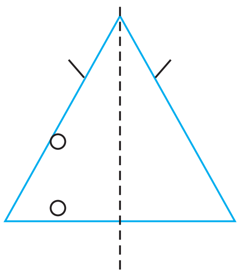
(d) Rhombus (e) Equilateral triangle (f) Regular pentagon

(g) Scalene triangle (h) The English Alphabet 'X' (i) A star with six vertices

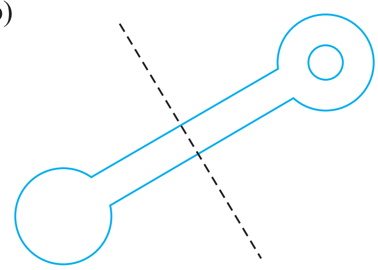
(j) Semi-circle (k) Kite (l) Isosceles triangle

2. **Provide the missing parts in the following figures to make them symmetrical.**

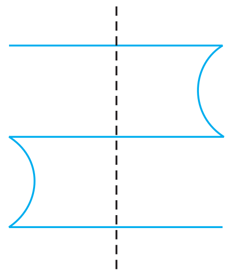
(a)



(b)



(c)



3. **Draw the lines of symmetry for the following letters of the English Alphabet ?**

A	B	C	D	E	H	M
O	T	U	V	W	X	Y

4. **Write 'T' for True and 'F' for False for the following statements.**

(a) A semi-circle has an infinite number of lines of symmetry. ☐

(b) The line of symmetry is different from the axis of symmetry. ☐

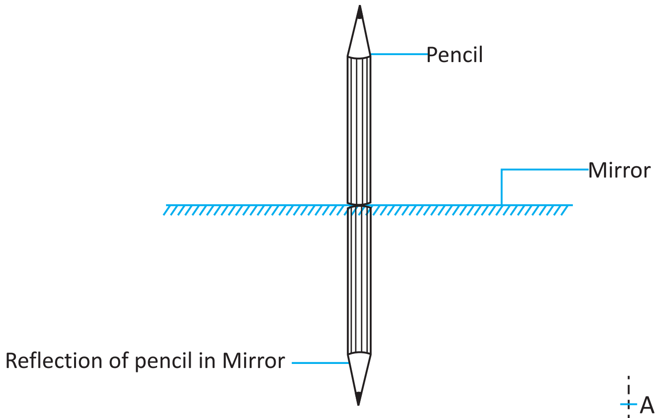
(c) The two halves created by an axis of symmetry are congruent. ☐

(d) The shape of heart is symmetrical about only one axis. ☐

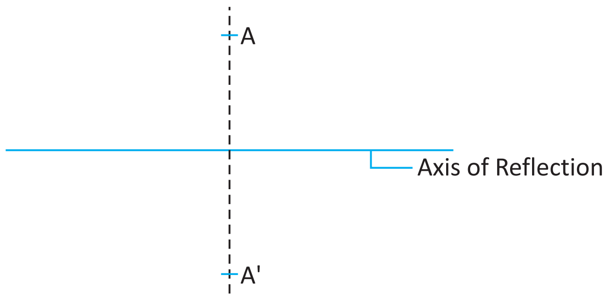
(e) The letter Z is not symmetrical about any axis. ☐

Reflection

Let us assume that a pencil has been placed on a mirror in the erect position. The mirror is lying flat on the table. Look at the figure shown here.



The pencil shown here is exactly equal, in terms of shape and size, to its reflection in the mirror. This reflection is also called image. This mirror is acting as on axis of reflection or line of reflection for the pencil. You have learnt in Class VI that you can construct a point symmetric to a point with respect to a line of symmetry. Look at this figure. Now, you will be able to find out a point on the other side of line of reflection using the same principle.

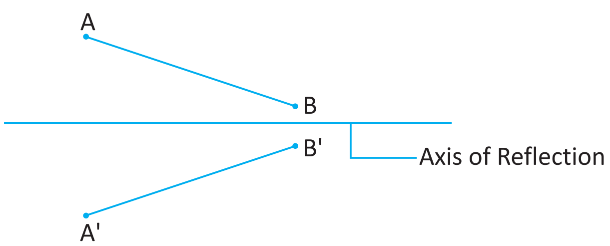


(i) Point Reflection (about x-axis)

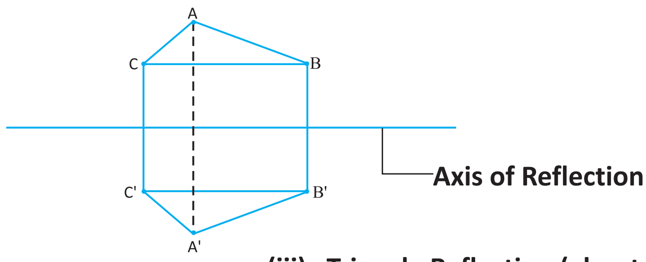
Here, A is a real object (like pencil). The point A' is a reflected image of the object A.

Look at this figure. We can use the same principle to get the reflection of a line segment AB, along an axis of reflection.

(ii) Line segment reflection (about x-axis)



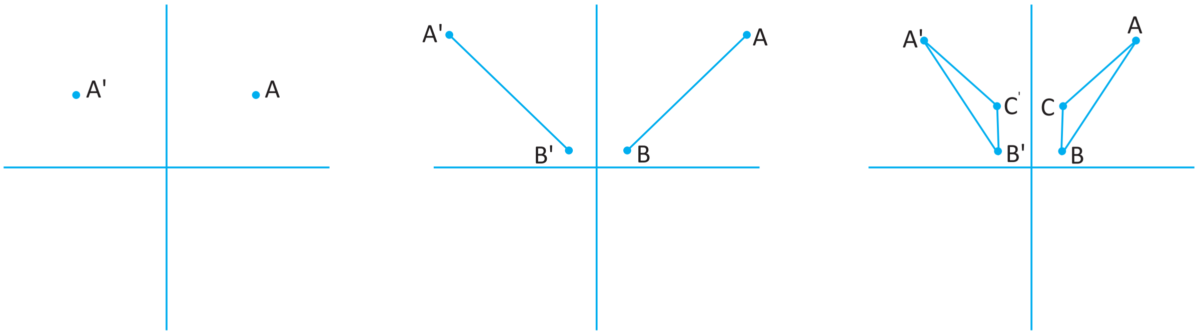
Look at this figure now. We can use the same principle to make the reflection of a triangle, ABC.



(iii) Triangle Reflection (about x-axis)

In these three cases, we have reflected a point, a line segment and a triangle along a mirror-like object, about the horizontal axis. It is the case of reflection along the x-axis.

We can get the reflection images of point, line segment and triangle about a vertical axis as well. Look at the figures shown ahead. They show us the three cases of reflection about the vertical axis.



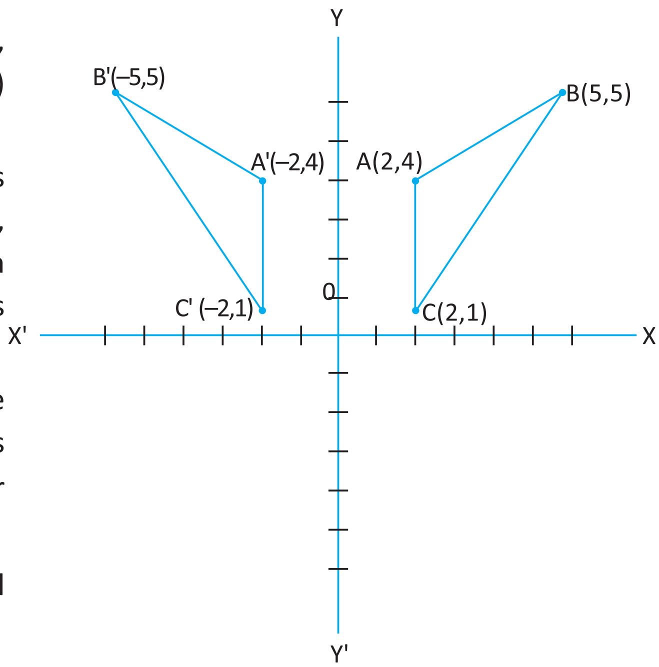
(i) (ii) (iii)

Vertical reflection of point line segment and triangle about vertical axis (y–axis)

Reflection on an XY–Plane

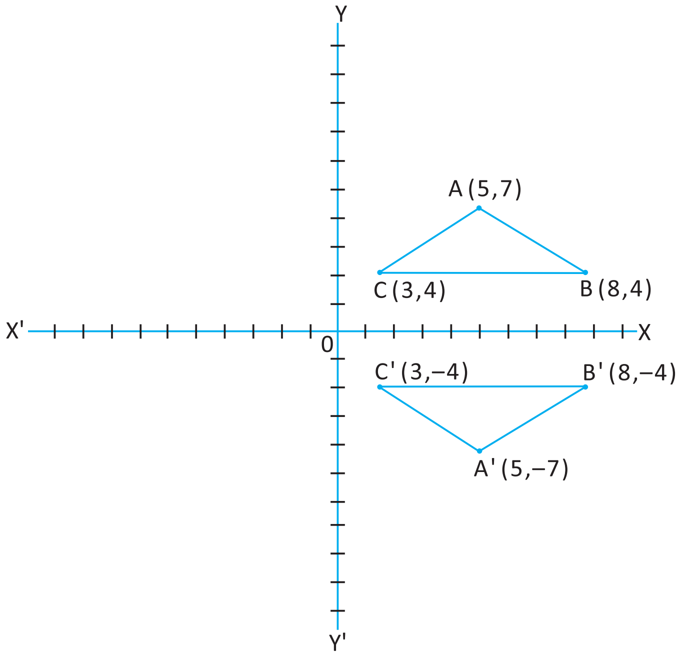
If we want to find out the coordinates of reflated points, line segments or shapes about the horizontal axis (x-axis) or vertical axis (y-axis), we can do so on the graph paper.

Use a graph paper. Draw (x-axis) and (y-axis) on it. Label its grades (0, 1, 2, 3, ...) on x-axis. Label its grades (0, 1, 2, 3, ...) on y- axis. Do the same on the negative sides of both axis. Now, plot the following figure about the y-axis (as has been shown). Draw the reflection of the figure as well.



Note that all values of abscissa (points on the x-axis) have changed to negative values. The values of ordinates remain the same. The ΔA'B'C' is a mirror image or reflection of ΔABC.

Now, let us see the reflection of ΔABC about the horizontal axis or x-axis.



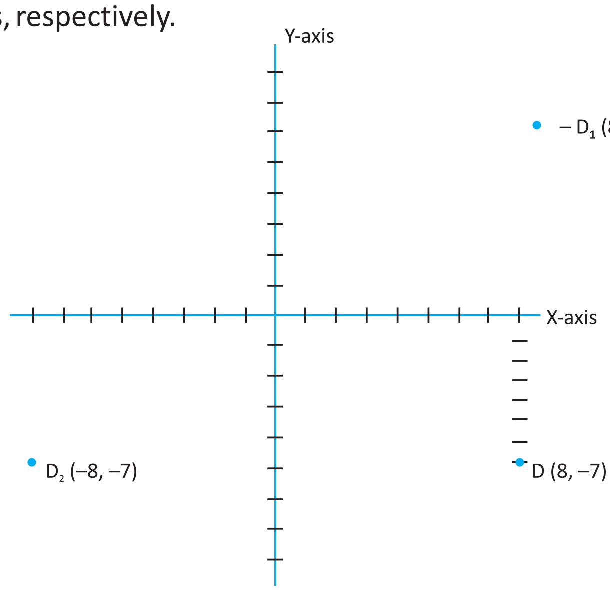
Here, we can see that the abscissa do not change in terms of sign. They remain the same. The values of ordinates are changed. ΔA' B' C' is a mirror image of ΔABC when the reflection is about the horizontal axis (x-axis).

Laws of Reflection

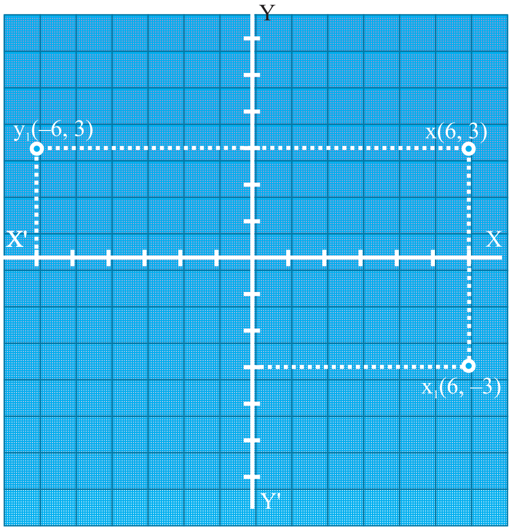
1. Every point on the axis of reflection remains unchanged upon reflection.
2. The shape and size of geometrical figure do not change.
3. If a line is parallel or perpendicular to the axis of reflection, its image will also be parallel or perpendicular to the axis of reflection.
4. The orientation of a figure is reversed upon reflection. Its LHS becomes RHS and vice-versa. If it is pointing upwards, its image points downwards. If it is facing LHS, it seems to be facing RHS in the reflection.

Example 6 : Plot a point D (8, –7) on the graph paper. Then, plot its reflection D_1, and D_2, along the x - axis and y - axis, respectively.

Solution :



Example 7 : **Draw a point x (6, 3) on a graph paper. Now, draw its reflection on the x-axis as x_1 and on the y-axis as y_1. What are the coordinates of x_1 and y_1 ? Can you find the coordinates of x_1 and y_1 without plotting them ?**



Solution : See the graph paper and the reflection of x as x_1 and y_1.

The coordinates of x_1 are (6, – 3).

The coordinates of y_1 are (– 6, 3).

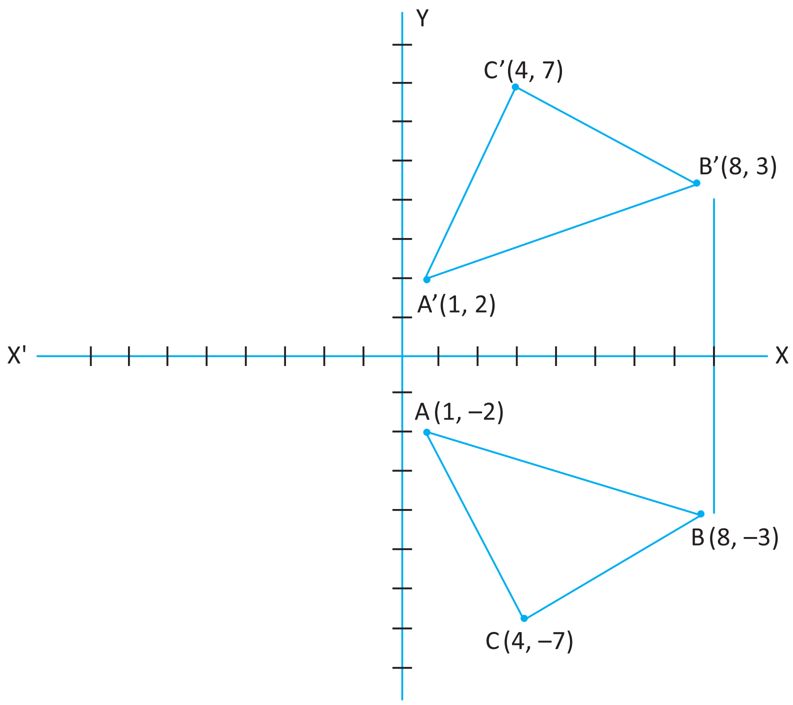
Yes, we can find out the coordinates of x_1 and y_1 without plotting them on the graph paper. For x_1, the value of ordinate will be negative, the value of abscissa remaining the same. Hence, $x_1 \leftrightarrow (6, -3)$.

For y_1, the value of abscissa will be negative, the value of ordinate remaining the same.

Hence $y_1, \leftrightarrow (-6, 3)$.

Example 8 : **Find out the coordinates of ΔABC where A ↔ (1, –2), B ↔ (8, –3) and C ↔ (4, –7) and the axis of reflection is the x-axis.**

Solution :



The reflection of ΔABC is ΔA'B'C'.

The coordinates of its vertices are as follows :

A' - ↔ (1, 2)

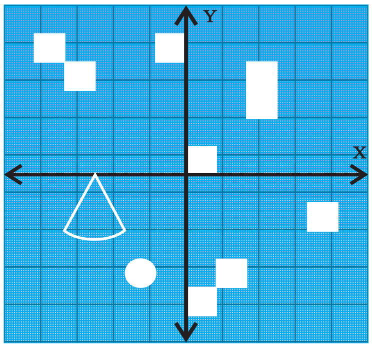
B' - ↔ (8, 3)

C' - ↔ (4, 7)

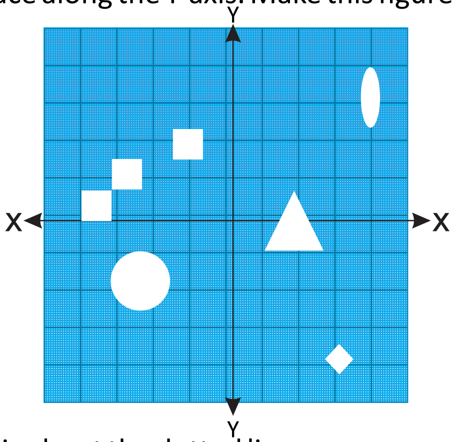
Exercise 14.2

1. Draw three points MNP on the graph paper such that M ↔ (4, –7), N ↔ (8, –9) and P ↔ (7, – 6). Draw the reflections of ΔMNP along horizontal and vertical axis.

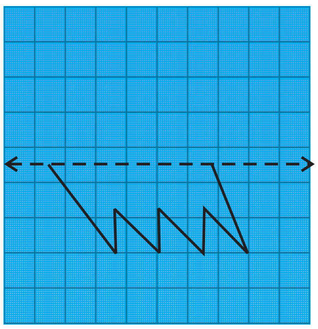
2. Assuming that reflection is taking place along the X-axis and making this figure symmetrical along the X-axis.



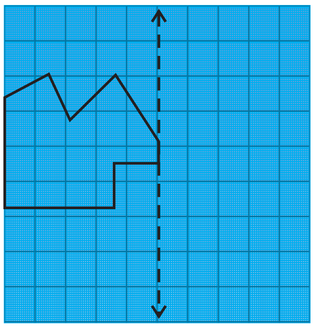
3. Assuming that reflection is taking place along the Y-axis. Make this figure symmetrical along the Y-axis



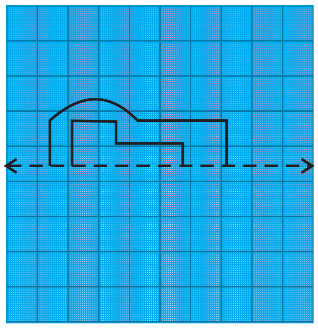
.4. Make the following shapes symmetric about the dotted lines.



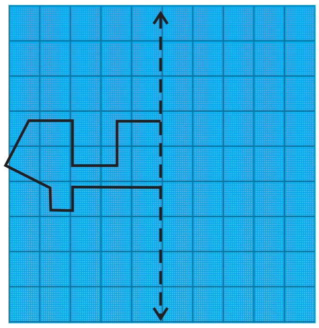
(a)



(b)



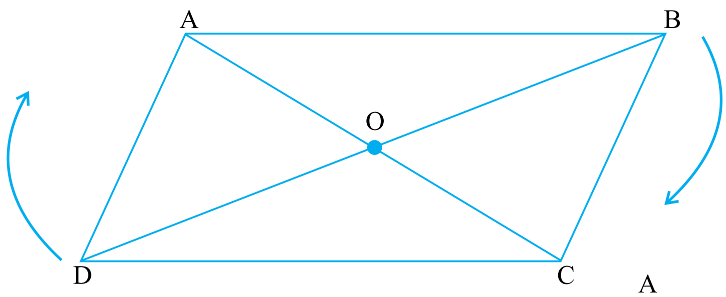
(c)

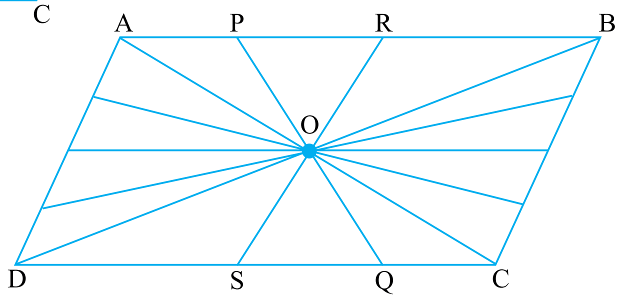


(d)

Point Symmetry

A parallelogram is not an symmetric figure. Because it is not symmetric about any one of its axes. Hence, it is asymmetric about its diagonals and bisectors of its sides.





Let us rotate the parallelogram about the point O, which is the point of intersection of its diagonals. When we do so, we find that it rotates evenly about point O. Hence, we can state that the parallelogram ABCD is symmetrical about point O.

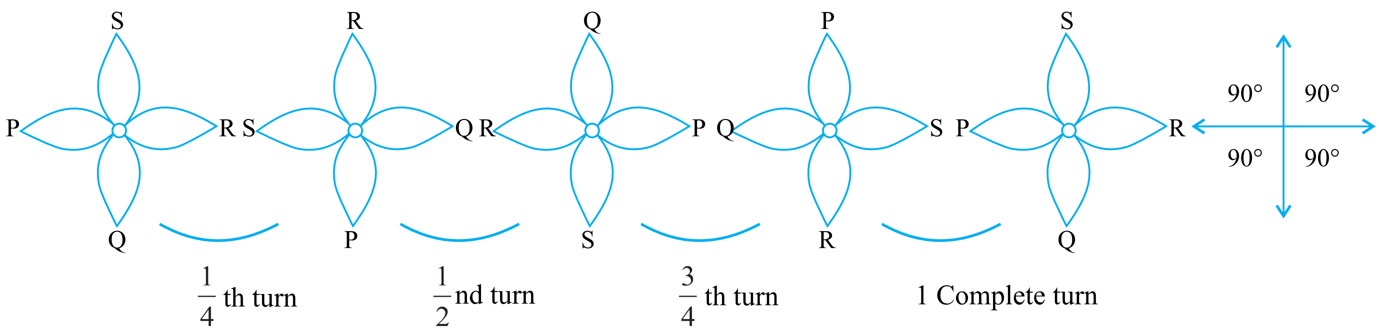
When we go deeper, we find that the point O bisects all line segments that pass through it. So, AO = OC, BO = OD, PO = OQ and RO = OS and so on.

Here, O is called point of symmetry.

The size and shape of geometrical figure remains unchanged, if it is rotated by an angle of 360°.

Rotation

You must have seen a fan rotating on the ceiling of your home. It has four blades. These blades evolve around one point. When this fan rotates around a fixed point, called axis of rotation, the blades change their position in a uniform manner in the space. So, let us draw a diagram to understand this movement.



With each quarter turn, the single blade moves ahead by an angle of 90°. All other blades naturally move ahead. They all are turning about a point. The axis of rotation passes through a point. The direction of the rotation can be clockwise or anti-clockwise.

Facts to Know

- When an object is rotated about a fixed point, every point on the object rotates through the same angle relative to the fixed point.

Rotational Symmetry and Rotational Order

A figure is said to have rotational symmetry, if it appears to be the same when rotated by an angle of x°, where $x \leq 180°$ Further, the quotient of $\frac{360°}{x°}$ is called Rotational Order of the figure.

Let us take up a few examples :

(a) Rotational order of square $= \frac{360°}{90°} = 4$

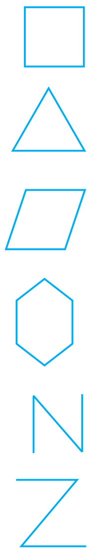
(b) Rotational order of triangle $= \frac{360°}{120°} = 3$

(c) Rotational order of parallelogram $= \frac{360°}{180°} = 2$

(d) Rotational order of hexagon $= \frac{360°}{180°} = 2$

(e) Rotational order of letter N $= \frac{360°}{180°} = 2$

(f) Rotational order of letter Z $= \frac{360°}{180°} = 2$



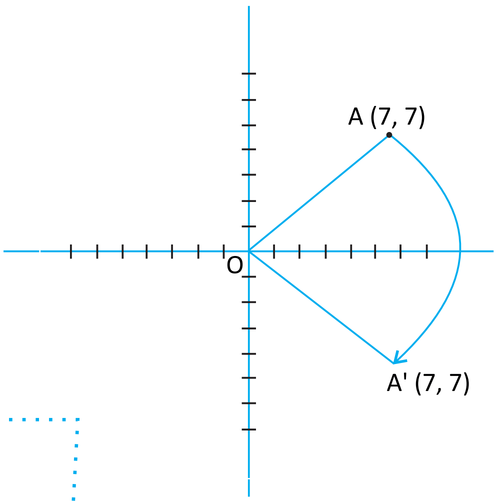
Facts to Know

- When an object rotates, its size and shape remain unchanged.

Rotation of a Point

If a point is rotated about a centre, its distance from the centre of rotation remains unchanged. Look at the graph shown here. The point A (7, 7) is rotating in a circle. The centre of rotation is O. The distance OA does not change even as the point A rotates about O.

In the figure given here, the point A is moving in a clockwise direction. Note that anti-clockwise movement of this point about centre O is also possible.

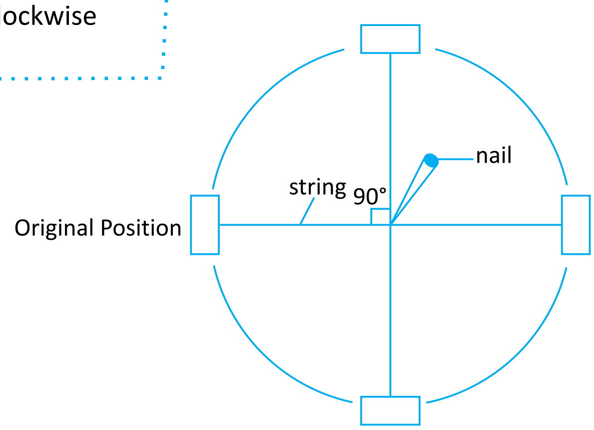


Facts to Know

- One complete turn of an object about the point of rotation covers 360°. So, $\frac{1}{4}$th turn = 90°.
- As already stated, this movement can be in clockwise and anti-clockwise directions. When A reaches A' it has covered an angle of 90°.

Rotation of Line Segment

Consider a piece of plastic in the rectangle shape for this activity. Now tie the plastic rectangle in the middle with a nylon rope. On a wooden table, nail down the other end of the string. The length of plastic rectangle can be 5 inches. The length of nylon rope can be 10 inches. That depends on the size of table top you have.



Rotate the plastic rectangle by 90° in each turn. The position and size of line segments do not change when they rotate about a fixed point. The plastic rectangle gets inverted when it is rotated by an angle of 180°. The same is true for triangles, pencils and polygons.

Laws of Rotation of Line Segements

1. The size and shape of the rotating figure do not change.
2. The centre of rotation remains fixed.
3. The distance between the figure and centre of rotation remains fixed.
4. If the figure is a solid object, it is inverted upon turning by an angle of 180°.

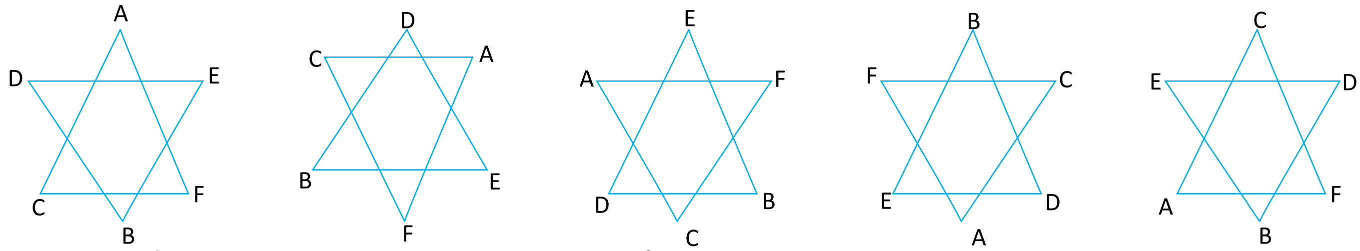
Facts to Know

- If you rotate a figure through 90° in the clockwise direction, it is the same as rotating the same figure through an angle of 270° in the anti-clockwise direction.

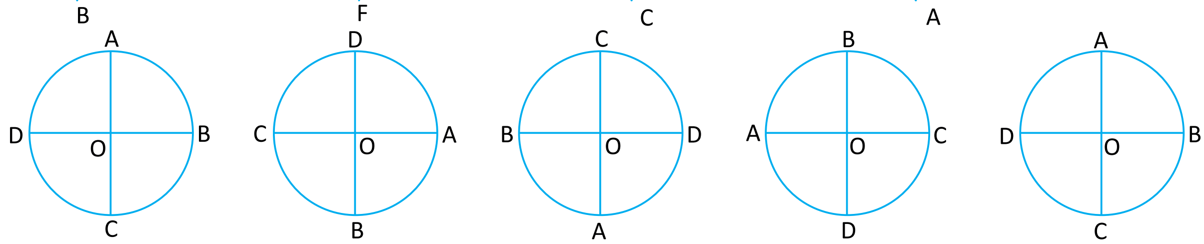
Example 9 : **Write the centre of rotation, angle of rotation, direction of rotation and order of rotation for the following figures.**

(a)

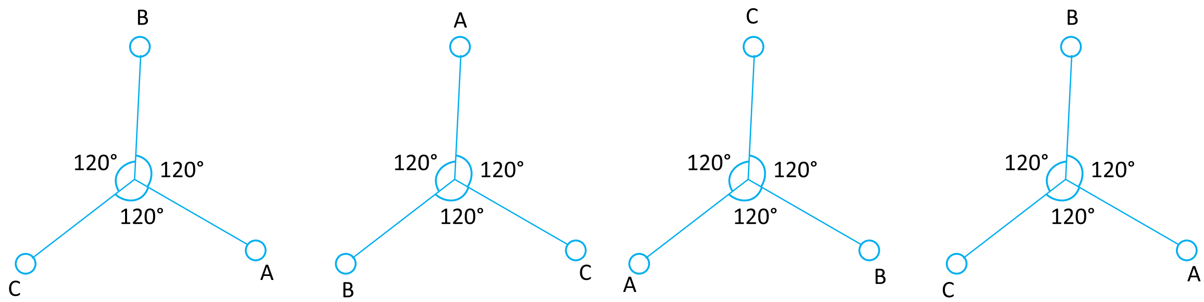
(b)



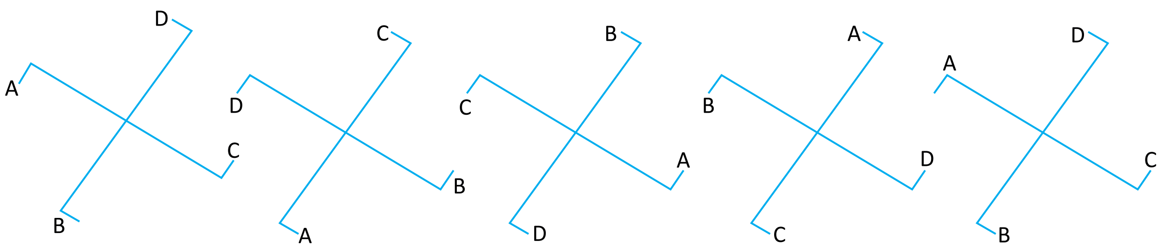
(c)

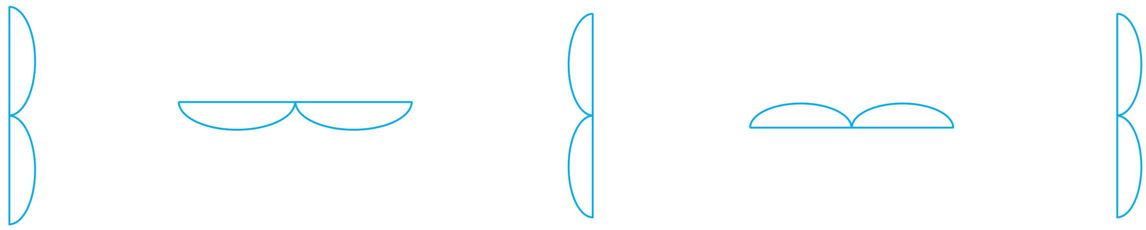


(d)



(e)



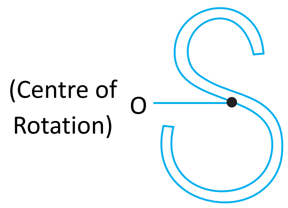
(f) 

Facts to Know

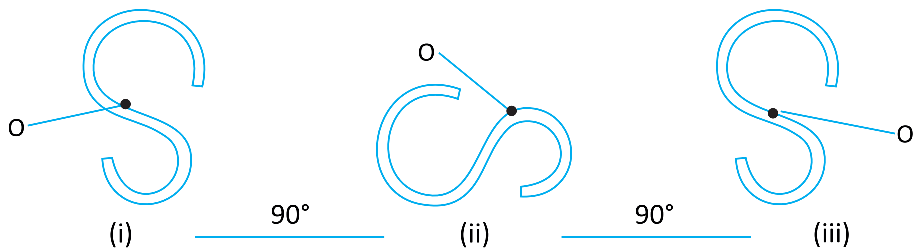
- When the direction of rotation has not been mentioned in the question, assume that the figure is rotating in the anti-clockwise direction.
- Rotational order or order of rotation are the same concept.

Solution :

(a) The letter S has the centre of rotation as shown here.



The angle of rotation is the angle through which a figure must rotate to look like the original figure. Note that the figure should look alike the figure at the first (original) position. It is not necessary that it may come to its original state. If we apply this rule, we find that S has to turn through an angle of 180° to look like the original position (shown by (i) here). The position of the centre of rotation is clearly visible here (in all three figures).



The direction of rotation is clockwise.

Order of Rotation = $\frac{360^\circ}{x}$

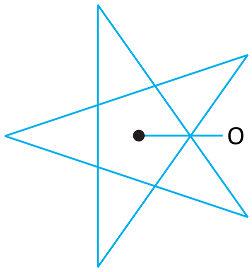
where x = angle rotated in degrees by the figure to come to a state in which it looks exactly like the figure in the original position (the figure may not have come to its original state).

So, order of rotation for S = $\frac{360^\circ}{120^\circ} = 3.$

(b) The centre of rotation for this figure has been shown ahead.

If this figure rotates by an angle of 72°, then it comes to a state in which it looks like the original figure.

Hence, We have



Angle of rotation = 72°

Direction of rotation = clockwise

Order of rotation = $\frac{360^\circ}{x}$

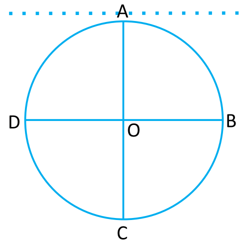
Hence, x = 72°

$\Rightarrow$ Order of rotation = $\frac{360^\circ}{72^\circ}$ = 5

Facts to Know

- If we rotate a figure by 180°, the centre of rotation remains unchanged. Also, the size and shape of the figure remain the same. Every point of figure remains equidistant from the centre of rotation.
- A circle is a polygon with an infinite number of sides.

(c) The centre of rotation of this figure is O, which has been shown below.



It is clear from the figure that point A has moved ahead by an angle of 90°.

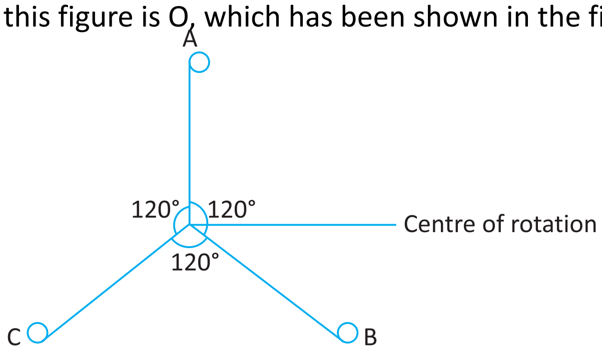
So, all other points are also moving ahead by an angle of 90° each. The figure remains the same when the angle of rotation is 90°. The direction of rotation is clockwise.

Order of rotation = $\frac{360^\circ}{x}$

x = 90°

$\Rightarrow$ Order of rotation = $\frac{360^\circ}{90^\circ}$ = 4

(d) The centre of rotation of this figure is O, which has been shown in the figure given here.



It is clear that this figure is moving ahead by an angle of 120° with each step.

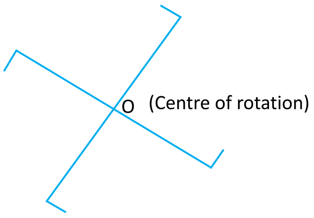
The direction of rotation is clockwise.

Order of rotation = $\frac{360^\circ}{x}$

Here, x = 120°

$\Rightarrow$ Order of rotation = $\frac{360^\circ}{120^\circ}$ = 3

(e) The centre of rotation of this figure is O, which has been shown in the figure given here.



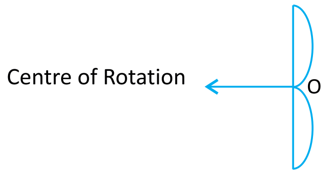
It is clear from the set of given figure that the figure is rotating by an angle of 90°. After turning through this angle, the figure remains similar to its original state. Further the direction of rotation of this figure is anti-clockwise.

Order of rotation $= \frac{360°}{x}$

Where $x = 90°$

$\Rightarrow$ Order of rotation $= \frac{360°}{90°} = 4$

(f) The centre of rotation of this figure has bean shown as point O in the figure given here.



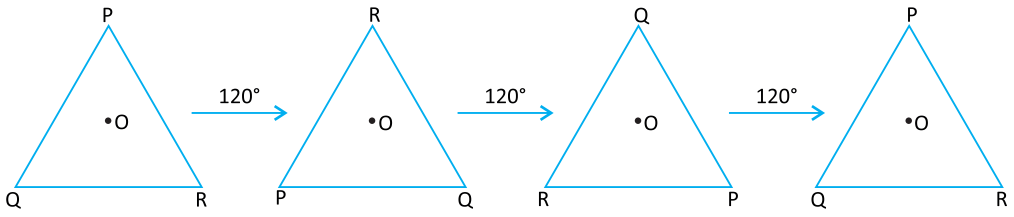
Further, the figure is moving by an angle of 360° to come to its original or similar-to-original state. So, $x = 360°$

But, $x \leq 180°$ for calculating order of rotation.

Hence order of rotation of this figure cannot be calculated.

Example 10 : Discuss the rotational symmetry of an equilateral triangle.

Solution : Look at the figure shown here. It shows an equilateral triangle ΔPQR.



It is clear that ΔPQR is rotating in an anti-clockwise direction.

The angle of rotation is 120°. It means that when the triangle rotates by an angle of 120° in the anti-clockwise direction, it assumes the same state as the original position (although its vertices are not at their original positions.

The centre of rotation ΔPQR is O, which has shown in the figure given here. When ΔPQR rotates through an angle of 360° (after three turns of 120° each), then its vertices come back to their original (initial) position. But it assumes the shape of the original state by turning through an angle of 120°.

Order of rotation $= \frac{360°}{x}$

Where, x = angle turned per time of rotation

= 120°

Order of rotation $= \frac{360°}{120°} = 3$

Example 11 : Give the tabular format of line symmetry, number of lines of symmetry, rotational symmetry and order of rotational a symmetry of the following letters of the English Alphabet.

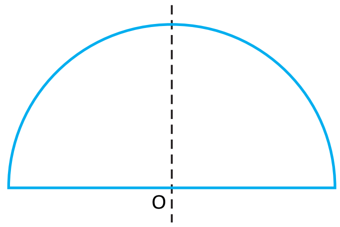
S	O	N	H	C	E

Solution : The data is as follows .

Alphabet	Line symmetry	No. of lines of symmetry	Rotational Symmetry	Order of Rotational Symmetry
S	No	0	Yes	2
O	Yes	2	Yes	2
N	No	0	Yes	2
H	Yes	2	Yes	2
C	Yes	1	No	0
E	Yes	1	No	0

Example 12 : (a) Is there any symmetry for a semicircle?

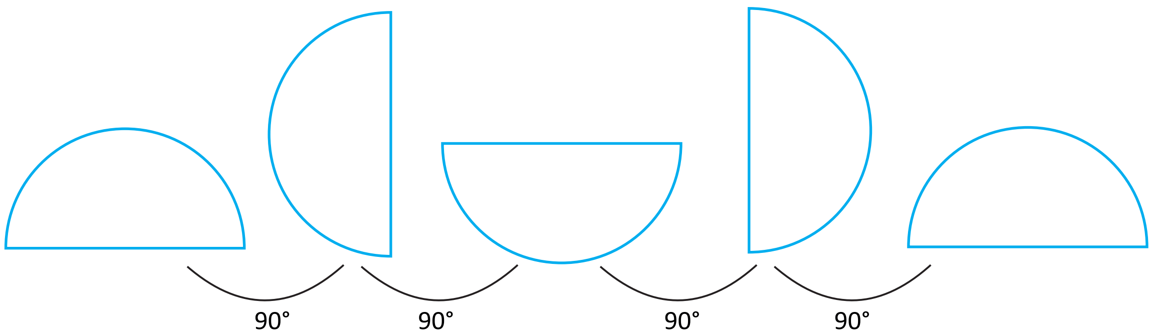
(b) Does a semicircle have rotational symmetry?



Solution : (a) The semicircle has symmetry about the right bisector of its diameter. Now look at the figure shown here.

If it is folded along the right bisector at O (dotted line), the two halves so obtained would be congruent to each other.

(b) By definition a figure has rotational symmetry, if it obtains its original state or a state exactly similar to its original state by rotating through an angle of x, where x $\leq$ 180°. In



case of the semi-circle the rotation has to be done through an angle of 360° (90°×4) to obtain its original or similar to original state. Hence, the semicircle does not satisfy the condition for rotational symmetry.

Exercise 14.3

1. Give the centre of rotation of isosceles rectangle, circle, square, rhombus, equilateral triangle, regular pentagon and regular hexagon.
2. **What is the order of rotational symmetry of a ceiling fan with :**
   (a) 3 blades (b) 4 blades (c) 2 blades
3. **Name the shapes of the figures that have rotational symmetry and whose angle of rotation are as follows.**
   (a) 45° (b) 60° (c) 90° (d) 120°
4. A line segment has coordinates A (4, 10) and B (0, 0). Rotate $\overline{AB}$ by 90° in the clockwise direction about the origin and find the coordinates of the image ($\overline{A'B'}$) of $\overline{AB}$.
5. **Find the coordinate of the images of the following points when they are rotated about the origin by 90° in the anti-clockwise direction.**
   (a) $P \leftrightarrow (-2, 3)$ (b) $Q \leftrightarrow (-5, -4)$ (c) $R \leftrightarrow (4, 2)$ (d) $S \leftrightarrow (5, -3)$
6. **Find the coordinates of the images of the following points when they are rotated about the origin by an angle of 180° in the clockwise direction.**
   (a) $P \leftrightarrow (-2, 3)$ (b) $Q \leftrightarrow (-5, -4)$ (c) $R \leftrightarrow (2, 4)$ (d) $S \leftrightarrow (4, -6)$
7. **Plot a quadrilateral with points as follows :**
   $A \leftrightarrow (-6, 5)$
   $B \leftrightarrow (8, 4)$
   $C \leftrightarrow (7, -6)$
   $D \leftrightarrow (-8, -4)$
   Plot its image if it is rotated by an angle of 180° in the anti-clockwise direction.

Points to Remember

- Figures can be symmetrical or asymmetrical.
- A figure has line symmetry if there is a line about which the figure may be folded so that its two half parts coincide with each other exactly and are congruent with each other in terms of shape and size.
- The axis of symmetry is the line around which figure shows line symmetry.
- A figure that shows line symmetry about at least one line or axis is called **Symmetrical figure**.
- Curved figures can also be symmetric, besides straight lines and line sketches.
- The total number of lines of symmetry of a regular polygon is equal to the number of its sides.
- Figure can be reflected about horizontal, vertical or slant axis (lines) of reflection.
- Laws of reflection are followed with reflecting objects or figures about axis. There are four laws of reflection.
- A figure is symmetrical about a point if that point bisects all line segments of the figure. That can be drawn in it. A figure may not be symmetrical about any axis but it may be symmetrical about a point.
- Rotation of a point around a fixed centre does not change the distance between the point and that centre.
- When a figure like rectangle is rotated about a fixed point, it gets inverted upon rotating by an angle of 180°.
- Four laws of rotation govern the rotation of line segments around a fixed point.
- Rotational order = $\frac{360°}{x}$, where $x \leq 180°$
- When an object rotates, its size and shape remain unchanged.

EXERCISE

1. MULTIPLE CHOICE QUESTIONS (MCQs):

Tick (✓) the correct options :

(a) The reflection of point P ↔ (3, –8) along y-axis is
- (i) P' ↔ (–3, –8) ☐ (ii) P' ↔ (–3, 8) ☐
- (iii) P' ↔ (3, 8) ☐ (iv) P' ↔ (8, 3) ☐

(b) The rotational order of a regular octagon is
- (i) two ☐ (ii) four ☐
- (iii) six ☐ (iv) eight ☐

(c) A parallelogram has
- (i) No point of rotation ☐ (ii) One axis of rotation ☐
- (iii) One point of rotation ☐ (iv) None of these ☐

(d) An equilateral triangle, if rotated by an angle of 180° will become a/an
- (i) rhombus ☐ (ii) isosceles triangle ☐
- (iii) scalene triangle ☐ (iv) none of these ☐

(e) If a small scale is rotated about the point (0, 0) on a graph paper, after covering an angle of 180°, it will
- (i) remain as such ☐ (ii) get inverted ☐
- (iii) become a square ☐ (iv) become a point object ☐

(f) The orientation of a figure is reversed upon
- (i) drawing ☐ (ii) rotating ☐
- (iii) reflection ☐ (iv) deletion ☐

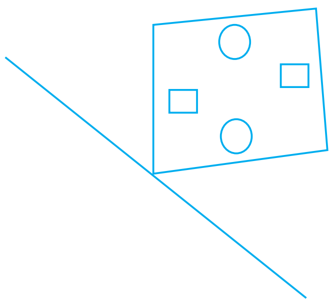
(g) The number of lines of symmetry for a rectangle is
- (i) 4 ☐ (ii) 3 ☐
- (iii) 1 ☐ (iv) 2 ☐

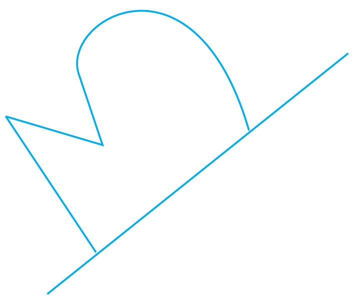
(h) The number of lines of symmetry in a circle is
- (i) 1 ☐ (ii) 2 ☐
- (iii) 3 ☐ (iv) None of these ☐

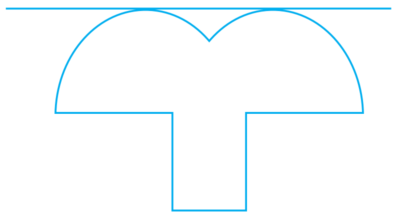
2. Draw all lines of symmetry for the following.

V W X Y M U B A T

3. Draw the reflections of the following figures along the dotted lines.







4. **Find the coordinates of the images of the following points when they are rotated as has been described.**

(a) P ↔ (–7, 6) ; clockwise 90°

(b) T↔ (–18, –12) ; anti-clockwise 180°

(c) R ↔ (6, –20) ; clockwise 90°

(d) A ↔ (7, 21) ; anti-clockwise 180°

(e) B ↔ (0, –4) ; anti-clockwise 180°

(f) D ↔ (–11, 8) ; anti-clockwise 180°

5. **Find the coordinates of the images of the following points when rotated about the origin by an angle of 180°.**

(a) M ↔ (6, 18)

(b) T ↔ (–4, –9)

(c) P ↔ (–7, 2)

(d) D ↔ (10, –5)

(e) X ↔ (–7, 0)

(f) Y ↔ (0, 11)

6. **Write T for true and F for False for the following statements.**

(a) The rotational order of a parallelogram is zero. ☐

(b) If a triangle is reflected along y-axis, the abscissae of its points would get opposite signs. ☐

(c) If a quadrilateral is reflected along x-axis, the ordinates of its point would get opposite signs. ☐

(d) A parallelogram is asymmetrical about any axis. ☐

(e) A point P ↔ (– 4, 12) is reflected along y-axis, its reflection P' has the coordinates (12, – 4). ☐

(f) The point x (–6, –3) has bean rotated by an angle of 180°. Hence, its reflection is *x*' (6, 3). ☐

(g) The total number of lines of symmetry of a regular polygon is equal to the number of its sides. ☐

(h) Rotation of a figure around a point changes the shape and size of such a figure. ☐

(i) Rotational order = $\frac{360^\circ}{x}$, where $x \leq 180^\circ$ ☐

(j) If, after rotation, the object looks exactly the same as it was in its original state, it is said to have rotational symmetry. ☐

7. **Draw the lines of symmetry for the following.**

(a) straight line

(b) line segment

(c) kite

(d) inverted heart

(e) rhombus

(f) the letter, J

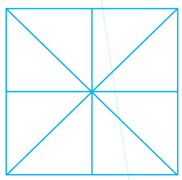
HOTS

Make 3 copies of the shape alongside. Shade triangles in such a way as to create shapes with:

(a) 1 line of symmetry

(b) 2 lines of symmetry

(c) Rotation symmetry of order 2



Lab Activity

Objective : To make designs with rotating shapes

Materials Required : Chart papers, a pair of scissors, sketch pens, fevicol

Procedure :

Step 1. Draw a rectangle and make a shape as shown :

Step 2. Cut out the shape as shown. Make 10 identical shapes :

Step 3. Take another chart paper and mark a dot. Place one cutout shape on the dots as sown :

Step 4. Rotate the shapes to a new position and paste it, keeping the same point of the shape on the dot as shown :

Step 5. Continue rotating and pasting the shapes until you complete the circle.

Step 6. Colour the design, and make it more attractive.

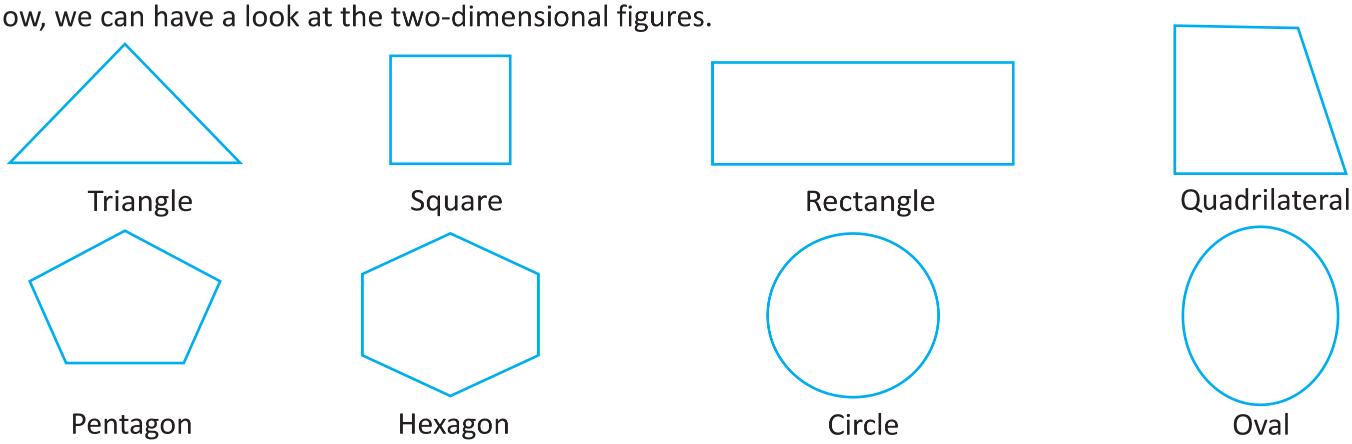
15 Representing 3-D in 2-D

In previous classes, we studied length, breadth, height and area. These properties were studied, for two dimensional figures. Example : Land field, table top, floor, sheet of paper etc. All these have **length** and **breadth** or breadth and **height** as their two vital parameters. But in our everyday life, we do not see flat objects at all times. We see ice-cream cone, football, gas cylinder, room, hall, tumbler, tiffin box, book, pen, cake etc. All of these are solid shapes, not flat shapes. So, we have three types of things around us. We can have things having only length (like straight line). Then, we can have things having two dimensions, which can be **length** and **breadth** or breadth and **height** (like table top, wall of classroom, blackboard surface, notebook page, book page, top view of a frisbee etc.). Finally, we can have things having three dimensions, which are **length**, **breadth** and **height**. The third category is also called **solid objects** or **three-dimensional shapes**.

Let us recall the shapes. The figures with only one-dimension have been shown here.

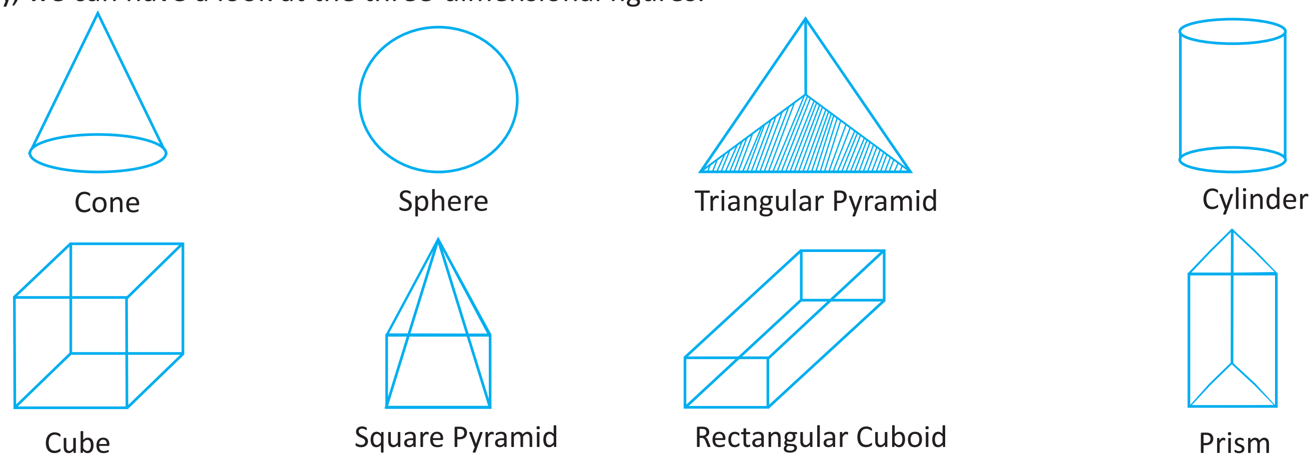
One-dimensional figures

Now, we can have a look at the two-dimensional figures.



Two-dimensional figures

Finally, we can have a look at the three-dimensional figures.

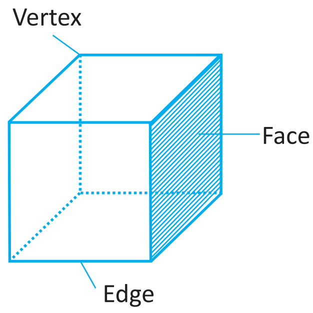


Three-dimensional (solid) figures

One-dimensional figure is a simple figure.
Two-dimensional figure is a 2-D figure.
Three-dimensional figure is a 3-D figure.
In this chapter, we shall discuss 3-D figures and their representation in terms of 2-D figures.

Definitions Regarding Solid Objects

Look at this figure. It is a cube. Let us discuss its various features.



We can see that a three-dimensional object like cube can be made on a sheet of paper. This cube has the following features:

1. It has six surfaces.
2. It has twelve edges. An **edge** is formed when two adjacent surfaces of a solid intersect.
3. It has six square-shaped faces. A plane surface enclosed by an edge or edges is called **face**.
4. It has eight vertices. A **vertex** is a point where three surfaces meet. A vertex of a solid can also be called **corner**.

We have done this exercise for the cube. We can find out these properties for other solids as well. Read the table that follows:

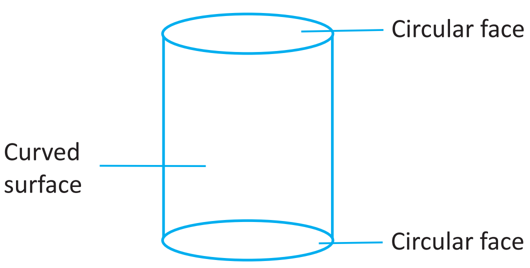
Solid	Prism	Square pyramid	Cuboid	Triangular pyramid
Edges	9	8	12	6
Faces	5	5	6	4
Vertices	6	5	8	4

Facts to Know

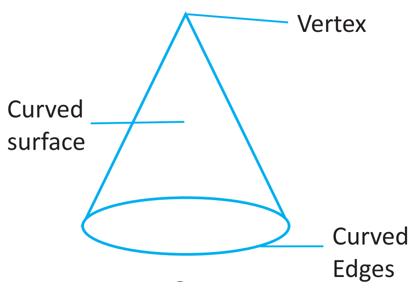
The 3-D shapes whose faces are polygons are called **Polyhedrons**. Example : cube, cuboid etc. The faces can be triangle, rectangle, square and so on. These objects have straight surfaces, not curved ones.

Definitions Regarding Solid Curvilinear Objects

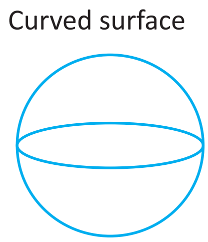
The cylinder, cone and sphere have curved surface. So, we have the concept of curved surface, curved edge, curved face and circular face in case of curvilinear solid shapes. Look at the figure shown here :



Cylinder



Cone



Sphere

Read the table that follows :

Solid	Cone	Cylinder	Sphere
Curved Edges	1	2	1
Curved Surfaces	1	1	1
Circular Faces	1	2	1
Vertices	1	0	1

Facts to Know

For any polyhedron (with straight surfaces), we have :

$F - E + V = 2$,

where, F = no. of faces of polyhedron

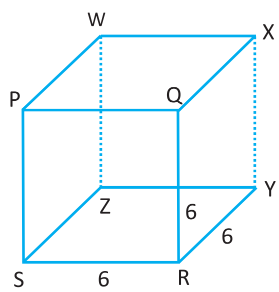
E = no. of edges of polyhedron

V = no. of vertices of polyhedron

Showing 3-D Objects in 2-D (Oblique Sketches)

We can represent 3-D objects in 2-D very easily. Let us start with the cube. Look at this figure. It is a cube of side 6 cm. Now, let us represent it on a sheet of paper in a 2-D format.

Take the following steps :



(i) Draw a square PQRS , so that each one of its sides is equal to 6 cm in length. Label the vertices too.

(ii) With PQ as a base, draw a parallelogram PQXW. You can use any acute or obtuse angle. But it cannot be a right angle. The sides of the parallelogram are also equal, i.e., 6 cm each.

(iii) From W and X, draw dotted vertical lines. Thus, draw WZ = 6 cm and XY= 6 cm.

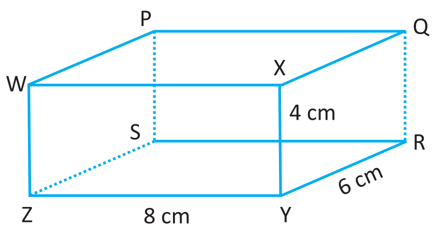
(iv) Join Z with Y, R with Y and S with Z.

(v) This is the required cube. Its vertices are P, Q, R, S, X, Y, Z, W.

(vi) It has 6 faces, 8 vertices and 12 edges. Each one of its edges is 6 cm long.

Now, let us draw a cuboid on a sheet of paper in the same manner. Look at the figure shown here.

Take the following steps :



(i) Draw a rectangle WXYZ on the plane sheet of paper, so that ZY = WX = 8 cm, XQ = WP = 6cm and WZ = XY = 4 cm.

(ii) With WX as the base, draw a parallelogram WXQP, so that WX = PQ = 8 cm and XQ = WP = 6 cm. You need not make a specific angle, it can be an acute or obtuse angle. It cannot be a right angle in any case (because parallelograms do not have right angles).

(iii) From P and Q, draw dotted vertical lines. Thus, draw PS = 4 cm and QR = 4 cm.

(iv) Join Z with S, Y with R and S with R.

(v) This is the required cuboid. Its vertices are : W, X, Y, Z, P, Q, R, S.

(vi) It has 6 faces, 8 vertices and 12 edges. Its length is 8 cm, breadth is 6 cm and height is 4 cm.

Similarly, we can draw the objects, which are in 3-D, on a sheet of paper in a 2-D format. In this chapter, we shall draw many 3-D objects on sheets of paper (in a 2-D format) to clear the concept.

From these two examples, it is clear that 3-D objects can be drawn on plane surfaces, in a 2-D format.

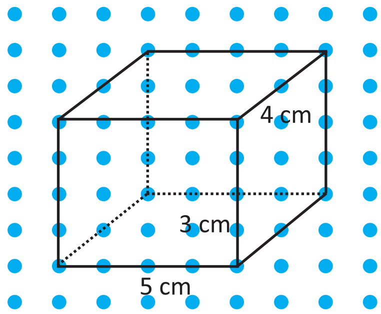
The following points can be noted in this context :

1. The images on plane surfaces (for all 3-D objects) are distorted.
2. The angles between various edges may be 90° in some 3-D objects but on paper (in a 2-D format), there may be oblique angles to represent 90°.
3. All faces of the 3-D object are not visible. It is obvious because the sketch on a plane surface cannot show more than two dimensions.
4. All the lengths are not equal to those in the original 3-D object. That is because of the limitations of the length and breadth of the plane surface.

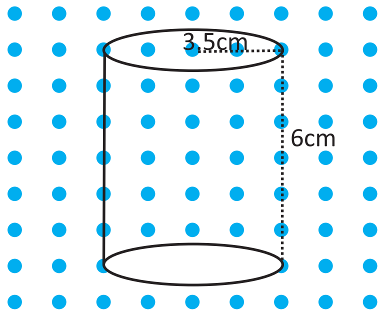
Showing 3-D Objects in 2-D (Isometric Sketches)

There is another way of making 3-D images on 2-D (plane) surfaces. Let us use the dotted paper to draw some sketches. It is available in markets. Alternatively, you can punt dotted paper from the punter. This paper has small dots in the form of equilateral triangles.

Your computer lab assistant in school can help you get these sheets on printer. In this case, the dimensions of the 3-D objects are exactly the same on this type of paper, just as they actually are. Let us draw a cuboid of length 5 cm × 4 cm × 3 cm on the dotted paper.



Let us draw a cylinder on the dotted sheet of paper. The height of cylinder is 6 cm. The radius of its upper and lower bases is 3.5 cm. Let us draw this 3-D figure in 2-D.



The dotted paper we used here is called **Isometric Paper**. In this sketch, the dimensions of the object remain the same on the 2-D (dotted) sheet.

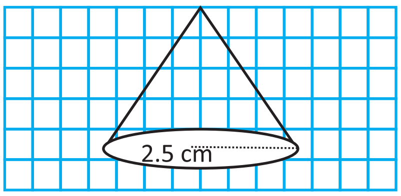
The sketch of a solid in which measurements are the same as the original ones (in 3-D) is called **Isometric Sketch**. The 2-D isometric sketches are exactly same as the 3-D solids but they are in 2-D format.

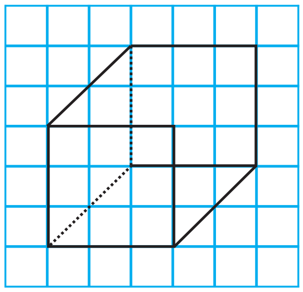
Showing 3-D Objects in 2-D (Squared Paper)

We can also show 3-D objects in a 2-D format on a squared paper. This sheet is available in stationary shops. Alternatively, your school's computer lab assistant would get it printed.

Let us draw the shape of a cone with radius 2.5 cm and height 3 cm on the squared paper, which has small squares of equal size. We can show only the 2-D format of cone on this squared paper.

Further, let us draw a cube of side 3 units on the squared paper.





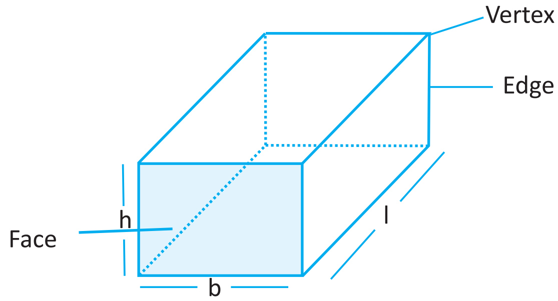
Facts to Know

We can represent 3-D shapes on plain paper, in 2-D format, in three ways:

(i) As oblique sketches
(ii) As isometric sketches
(iii) On squared paper

Example 1 : **Describe various parts of a cuboid by making a sketch and label it neatly.**

Solution :



Example 2 : **Give the specifications and line diagram of a square pyramid.**

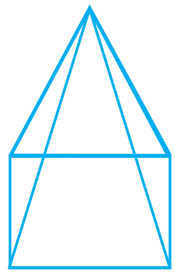
Solution : A square pyramid has been shown here. We can note the following specifications of this pyramid from the figure drawn here:

No. of edges = 8

No. of faces = 5

No. of vertices = 5

Shape similar to the pyramids is located in Giza (Egypt)



Example 3 : **Give the shapes of the following objects:**

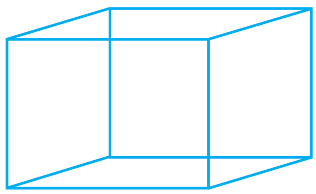
(a) Football	(b) Candle	(c) Train	(d) Wire
(e) Book	(f) Dice	(g) Lunch box	(h) Coin
(i) Cake of soap	(j) Joker's hat	(k) Laddu	(l) Eraser

Solution :

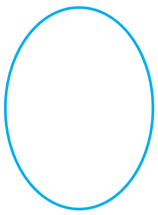
(a) Sphere	(b) Cylinder	(c) Cuboid	(d) Cylinder
(e) Cuboid	(f) Cube	(g) Cuboid	(h) disc
(i) Cuboid	(j) Cone	(k) Sphere	(l) Cuboid

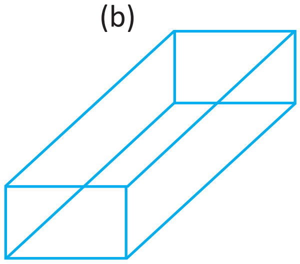
Example 4 : **Name the shapes of the following 3-D objects.**

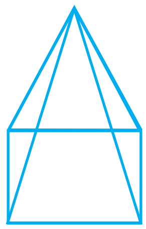

(a)


(b)


(c)


(d)


(e)

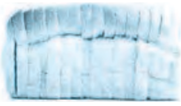
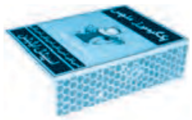
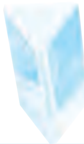

(f)

Solution :

(a) Cone	(b) Cube	(c) Cylinder
(d) Oval	(e) Cuboid	(f) Square pyramid

Exercise 15.1

1. Give the shapes of the following 3-D objects:

(a) 
(b) 
(c) 
(d) 
(e) 
(f) 
(g) 
(h) 

2. Draw the shape of an object whose length is 7 *m*, breadth is 5 *m* and height is 2 *m.* Draw it on a squared paper. Use the scale 2 *m* = 1 cm. What do you see?

3. Draw the shape of a room whose length, breadth and height are equal. Identify the edge, vertex, face and total number of faces of this 3-D object. Draw this object as an oblique sketch.

4. Write the number of vertices of the following :

(a) Sphere (b) Cylinder (c) Cube (d) Cone (e) Square Pyramid

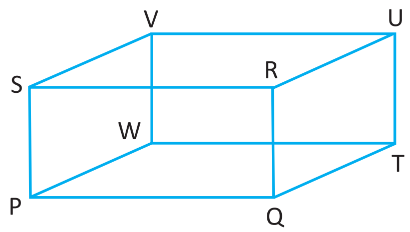
Using Nets to Depict 3-D Objects

We all write, read or view in 2-D environments. The book is single-dimensional. The TV is flat. The figures of geometry on our classroom's blackboard are in 2-D. Then, how can we represent 3-D objects in 2-D environment. This is a big problem. However, this problem can be solved with the help of nets. We can show three dimensions of a solid object in 2-D very easily. When we try to show the 3-D images on the sheet of paper (which is a 2-D object), we are unable to show all vertices, edges and faces. We shall use nets to show those hidden parts of 3-D objects.

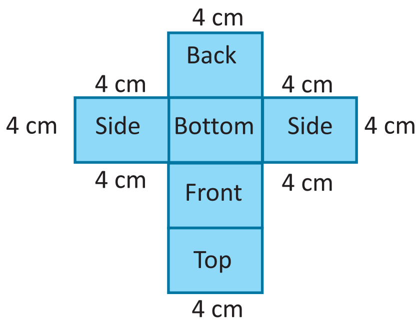
We use the shapes (that can be used on sheets of paper) to depict 3-D objects. For example, we can use squares, triangles, circles, rhombuses etc. to show solid (3-D) objects. The tool that we shall use is called **net**.

Using Net to Show a Cube

Look at this figure. It is a cube. It is a 3-D object. We have to represent it in 2-D. The length of each side of this cube is 4 cm.



Its net representation is as follows:



We can easily form a cube with the help of this net. Simply fold the square-shaped surfaces along the edges and the cube would be ready. You can try this by making this net on the sheet of cardboard. Cut it from the main cardboard. Now, fold it along marked edges. When the sides are lifted, the cube will be formed. Use cello-tape to fix the sides on the sides of their edges.

Using Net to Show a Cylinder

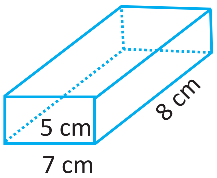
A right cylinder can be represented in the form of a net as follows :

It is clear that when the cylinder is cut open (from one side), its height remains the same and its circular edges become straight lines.

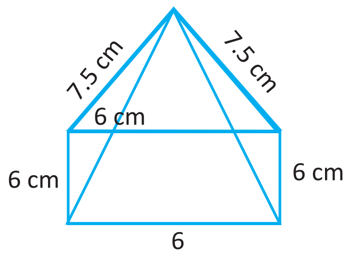
Facts to Know

We can represent all objects in 3-D on a sheet of paper. The hidden dimensions or edges can be shown with the help of dotted lines.

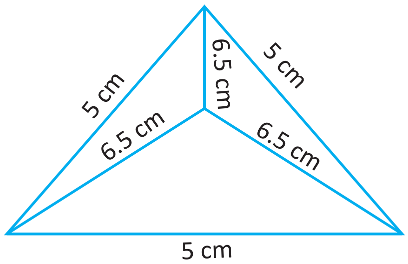
Example 5: **Draw the nets of the following 3-D object.**



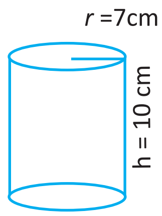
(a)



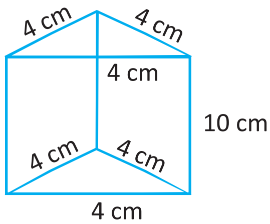
(b)



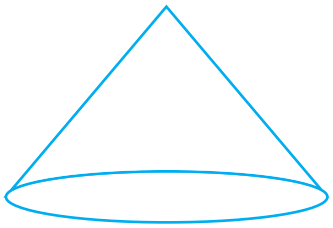
(c)



(d)

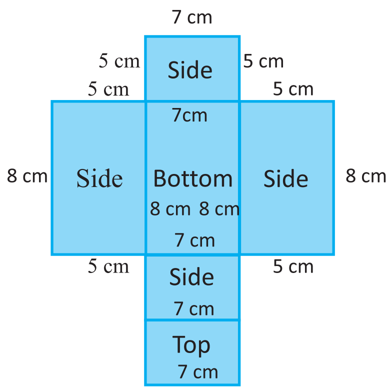


(e)

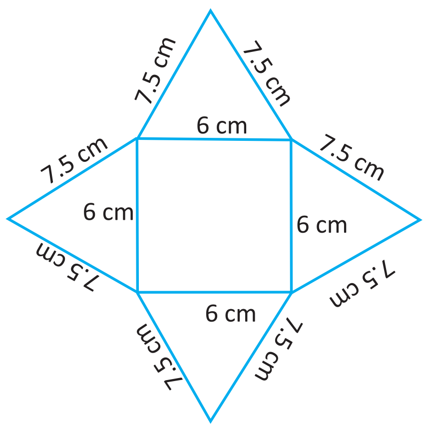


(f)

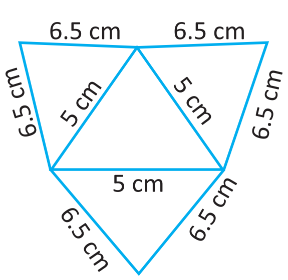
Solution :



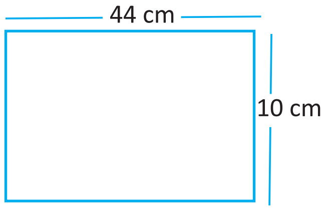
(a)



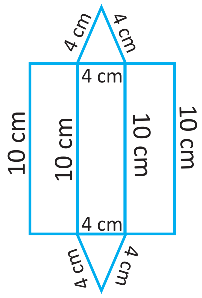
(b)



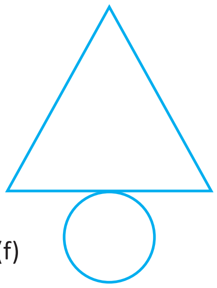
(c)



(d)



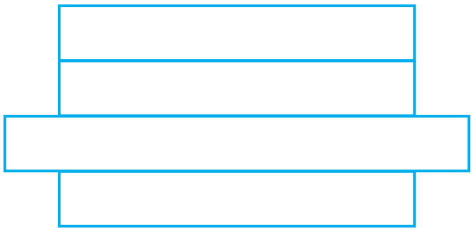
(e)



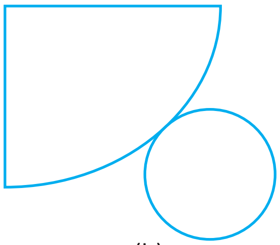
(f)

Exercise 15.2

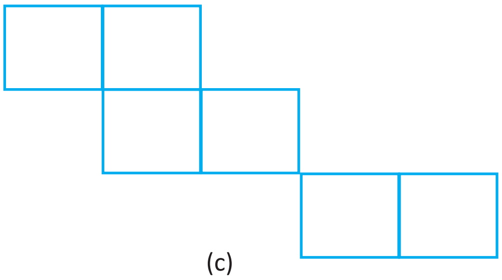
1. The nets of the 3-D images of some objects have been shown here. Draw the 3-D object for the same.



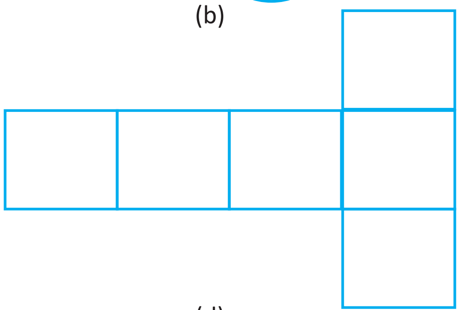
(a)



(b)

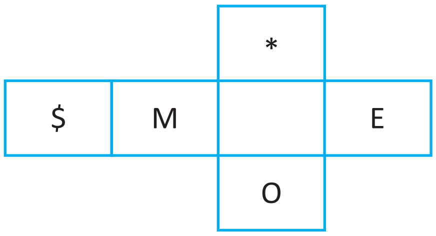


(c)



(d)

2. Given below is the net of a cube. Some signs have been marked on each one of the parts of the net, which are, in fact the faces of the cube. Now, answer the following questions :



(a) What is seen at the bottom?

(b) What is seen at the front?

(c) Which thing can you see at the top?

(d) What is located at the back?

(e) Who are the two vertically faced neighbours of M?

(f) Which one out of M,*, E and $ is not on the vertical face?

(g) Which faces are adjacent to the one having circle (O) ?

(h) Name the face pairs of the cube.

3. Draw the nets for the following objects :

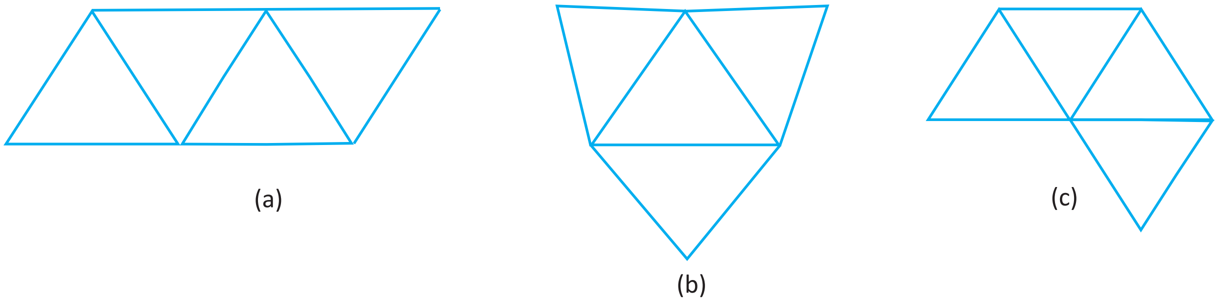
(a) A water pipe

(b) A dice of dimensions 2 cm × 2 cm × 2 cm

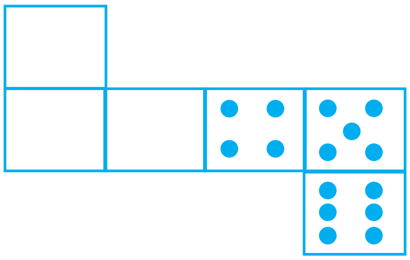
(c) A book of size 20 cm × 12 cm × 4 cm.

(d) A copper wire of radius 14 cm and length 112 cm.

4. **Find the odd one out.**

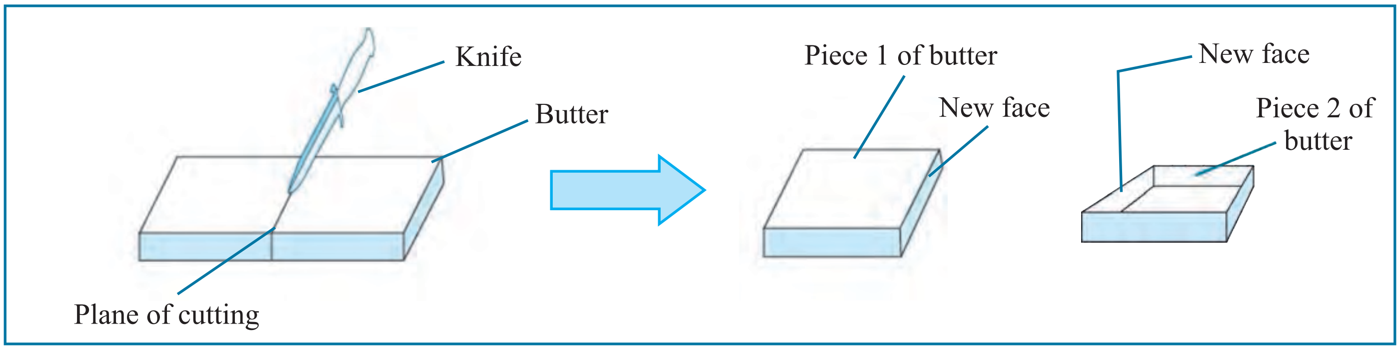


5. **The sum of the number of dots on the opposite faces of a dice is always equal to seven. Fill up the dots in the empty squares of the net of this dice.**



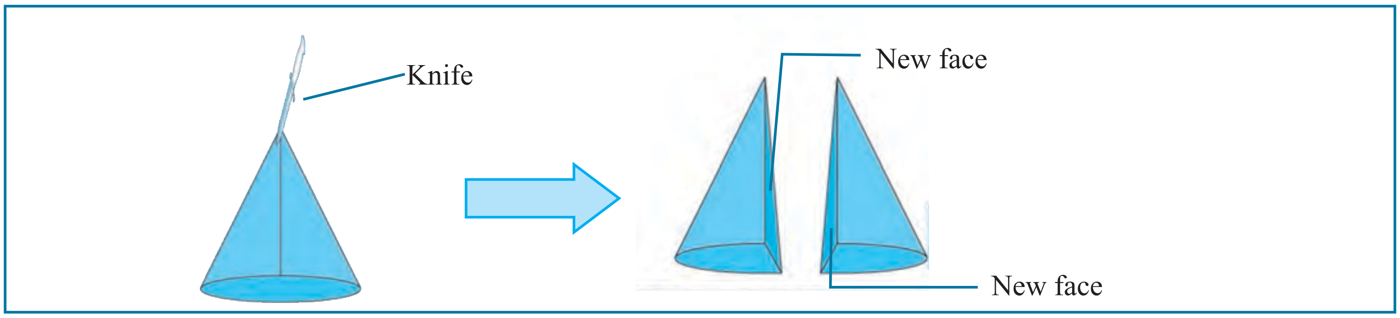
Visualising 3-D Objects : Cutting Through an Object

A 3-D object can be cut into a numbers of parts. When we cut a 3-D object with a knife, we get two parts. We also get two new faces at the cut portion (where we had used the knife). These new faces are known as **cross-sections of solid or 3-D shape**. Let us cut through a piece of butter, which is in the shape of a cuboid. We are cutting it into exactly two equal parts.



The new faces (cross-sections) are in the shape of rectangle.

Now, let us cut a solid cone in a vertical direction.

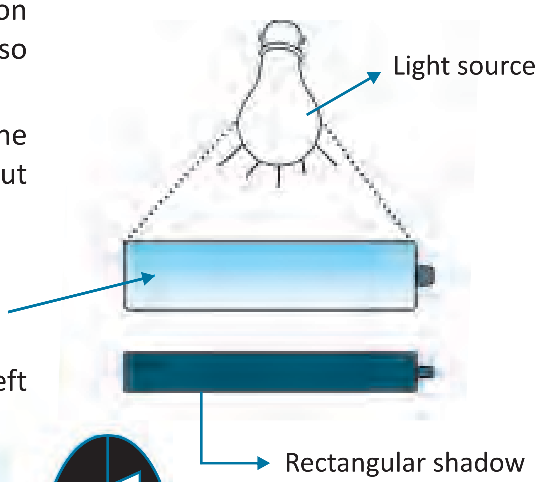


The new faces so obtained (in the case of cone) are in the shape of rectangle.

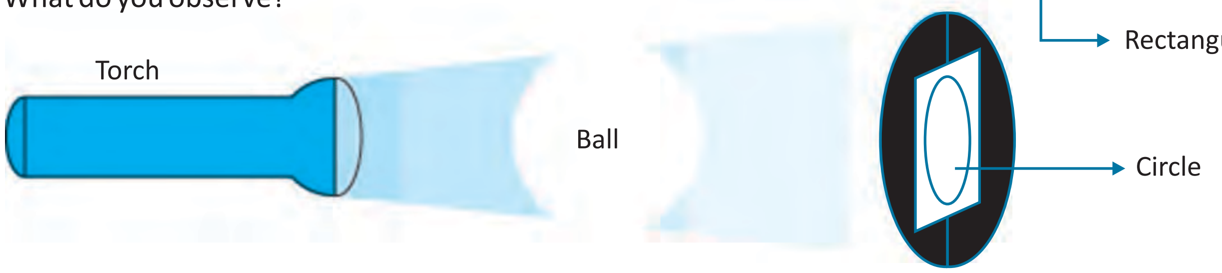
Visualising 3-D Objects : Creating Shadows of 3-D Solid Shapes

The shadow of a 3-D object (solid) is a 2-D image. When we throw light on a 3-D solid from one of its sides, we obtain an image. The shadow so obtained would depend upon which side we are throwing light from.

Let us throw light on a pencil-like cell (AA battery) from the top. At the bottom, we shall get a shadow that is not cylindrical (like the battery) but a rectangle.



Now, take a ball and throw light on it with the help of a torch from its left side. What do you observe?



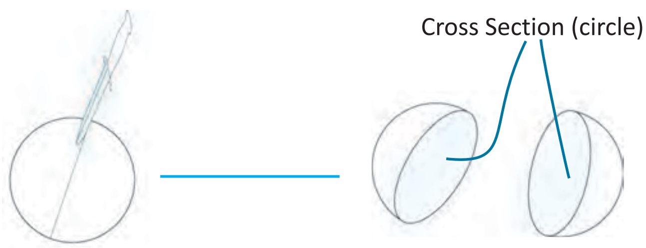
It is obvious that the shadow so obtained is a circle. In this case, we can throw light on the ball from any side. The shade obtained in any case would be a dark circle. So, the shape is also important while getting shades of 3-D objects.

Facts to Know

When we get shadows of 3-D objects, we observe the following:

1. The shades are 2-D images, they are dark too.
2. The side (face) from which we throw the light on the object is important.
3. The shape of object is equally important.

Example 6 : Cut through a ball and inform about the cross-sections so obtained.

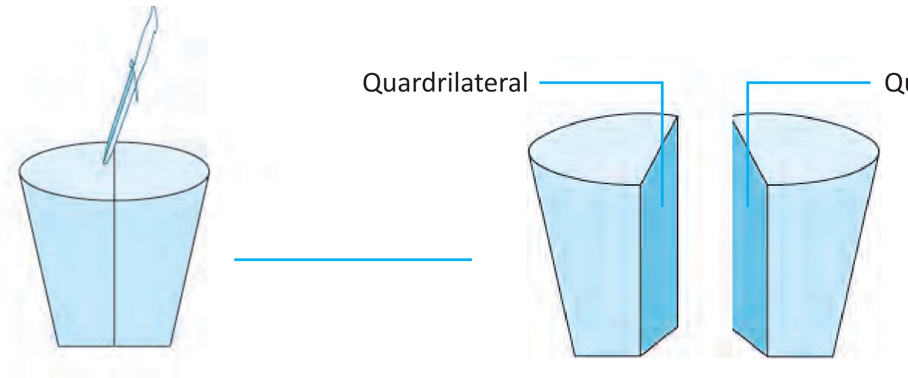


Solution : When we cut through the ball, we get two hemispheres. The cross-sections obtained after cutting are two circles. If the ball is not cut at any of its diameters, the two parts are not equal to each other. But cross sections of these parts are circles only.

Example 7 : Cut through a solid tumbler and inform about the cross-sections so obtained.

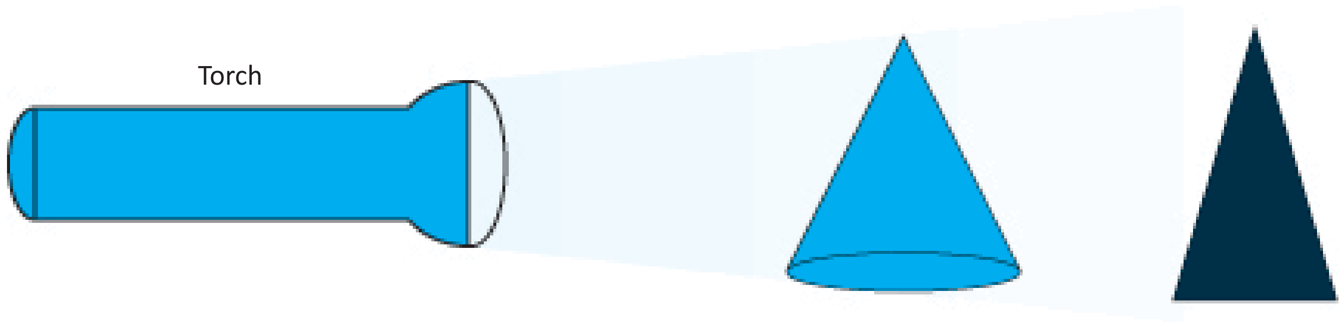
Solution : Look at the figure shown here. We assume that it is a solid tumbler.

Thus, we observe that when a solid tumbler is cut, its cross-sections are quadrilaterals. If the tumbler is not cut at any of the diameters of the top of tumbler, the quadrilaterals (of the two cross-sections) would not be equal to each other.



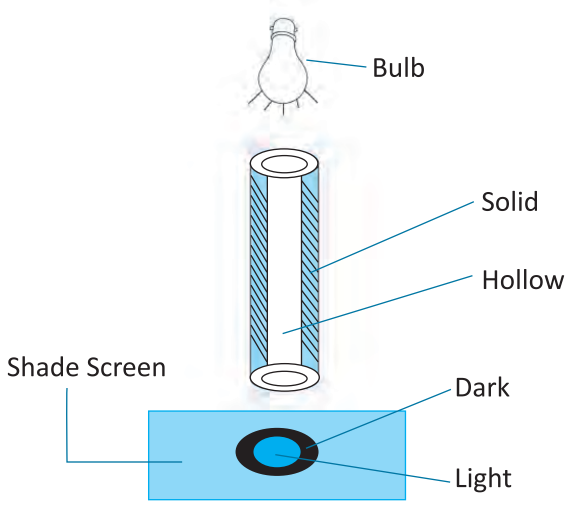
Example 8 : What shadow would you get on throwing light on a cone from its slant side?

Solution : If we throw light on a cone from its slant side, we get a triangle on the shade screen.



Example 9 : What would be the shadow if a water pipe is shown light from its one end?

Solution : The shadow of the pipe would be a dark circle in the middle of the light. The dark circle is the shadow of the pipe.



Exercise 15.3

1. Cut a cylinder across its length. What are the shapes of two cross-sections so obtained?
2. A pyramid has a pentagon at its base. Its height is 18 cm. It is cut by a knife at a point 7 cm from its top, parallel to the pentagon base. What are the shapes of the cross-sections so obtained?
3. Light is thrown on a dice from its bottom and the shade is obtained on a shade screen above the dice. What is the shade so obtained?
4. What is the shape of the shadow of a book, if it is subjected to torch light from its top and the shadow is obtained at a shade screen located exactly below it?

Points to Remember

- One-dimensional objects are — line, line segment and ray etc.
- Two-dimensional objects are — square, triangle, circle, rectangle, rhombus etc.
- Three-dimensional object are cube, cuboid, prism, pyramid, cone etc.
- The corners of a solid object are called vertices. Its line segments, formed due to faces, are called edges. Its faces are the surfaces that are present on all of its sides.
- A 3-D object can be draw — as a 2-D figure in there ways — (a) as an oblique sketch (b) as an isometric image (c) on a squared paper.
- The dimensions of a 3-D object on an isometric sheet of paper are exactly equal to its original dimensions.
- Solid curvilinear objects have curved surface, curved edge, curved face and circular face. Not all of them may be present in them.
- We can use nets to depict 3-D objects.
- We can cut through a 3-D object. The cut portions of a 3-D object are called cross-sections.
- We can create shadows of 3-D objects.
- The type and size of shadow depend upon which side of the 3-D object we are throwing light from. The shape of the object is also equally important.
- For solids, the Euler's Formula is $F + V - E = 2$, where F = number of faces, V= number of vertices and E = is number of edges.

EXERCISE

1. **MULTIPLE CHOICE QUESTIONS (MCQs) :**

   Tick (✓) the correct options.

   (a) We cannot get the 2-D image of the following on a sheet of paper.

   (i) cube ☐ (ii) cone ☐

   (iii) pyramid ☐ (iv) circle ☐

   (b) If light is thrown on a rectangular block from the top, the shade at the bottom would be a

   (i) circle ☐ (ii) square ☐

   (iii) prism ☐ (iv) none of these ☐

(c) The dimensions of a 3-D object on an isometric paper are the same as the ones

(i) on a squared paper (ii) of the 3-D object itself

(iii) of the plain sheet of paper (iv) of another 3-D object

(d) The shadow of a ball is always a

(i) circle (ii) square

(iii) pyramid (iv) straight line

(e) The Euler's formula is

(i) $V - E + F = 6$ (ii) $V + F - E = 4$

(iii) $V - F - E = 2$ (iv) $V + F - E = 2$

(f) The 3-D shapes, whose faces are polygons, are called

(i) polyholes (ii) polycites

(iii) polyhedrons (iv) tetrahedrons

(g) If a rectangular room is cut into exactly two equal haves, its cross-section would be

(i) circles (ii) rectangles

(iii) cones (iv) areas

(h) A circular prism has no

(i) edge (ii) vertex

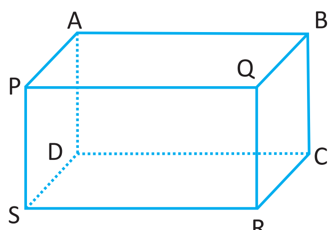
(iii) base (iv) none of these

(i) A triangular pyramid is also called

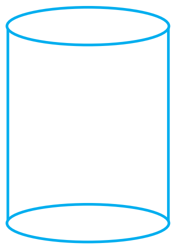
(i) polyhedron (ii) tetrahedron

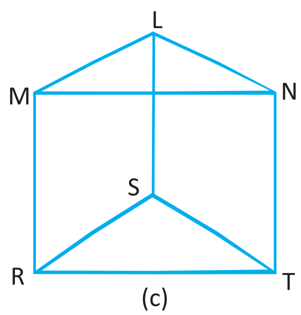
(iii) pyramid (iv) 3-D shape

2. Name the vertices, faces and edges of the following figures. Then, check whether Euler's formula is applicable to them.



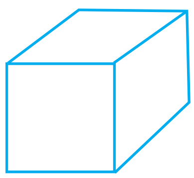
(a)

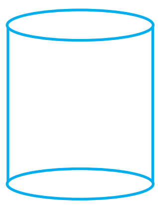

(b)

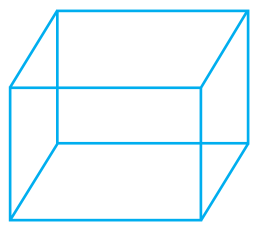


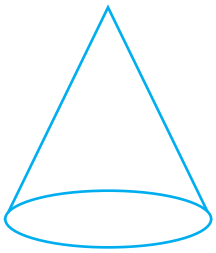
(c)

3. Draw the nets for the following.

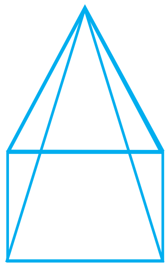

(a)


(b)

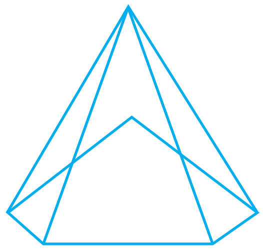

(c)



(d)



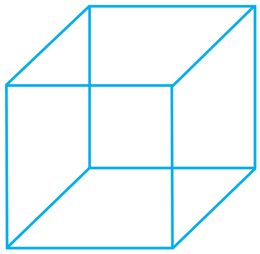
(e)



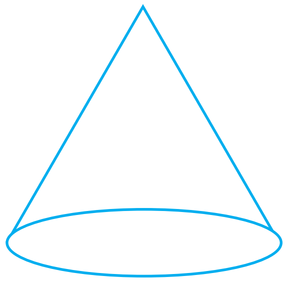
(f)

4. What shadows would emerge if light is thrown on the following 3-D shapes? The direction of light has been shown.

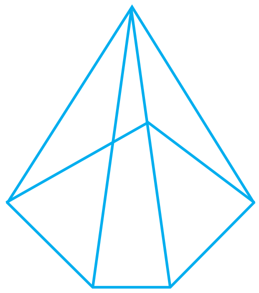
(a)



(b)

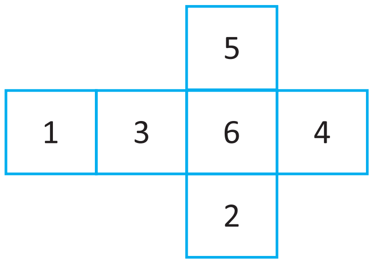


(c)



(d)

5. Look at the net of a cube. Then answer the following questions:



(a) Which ones of the faces are not exactly opposite to each other?

(i) 5 and 2 (ii) 4 and 6

(iii) 6 and 1 (iv) 3 and 4

(b) Which face is not adjacent to 6?

(i) 1 (ii) 5

(iii) 3 (iv) 2

(c) If face 1 is put on the RHS of face 4, which face would face in that case?

(i) 5 (ii) 2

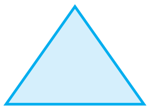
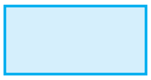
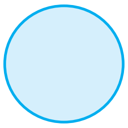
(iii) 6 (iv) none of these

(d) Which face is exactly opposite to face 2 ?

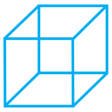
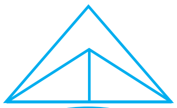
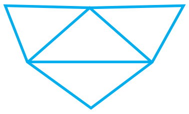
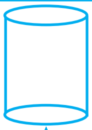
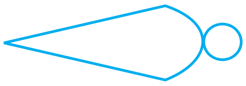
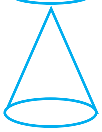
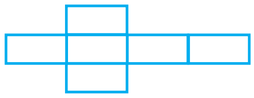
(i) 4 (ii) 3

(iii) 1 (iv) 5

6. **Match the shadow with their 3-D objects if the light is thrown from the top :**

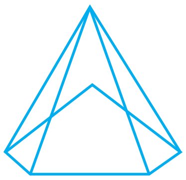
Column A		Column B	
(a)		(i)	volleyball
(b)		(ii)	tetrahedron
(c)		(iii)	cone
(d)		(iv)	cube

7. **Match the nets with solids:**

Column A		Column B	
(a)		(i)	
(b)		(ii)	
(c)		(iii)	
(d)		(iv)	

8. Using a paper of 16 cm × 12 cm, how many cylinders can be made ? What can be the height of these cylinders?

9. (a) What is the difference between oblique sketch and isometric sketch?
   (b) Prove that Euler's formula is applicable to the following figure.
   (c) What is the difference between the circular face and curved surface of a right cylinder?
   (d) What is the sum of number of dots on any two opposite faces of a dice?
   (e) Can we have more than one net for a 3-D object? Give examples.



10. **Write T for True and F for False for the following statements :**
   (a) All faces of a prism are triangles. ☐
   (b) The horizontal cross-section of a cone is a circle. ☐
   (c) A triangular pyramid has five vertices. ☐
   (d) If we cut through a cuboid, we get two cross-sections, both being rectangles. ☐
   (e) The isometric sketches of a 3-D object have the same dimensions as the object. ☐
   (f) An ice-cream cone is a type of prism. ☐

(g) There can be more than one net for a 3-D object.

(h) A cuboid has 6 faces and 8 vertices.

(i) We cannot draw 2-D sketches on a squared paper.

(j) A birthday cap is a hollow cone.

HOTS

If 3 cubes of dimensions 2 cm by 2 cm by 2 cm are placed side by side, what would be the dimensions of the resulting cuboid? Represent it with the help of
(i) oblique sketch (ii) Isometric sketch

Lab Activity

Identifying 3D shapes

Objective : To identify 3D shapes through verbal reasonings

Materials Required : Paper, pencil, ruler

Preparation : Students play in pairs.

Step 1 – Let Student A observe all the solids studied and select one of the solids. Write the name of that solid on a piece of paper and fold it up. Student B should not be told what A has selected.

Step 2 – Keep this folded piece of paper at the centre.

Step 3 – Student B now asks suitable questions of student A in order to find out the solid selected by A.

Step 4 – Student A will answer B's questions only in 'yes' or 'no'.

Step 5 – The aim of Student B is to ask such questions so as to get 'yes' or 'no' answer. He should try to find the name of the solid by asking the least number of questions.

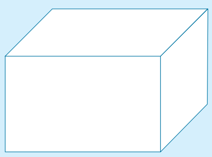
Step 6 – When B succeeds in getting the name of the solid, the roles of A and B are reversed and the game continues.

Example After student A has selected a solid, Student B could ask the following questions :

(a) Does the object have any curved surface?

Ans : No

(That means it is not a cone or cylinder.)



(b) Does it have 6 equal faces?

Ans : Yes

Then the object is a cube. If the answer is 'no' it is a cuboid.

So, we have reached a conclusion by asking only 2 questions.

Record the activity –

Solid	Questions	Answer
Cube		
Cone		
Cylinder		
Cuboid		

Copy this table in your note book or on a chart paper and play the game for all the solids you have studied so far.

16 Data Handling and Probability

Data is the food of Statistics. Statistics is the branch of Mathematics in which we draw many useful facts or inferences from data. For data to be meaningful and useful the items (of data) must be gathered or captured and recorded in a systematic manner. This is called data handling.

The collection, recording and presentation of data help us to organise our experiences and draw inferences from them. The collection and analysis of data is becoming more and more important nowadays as different people in different sectors are dependent on this type of data handling. Let us look at some common forms of data.

Data showing monthly earning (in ₹) of overtime of an employee	
August	• • • •
September	• •
October	• • • • •
November	•

• represents ₹ 500

Temperatures of Delhi on any five days of March 2011		
	Maximum	Minimum
1st March	38°C	29°C
2nd March	37°C	26°C
3rd March	28°C	26°C
4th March	30°C	29°C
5th March	32°C	28°C

Collection of Data

Data can be classified into :

- Primary data
- Secondary data

You have already studied about this data type in your previous class. Before collecting data, you should know which type of data is required ?

The data about temperatures of Delhi (as given above) on any five days of March 2011 give us only a bit of some information. Can we get information about :

- Which month of the year 2011 has maximum number of hot days?
- On which day the minimum temperature of the year was recorded?

Certainly, we cannot get this information using the table given above. It means that *collection of data must be done keeping in mind the specific information which we want to acquire*. So, you can see that collecting relevant data or information is very important.

Let us take another example, suppose you collect marks of 25 students of your class in a monthly test of Mathematics marks out of 50 in the month of August.

45, 26, 40, 26, 25, 38, 26, 48, 45, 30, 19, 40, 30
34, 30, 45, 20, 18, 20, 15, 30, 18, 45, 38, 15

Observer the marks obtained. Can you answer in a quick response, which student got the lowest marks. How many students got the marks between 30- 45 ? Obviously, you cannot.

You can answer these questions at a first glance only if you organise and tabulate the data in a systematic manner. If you put data in descending order you will get

48, 45, 45, 45, 45, 40, 40, 38, 38, 34, 30, 30, 30, 30, 26, 26, 26, 25, 20, 20, 19, 18, 18, 15, 15

By observing this data you can easily answer :

- Which student got the maximum (highest) marks?
- How many students got 30 marks each ?

The data (marks of students) which is collected and put as it is without any specific arrangement is called raw or ungrouped data. To get better understanding, this raw data is organised in either ascending order or descending order.

In this organised data, you can observe more than one values which repeat themselves. The number of times a particular value repeats itself is called its frequency.

The organised collection of data can be put in tabular form. When we put (organise) data in tabular form it is called frequency distribution table.

Let us consider another example. Suppose the numbers of newspapers sold at a local shop over the last 10 days are :

22, 20, 18, 23, 20, 25, 22, 20, 18, 20

The frequency distribution table for this data can be shown as :

Papers sold	Tally marks	Frequency
18	||	2
19		0
20	|| ||	4
21		0
22	||	2
23	|	1
24		0
25	|	1

Various types of data (that we come across in our daily life) are put in tabular form. Our school roll numbers, progress reports, index in notebook, temperature as and many others are all put in tabular forms.

Representation of Data

Any collected information can be easily arranged in a frequency distribution table. This information can be put as a visual representation in the form of Pictograph, Line graph, Bar graph, Histogram and Pie chart. *Graphs are visual representation of an organised data*.

Pictograph : Data (numerical data) is represented in the form of pictures.

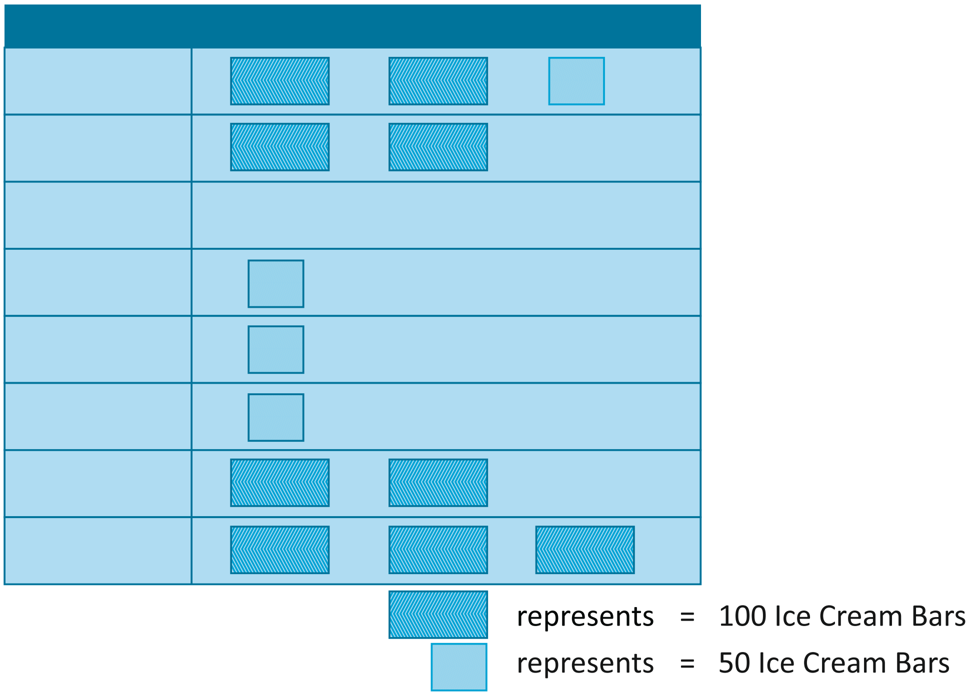
Line graph : Data is represented by lines, where different points (values) are joined by lines.

Bar graph : Data is represented using rectangular bars of uniform width. The length of bar depends upon frequency and scale choosen.

Histogram : Frequency distribution is represented by joining adjacent bars.

Pie chart : Data (numerical data) is represented in a circular form.

You have already studied about pictograph in class VI. Let us recapitulate with the help of an example.



In which month was the least quantity of ice cream sold?

In which month was the highest quantity of ice cream sold?

Exercise 16.1

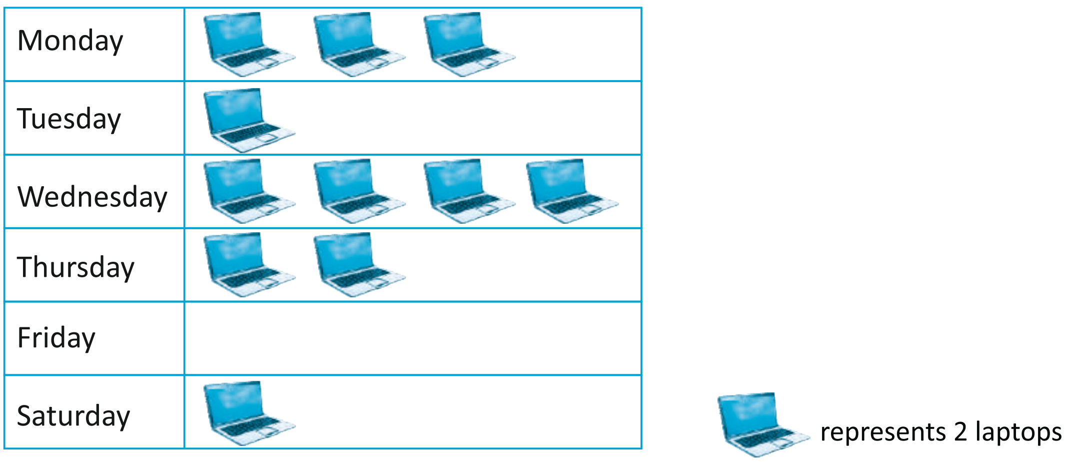
1. Make a frequency distribution table for the following marks out of 40 of 25 students.

15, 35, 20, 15, 25, 30, 35, 20, 19, 25, 25, 20, 15, 38, 36, 19, 18, 20, 25, 35, 18, 19, 15, 19, 19

2. Draw a pictograph for number of absentees in a firm on weekdays.

Monday – 5, Tuesday – 3, Wednesday – 3
Thursday – 2, Friday – 6, Saturday – 5

3. Following pictograph represents the number of laptops used in an office on weekdays.



Answer the following :

(a) How many laptops were used on Tuesday?

(b) What is the total number of laptops used for one complete week ?

(c) On which day the maximum number of laptops used ?

Measures of Central Tendency

After collecting and tabulating the data, the next step is to calculate a single number (value) which can represent or summarise the whole data. This single value is nothing but a measure of central tendency.

Suppose Sohan rides on an average 8 hours daily as sales executive. It means that he may ride more than 8 hours on some days and less than 8 hours on some other days. Thus, the calculated average shows central value of a group of organised data. There are three types of such measures (averages) : Mean, Median and Mode.

Mean : The mean (arithmetic mean) is the sum of all observations divided by total number of observations i.e.,

$$\text{Mean} = \frac{\text{Sum of all observations}}{\text{Total number of observations}}$$

Example 1 : **The weights (in kg) of 10 players of India Hockey team are as follows :**

65, 55, 75, 70, 66, 72, 65, 73, 59, 60

Find the mean (arithmetic mean) of weight of these players.

Solution : $\text{Mean} = \frac{65+55+75+70+66+72+65+73+59+60}{10} = \frac{660}{10} = 66 \text{ kg}$

The mean of weights of 10 players is 66 kg.

Median : Median of a group of numbers (observations) is the number (observation) in the middle (centre), when the numbers are arranged in an order (either ascending or descending order).

If the number of observations is odd, then the middle number is the median. For example, the prices (in ₹) of eleven kitchen items are arranged in ascending order as

94, 96, 98, 100, 110, 112, 115, 115, 117, 120, 123

Here, median is 123.

If the number of observations is even, then the median is the average (central value) of two middle numbers. For example, the heights (in cm) of 10 students of your class have been arranged in ascending order as

90, 95, 96, 97, 110, 110, 112, 112, 120 125

Here, median is $\frac{125+120}{2} = 122.5$ cm

Mode : Mode of a group of observations is the number with maximum frequency i.e., the most frequent occurring value.

Example 2: **The prices (in ₹) of T-shirts sold in a shopping mall on a particular day are as follows :**

330, 250, 150, 330, 150, 200, 250, 250, 330, 250, 260, 360, 260, 200, 150, 360, 330, 150, 250, 260

Facts to Know

- In some cases, Median and Mode may coincide i.e., they have same value.

Find the mode.

Solution : We can tabulate it.

Prices (in ₹)	360	330	260	250	200	150
Number of T-shirts sold	2	4	3	5	2	4

We can see that mode of this data is ₹ 250. This value is the most frequent value.

Range : Range of observations gives us an idea about the spread distribution of observations So, range is the difference between the highest and lowest observations.

Range = Highest observation – Lowest observation

Facts to Know

- The data having two modes is called Bimodal data.
- The data having many modes is called Multimodal data.

Example 3 : **The runs scored by 10 players of a team in Delhi Ranji Cricket One Day Domestic match are as follows :**

8, 90, 70, 72, 60, 40, 10, 5, 20, 85

Find the range.

Solution : The lowest value (runs) = 5

The highest value (runs) = 90

Range = 90 – 5

= 85

Hence, range is 85

In all, we conclude that it is better to find range, mean, median and mode of data in order to get the complete picture.

Exercise 16.2

1. Popular books read by bookreaders of Delhi are as given below :

Book	Geeta	Ramayana	Midnight drama	True lies
No. of Readers	80	150	16	46

Find the mean.

2. Find median, mode and range of the data.

36, 40, 45, 46, 50, 46, 58,45, 46, 35

3. The heights (in cm) of 20 children in a group are as :

96, 98, 90, 96, 105, 90, 96, 120, 115, 96, 98, 112, 90, 88, 110, 90, 96, 90, 110, 92

Find the range, mean, median and mode of the data.

4. Find mean of first five odd numbers.
5. Find the value of x, if the mean of data is 12:

15, 17, 6, 2, 1, 3, 6, x, 10, 7, 3

Bar Graphs

You are already familier with bar graph. Bar graph represents numbers using bars (rectangles) of uniform width, drawn horizontally or vertically with equal spacing in between them. In a bar graph, the following aspects are important.

(1) All bars should be of same width

(2) The distance between any two consecutive bars should be same.

(3) The scale should be clearly and carefully written on the graph.

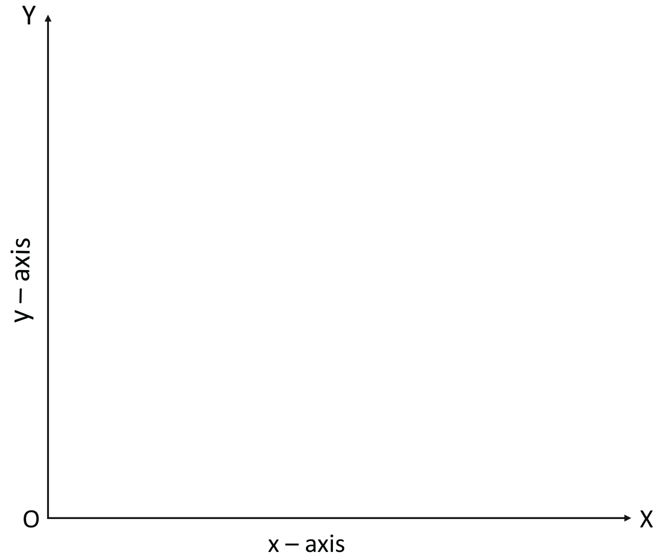
(4) Each rectangle (bar) indicates only one numerical value of the data. Therefore, you can draw as many bars as the number of values in data.

We draw some conclusions from the tabular representation of large amount of numerical data using bar graph.

To draw a bar graph, follow the steps.

(1) Draw two mutually perpendicular lines on graph paper.

(2) Name horizontal line as *x*-axis, vertical line as *y*-axis and point of intersection as origin (generally denoted by O).



You can understand it more clearly by going through the following examples.

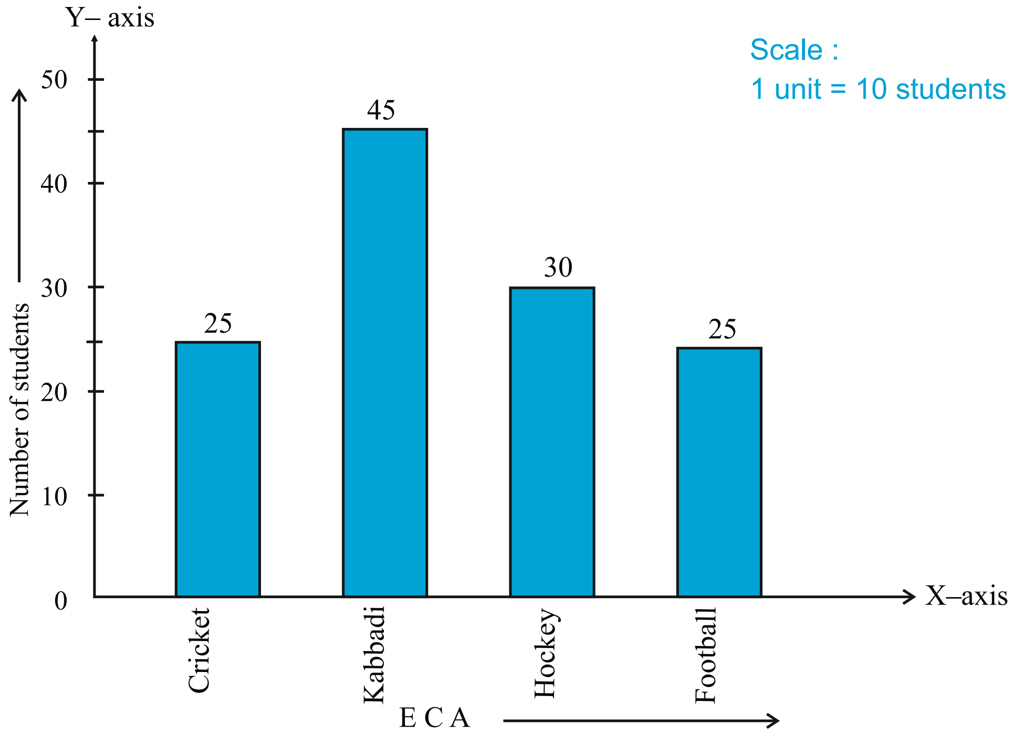
Example 4 : Draw a bar graph to represent number of students taking part in extra curricular activities held in your school.

Extra Curricular Activities (ECA)	Cricket	Kabbadi	Hockey	Football
Number of students	25	45	30	25

Solution : Before drawing the graph, fix the scale. Here, maximum value is 45, So we must mark units up to 50. Let us assume 1 unit as 10 students.

Draw two perpendicular axes. Along the X-axis, mark spaces for 4 bars equal distance apart. Write the names of extra curricular activities.

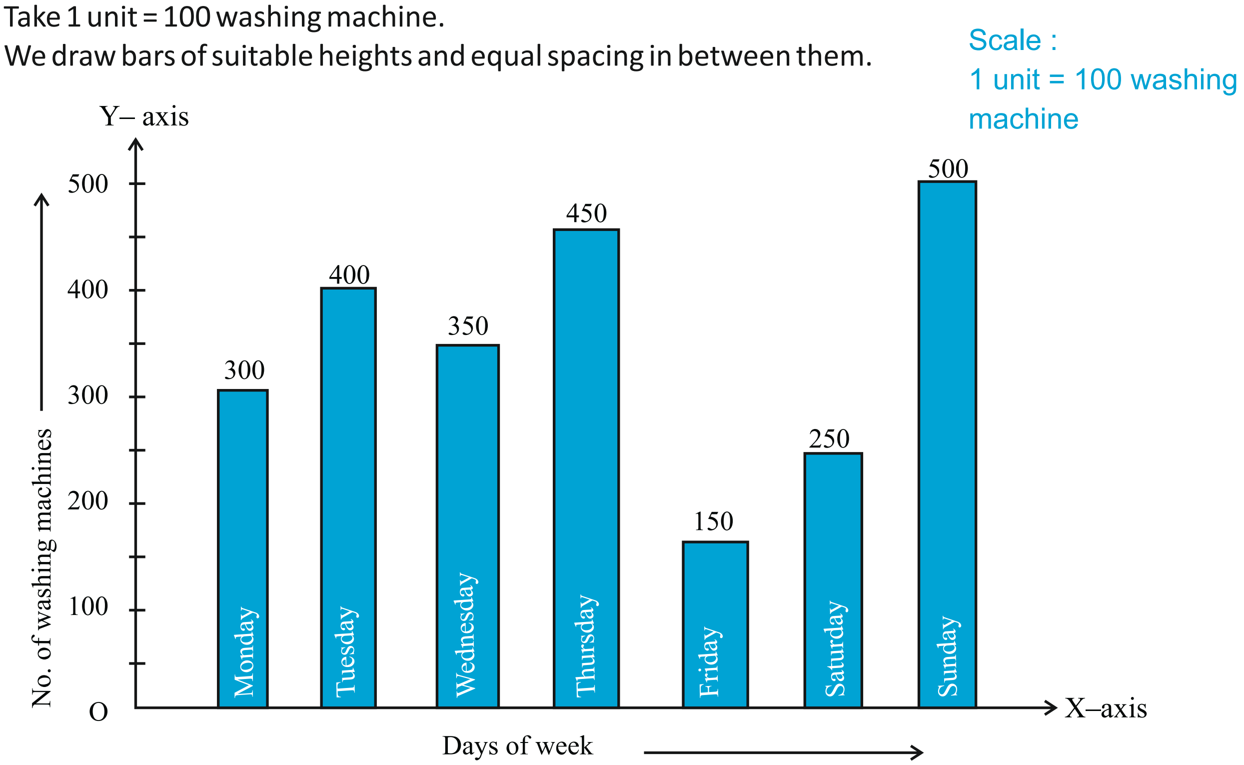
Draw bar graph as shown below. We see that maximum number of students play kabbadi. So, this is the mode of data.



Example 5 : **Draw a bar graph for daily production of Washing Machines in a factory for 7 days a week.**

Days of week	Mon	Tue	Wed	Thur	Fri	Sat	Sun
No. of Washing Machines	300	400	350	450	150	250	500

Solution : We draw two perpendicular lines OX and OY, respectively intersecting each other at the origin O as shown in the figure.
The maximum value is 500. So, we must mark units up to 500.
Take 1 unit = 100 washing machine.
We draw bars of suitable heights and equal spacing in between them.



Double Bar Graphs

Double bar graph helps us to compare or represent more than one types of information, we can draw the bar graphs of both set of observations on the same graph.

To draw a double bar graph, do remember some basic procedures.

- Draw two bars for a single reading.
- Keep no space between two bars for single reading. Shade them differently.
- Keep equal space between two consecutive sets of double bar.

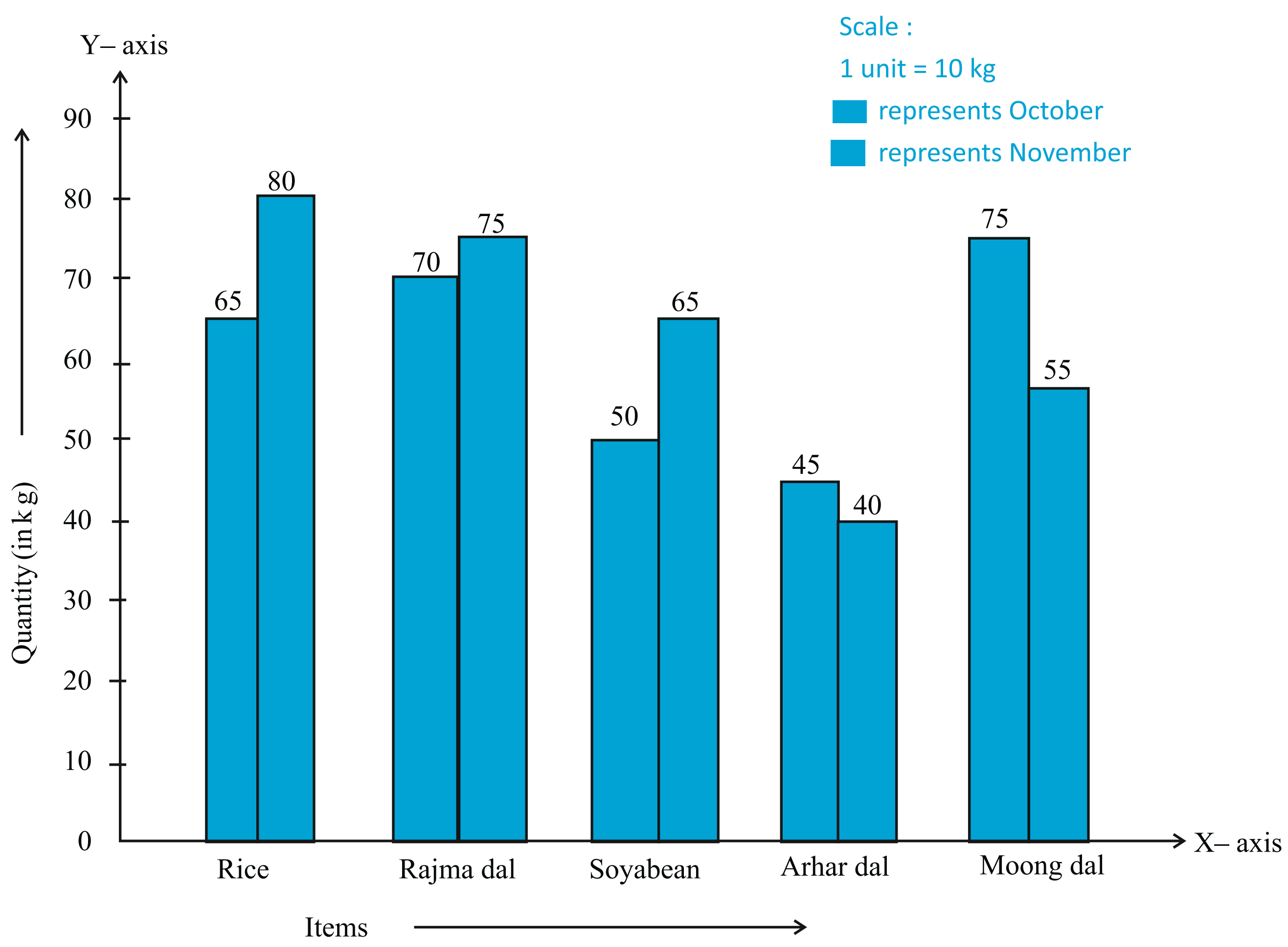
Example 6 : **A restaurant owner purchases following items for two consecutive months. Draw and compare it by using double bar graph.**

Item (in kg)	Rice	Rajma dal	Soyabean	Arhar dal	Moong dal
October	65	70	50	45	75
November	80	75	65	40	55

Solution : The highest value is 80, so we mark the units up to 90. We select scale of 1 unit as 10 kg.

Draw two axes. Mark the space for 5 double bars along X-axis.

Write the names of items. Draw bars and shade them differently.



Exercise 16.3

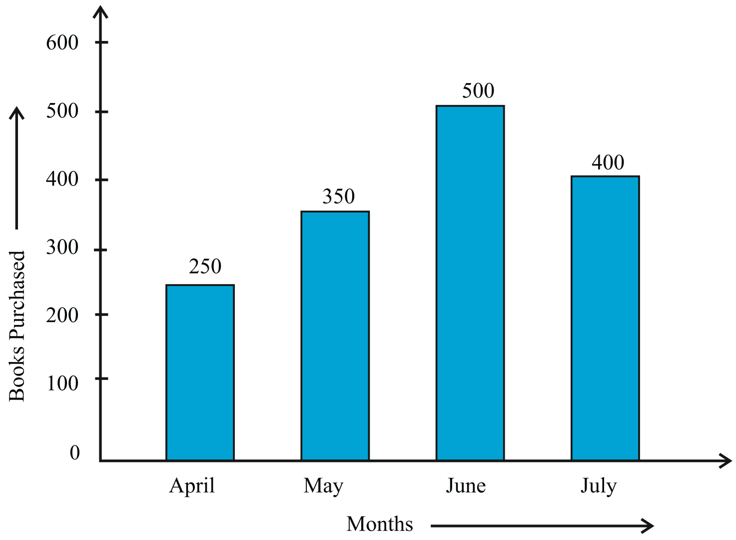
1. The following bar graph shows the purchase of books in four consecutive months. Answer the questions given below :

(a) In which month was the purchase minimum?

(b) How many books were purchased in July?

(c) Which month was the purchase maximum?

(d) Which month shows the purchase between 300 to 400?



2. Construct a bar graph to represent the data collected about the quantities of Rice used in 5 households :

Household	A	B	C	D	E
Quantities of Rice (in kg)	50	40	25	45	35

(a) What is the range of the data?

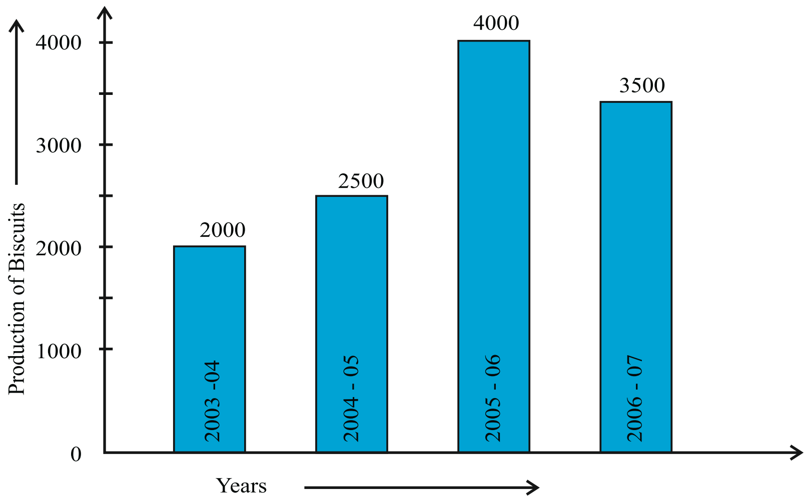
(b) What is the median of the data?

3. Observe the graph and answer the following questions:

(a) What was the production of biscuits in the year 2005-06?

(b) In which year, the production of biscuits was minimum?

(c) In which year the production of biscuits was in between 4000 kg to 3000 kg.

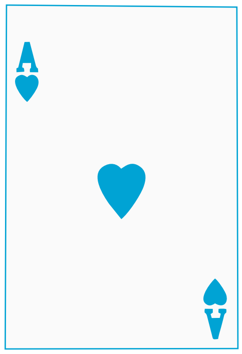


4. The income and expenditure of Amit for 5 consecutive months are given below.

Month	January	February	March	April	May
Income (in ₹)	7000	8000	6500	7500	8500
Expenditure (in ₹)	5000	4500	4000	6000	5500

Represent the data with the help of double bar graph.

Probability



Consider the following situations in our daily life.

★ There is no chance for you to go to Shimla today.

★ By tomorrow, your weight will increase by 5 kg.

★ You will go to picnic in the next hour.

★ I will earn ₹ 5000 in the next 24 hours.

In all the above situations, we are not sure about the happening of these events. There is always an uncertainty associated with these situations. All these situations can be dealt with the concept of probability.

Let us understand it mathematically.

Sandesh and Mohanty are the captains of their cricket team. One of them tosses the coin. What is the chance that one of the captains will win the toss? It is 50-50. As one of them flips the coin in the air, it will land either on its head or tail. The total number of outcomes (results) are 2, but only one outcome is possible to turn up at a time. So the probability of getting head is $\frac{1}{2}$ and the probability of getting tail is also $\frac{1}{2}$.

Hence, probability is the chance (likelihood) of happening of an event. i.e. probability of happening an event can be written as :

$$P = \frac{\text{Number of ways the event can happen}}{\text{Total number of all possible outcomes}}$$

Let us roll a die. It will always show a number on its upper face ranging from 0 to 6. Is it possible to get 7 when a die is rolled ? Obviously not !

Some events cannot occur at all. They are called impossible events. Probability of an impossible event is always equal to 0.

Similarly, some events will surely happen. Such events are called sure events. Probability of sure event is always equal to 1.

Probability of an outcome always lies between 0 and 1.

Facts to Know

- If the probability of occurence (happening) of an event is 'P' then the probability of non-occurence (non-happening) of this event is (1 –P).

Experiments, Outcomes and Sample Spaces

Suppose you draw one card from well shuffled pack of 52 playing cards. You can get any card from 52 cards. Therefore, the act of drawing one card is an experiment and the card (showing any number) in your hand is an outcome. Similarly, throwing a die is an experiment but getting 5 is an outcome.

Sample Space (S) : A set of all the possible outcomes in an experiment.

For example, sample space (s) = {H, T} where 'H' is the head and 'T' is the tail that we get on tossing coin.

S = {1, 2, 3, 4, 5, 6} is the sample space of six possible outcomes on rolling a die.

Example 7 : **What would be the probability of drawing**

(i) One face card?

(ii) King of any suit?

Solution : A pack of playing cards consists of 52 cards. There are 4 suits–Spade, Diamond, Heart and Club. Each suit has 13 cards. Ace is the first card. Three face cards are Jack, Queen and King. Spade and Club are of Black suit. Diamond and Hearts are of Red suit.

(i) There are 3 face cards in each suit respectively. So, they are 12 in all.

Probability of drawing one face card = $\frac{12}{52}$ or $\frac{3}{13}$

(ii) There are 4 kings in the pack of 52 cards.

Probability of drawing a king = $\frac{4}{52}$ or $\frac{1}{13}$

Exercise 16.4

1. A bag contains 2 blue, 3 red and 7 yellow balls. Find the probability of drawing.

(a) one red ball (b) one yellow ball (c) one blue ball

2. There are 12 marbles marked with numbers 1 to 12. Find the probability of drawing one marble marked with number.

(a) 5 (b) 7

3. Find the probability of the following if we draw a single card from well shuffled pack of 52 cards.

(a) a red jack (b) a queen of any suit

(c) ace of club (d) a card from diamond suit

4. There are 4 blue marbles, 5 red marbles, 1 green marble and 2 black marbles in a bag. Suppose you select one marble at random. Find probability that :

(a) marble is black

(b) marble is blue

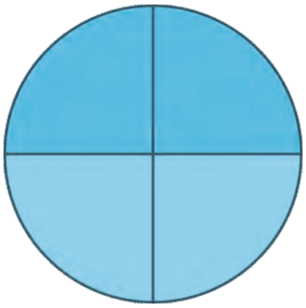
(c) marble is not green

5. A spinner is given here, determine the probability :

(a) getting green

(b) getting pink

(c) getting 'not red'



Points to Remember

- Data handling is the process of gathering, capturing and recording the data in a systematic manner. So that it could be meaningful and useful to us.
- When data is collected and put as it is without any specific arrangement, it is called raw or ungrouped data.
- The number of time a particular value repeats itself is called its frequency.
- Data can be organised in tabular form which is called frequency distribution table.
- Arithmetic mean is the sum of all observations divided by total number of observations.
- Median is the middle value of a group of observations when these observations are arranged in an order.
- Mode is the number (value) with maximum frequency.
- Range is the difference between highest observation and lowest observation.
- Bar graph represents numbers using bars (rectangles) of uniform width. These bars are drawn horizontally or vertically with equal spacing in between them.
- Double bar graph compares more than one types of information by drawing bar graphs on the same graph.
- Probability is the chance (likelihood) of happening of an event.
- Sample space is the set of all the possible outcomes in an experiment.

EXERCISE

1. MULTIPLE CHOICE QUESTIONS (MCQs):

Tick (✓) the correct options.

(a) The mean of 8, 12, 15, 20, 17 is

(i) 12.4 ☐ (ii) 14.4 ☐ (iii) 15.4 ☐ (iv) 18.4 ☐

(b) The median of 5, 7, 9, 3, 11, 12, 15 is

(i) 5 ☐ (ii) 15 ☐ (iii) 9 ☐ (iv) 12 ☐

(c) The mode of 2, 0, 3, 2, 5, 7, 2, 3, 0, 7, 2 is

(i) 0 ☐ (ii) 7 ☐ (iii) 3 ☐ (iv) 2 ☐

(d) The probability of an outcome always lies in between

(i) 1 and 2 ☐ (ii) 0 and 1 ☐ (iii) 3 and 4 ☐ (iv) 2 and 4 ☐

(e) The probability of a sure event is

(i) 0.5 ☐ (ii) 0 ☐ (iii) 1 ☐ (iv) All of these ☐

(f) If a dice is rolled, the probability of getting an odd number is

(i) 1 ☐ (ii) 2 ☐ (iii) 0.5 ☐ (iv) None of these ☐

(g) Probability of an impossible event is

(i) 1 ☐ (ii) 0 ☐ (iii) 1.5 ☐ (iv) 0.5 ☐

2. Arunoday studies for 5 hours, 4 hours, 7 hours and 8 hours, respectively on four consecutive days. How many hours does he study each day on an average?

3. The appointments in a company during five consecutive years were recorded as follow:

200, 450, 368, 196, 256

Find the mean of appointments.

4. Following are the runs scored by tenth (10th) player of a cricket team in two consecutive tournaments.

1, 3, 4, 2, 5, 6, 1, 2, 3, 3, 1, 4, 5, 3, 5, 6, 1, 3, 4, 3, 2, 3

Find the mode of this data.

5. A private mathematics tutor wants to judge, whether the new technique of teaching (that he applied after quarterly school test of students) was effective or not. He takes the scores (out of 25) of the 5 weakest students in the two successive quarterly tests.

Student	Vijay	Mohit	Neelam	Kanchan	Sanjay
Quarterly Test (Ist)	8	15	10	20	10
Quarterly Test (2nd)	18	10	16	15	14

Represent this information using double bar graph.

6. **One card is drawn from the pack of well shuffled 52 cards. Find the probability that :**

(a) card drawn is black jack

(b) card drawn shows a prime number

(c) card drawn is a multiple of 3

7. Choose a number at random from 1 to 5. (a) What is the probability of each outcome? (b) What is the probability that number choosen is even?

HOTS

Six friends were deciding where to go during the coming weekend. Everyone wrote their intentions on a slip of paper to show their choice.

Fishing | Boating

Water Park | Cricket match | Cinema | Hillside

Write the probability of the following.

P (water park) = P (Cricket match) = P (fishing) = P (Cinema) =

Lab Activity

Objective : To understand the concept of probability.

Materials Required : Two dice

Procedure :

Step 1 : Take two dice and roll them.

Step 2 : Note it down the number on both the dice separately.

Step 3 : The possible outcomes may be 1, 2, 3, 4, 5, 6.

So, the probability of getting any outcome is $\frac{1}{6}$.

Step 4 : Now, roll the die for at least ten times and record your observation for an even number and odd number.

Outcome	1	2	3	4	5	6	Probability
Even Number							______out of 10
Odd number							______out of 10

Answers

Ch-1 Knowing the Numbers

Exercise 1.1

1. (a) -28,-12,-7,-1,1,3,7 (b) -5,-4,1,3,6,8,11 **2.** (a) 15,9,6,3,-7,-10,-12,-14 (b) 14,13,8,6,2,0,-8,-10,-16

3. (a) 41 (b) -3 (c) 83 (d) 27 **4.** (a) 16 (b) 32 (c) -919 (d)-1728

5. (a) 16 (b) -137 (c) 7 (d) 484

6. (a) 7o c (b) 0o F (c) -14°C (d) 44°F

7. (a) 2,5,8,11 (b) 69,75,81,87

8. (a) The frog jumped 4 steps backward. (b) Today's temperature is 4o c above normal.
(c) Rahul reached at the platform 30 mintues after the arrival of train. (d) Start counting from 17 in descending order.

Exercise 1.2

1. (a) 0 (b) 144 (c) 0 (d) 132 (e) -77 (f) -120

2. (a) -80 (b) -54 (c) 0 (d) 90 (e) 1800 (f) 1000 (g) 0 (h) 600

3. (a) -96 (b) 16 (c) -10 (d) 75 (e) 4 (f) -3 (g) -6 (h) -7 (i) 3 (j) 100

4. (a) 20 (b) -12 (c) -2 (d) -12 (e) -21 (f) 37 (g) 9 (h) 1 (i) -9 (j) 10

5. (a) 12 (b) 6 (c) 2 (d) 100 **6.** 3 **7.** 12 **8.** -84

Exercise 1.3

1. (a) -20 (b) -25 (c) 0 (d) 26 (e) 16 (f) 0 (g) -5 (h) -5, -10 (i) -8, -4 (j) -5

2. (a) T (b) F (c) F (d) F (e) T (f) F

3. (a) 2540 (b) 735 (c) -25 **4.** (a) -8550 (b) 224 (c) 252 (d)-9100

5. (a) 1836 (b) -5015 (c) 1246 (d) -3589 **6.** -117 **7.** -9 **8.** 164M

Revision Exercise

1. (a) (iii) (b) (ii) (c) (iii) (d) (i) (e) (ii) (f) (ii) (g) (iv)

2. (a) 143 (b) -64 (c) 90 (d) -1000 **3.** (a) 4 (b) -21 (c) 0 (d)-21 (e) 36 (f) -8

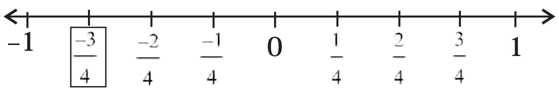
4. (a) 16 (b) -14 (c) 14 (d) 4 **5.**(a) -493 (b) -663 (c) -1034 (d) -1200 (e) 3400 (f) 7200

6. 12 moves **7.** 5400

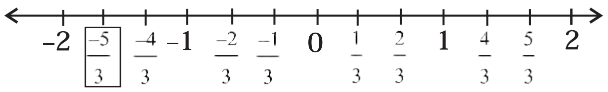
Ch-2 Rational Numbers

Exercise 2.1

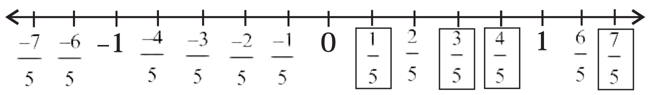
1. $\frac{6}{0}$ **2.** 4, -3, 2, -7 **3.** (a), (b), (e) **4.** Do it yourself **5.** (a) $\frac{-3}{4}$



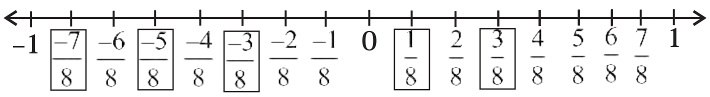
(b) $\frac{-5}{3}$



(c) $\frac{3}{5},\frac{1}{5},\frac{7}{5},\frac{4}{5}$



(d) $\frac{-7}{8},\frac{-5}{8},\frac{-3}{8},\frac{1}{8},\frac{3}{8}$



6. (a) $\frac{17}{56}$ (b) $\frac{1}{9}$ (c) $\frac{-1}{13}$ (d) $\frac{10}{51}$ **7.** (a) $\frac{-6}{14},\frac{-9}{21},\frac{-12}{28},\frac{-15}{35},$ (b) $\frac{4}{6},\frac{6}{9},\frac{8}{12},\frac{10}{15},$ (c) $\frac{-22}{14},\frac{-33}{21},\frac{-44}{28},\frac{-55}{35},$

(d) $\frac{12}{22},\frac{18}{33},\frac{24}{44},\frac{30}{55},$ **8.** (a) $x=\frac{-45}{4}$ (b) $x=-24$ (c) $x=-21$ (d) $x=20$ (e) $x=\frac{216}{5}$

9. (a) $\frac{-3}{21},\frac{-5}{23}$ (b) $\frac{+13}{35},\frac{15}{42}$ (c) $\frac{5}{10},\frac{6}{12}$ (d) $\frac{-7}{7},\frac{-8}{7}$ **10.** (a) $\frac{1}{6},\frac{1}{5},\frac{1}{2},\frac{2}{3},\frac{3}{4}$ (b) $\frac{-4}{7},\frac{-3}{7},\frac{-2}{7},\frac{6}{7},\frac{4}{4}$

11. (a) $\frac{7}{3},\frac{4}{3},\frac{2}{3},\frac{1}{3},\frac{-1}{3},\frac{-2}{3},\frac{-7}{3}$ (b) $\frac{7}{3},\frac{6}{3},\frac{2}{3},\frac{1}{3},\frac{-13}{3},\frac{-14}{3}$ (c) $\frac{4}{3},\frac{2}{5},\frac{2}{6},\frac{-1}{2},\frac{-3}{4}$ (d) $\frac{7}{10},\frac{2}{5},\frac{-2}{5},\frac{-6}{5},\frac{-7}{5}$

12. (a) $\frac{-4}{9}$ (b) $\frac{14}{27}$ (c) $\frac{-8}{15}$ (d) $\frac{-3}{4}$

Exercise 2.2

1. (a) $\frac{17}{5}$ (b) $\frac{-53}{42}$ (c) $\frac{-3}{7}$ (d) $\frac{-7}{11}$ (e) $\frac{29}{42}$ (f) $\frac{-919}{280}$

2. (a) $\frac{11}{12}$ (b) $\frac{-12}{7}$ (c) $\frac{44}{75}$ (d) $\frac{-5}{12}$ (e) $\frac{-1}{42}$ (f) $\frac{29}{15}$

3. (a) $\frac{-119}{8}$ (b) $\frac{13}{60}$ (c) $\frac{-48}{35}$ (d) $\frac{24}{35}$ (e) $\frac{-1}{2}$ (f) $\frac{-7}{24}$

4. (a) $\frac{-8}{25}$ (b) $\frac{35}{48}$ (c) $\frac{-35}{64}$ (d) $\frac{1}{2}$ (e) $\frac{-1}{5}$ (f) $\frac{9}{10}$

5. (a) $\frac{6}{7}$ (b) $\frac{-73}{168}$ (c) $\frac{1}{20}$ (d) $\frac{35}{64}$ (e) $\frac{-31}{30}$ **6.** $\frac{-23}{21}$ **7.** $\frac{-11}{28}$ **8.** $\frac{-2}{5}$

Exercise 2.3

1. (a) 2.6 (b) –0.28 (c) 0.024 (d) –0.175 **2.** (a) non-terminating (b) terminating (c) non-terminating (d) terminating (e) terminating (f) non-terminating (g) terminating (h) terminating

3. (a) $0.1\overline{3}$ (b) $0.\overline{428571}$ (c) 0.875 (d) 0.68 (e) 0.5 (f) 3.25 (g) 15.625 (h) 0.48

4. (a) $\frac{5}{4}$ (b) $\frac{281}{40}$ (c) $\frac{34}{9}$ (d) $\frac{2322}{90}$ (e) $\frac{7}{30}$ (f) $\frac{3353}{999}$ (g) $\frac{71}{90}$ (h) $\frac{307}{300}$

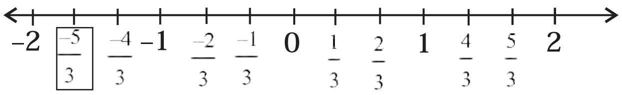
5. (a) $5\frac{7}{9}$ (b) $9\frac{112}{99}$ (c) $\frac{89}{198}$ (d) $10\frac{943}{990}$ **6.** $4\frac{1}{10}$ **7.** $x = 289$, $y = 999$

8. (a) ✓ (b) ✓ (c) × (d) ✓ (e) × (f) ✓

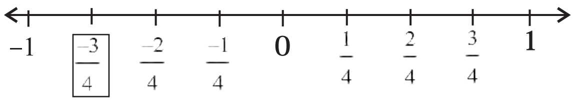
Revision Exercise

1. (a) (i) (b) (ii) (c) (i) (d) (iv) (e) (ii) (f) (iii) (g) (i) (h) (iv)

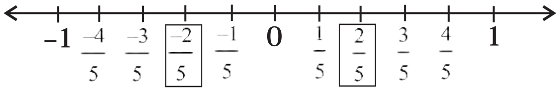
2. (a) $\frac{-5}{3}$



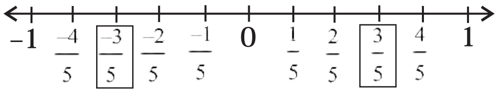
(b)



(c) $\frac{-2}{5}$ and $\frac{2}{5}$



(d) $\frac{-3}{5}$ and $\frac{3}{5}$



3. (a) $\frac{-13}{45}$ (b) $\frac{23}{49}$ (c) $\frac{-1}{8}$ (d) $\frac{11}{650}$ **4.** (a) $\frac{15}{14}$ (b) $\frac{11}{36}$ (c) $\frac{19}{15}$ (d) $2\frac{1}{5}$

5. (a) $\frac{3}{4}$ (b) $\frac{-5}{3}$ (c) $\frac{5}{11}$ (d) $\frac{3}{4}$ **6.** (a) $\frac{-7}{8}, \frac{-5}{6}, \frac{5}{8}, \frac{2}{3}, \frac{3}{4}$ (b) $\frac{-3}{7}, \frac{-5}{14}, \frac{-6}{35}, \frac{3}{10}$

(c) $\frac{-5}{6}, \frac{-2}{3}, \frac{-1}{3}, \frac{1}{3}, \frac{2}{3}, \frac{5}{6}$ (d) $-2, \frac{-3}{5}, \frac{-1}{2}, 0, \frac{1}{2}, \frac{3}{5}, 2$ **7.** (a) $\frac{-13}{7}$ (b) $\frac{11}{35}$ (c) $\frac{-13}{51}$ (d) $\frac{-11}{4}$

8. (a) $\frac{-7}{17}$ (b) $\frac{-9}{26}$ (c) $\frac{-16}{13}$ (d) 1 **9.** (a) $\frac{16}{75}$ (b) $\frac{-9}{10}$ **10.** $\frac{1}{4}$ **11.** $\frac{-144}{5}$ **12.** $\frac{5}{12}$

13. (a) $x = \frac{-1}{56}$ (b) $x = \frac{-14}{15}$ (c) $x = \frac{24}{35}$ **14.** (a) 0.4 (b) $0.\overline{857142}$ (c) 0.06 (d) $0.41\overline{6}$

15. (a) $\frac{1}{40}$ (b) $\frac{63}{20}$ (c) $\frac{230}{9}$ (d) $\frac{188}{495}$ **16.** (a) $8\frac{31}{990}$ (b) $\frac{35}{99}$ (c) $1\frac{461}{990}$ (d) $41\frac{649}{990}$

17. $\frac{100}{9}$ **18.** (a) $\frac{77}{27}$ (b) $\frac{113}{50}$

Ch-3 Fractions

Exercise 3.1

1. (a) $\frac{9}{7}$ (b) $\frac{18}{13}$ (c) $\frac{4}{5}$ **2.** (a) $\frac{3}{14}$ (b) $\frac{5}{34}$ (c) $\frac{83}{21}$ **3.** (a) $\frac{19}{2}$ (b) $\frac{95}{9}$ (c) $\frac{111}{10}$

4. (a) $\frac{8}{10}, \frac{12}{15}, \frac{16}{20}, \frac{20}{25}$ (b) $\frac{16}{22}, \frac{24}{33}, \frac{32}{44}, \frac{40}{55}$ (c) $\frac{26}{30}, \frac{39}{45}, \frac{52}{60}, \frac{65}{75}$ **5.** $7\frac{1}{2}$

Exercise 3.2

1. (a) $\frac{35}{11}$ (b) $\frac{19}{2}$ (c) $\frac{42}{5}$ (d) 6 (e) $\frac{56}{9}$ (f) $\frac{64}{3}$ **2.** (a) $\frac{21}{2}$ (b) $\frac{18}{11}$ (c) $5\frac{5}{6}$

(d) $8\frac{1}{2}$ (e) $22\frac{2}{9}$ (f) $\frac{56}{405}$ **3.** (a) $3\frac{1}{3}$ (b) $\frac{2}{5}$ (c) $\frac{1}{6}$ (d) $\frac{68}{665}$

4. ₹ 90 **5.** $22\frac{1}{2}$ km **6.** $7\frac{1}{2}$ litres

Exercise 3.3

1. (a) $\frac{6}{5}$ (b) $\frac{25}{6}$ (c) $\frac{31}{21}$ (d) $\frac{13}{22}$ **2.** (a) $\frac{2}{27}$ (b) $\frac{1}{10}$ (c) $\frac{3}{44}$ (d) $\frac{31}{49}$

3. (a) $\frac{44}{3}$ (b) $\frac{1000}{3}$ (c) $\frac{7}{5}$ (d) $\frac{84}{5}$ **4.** $\frac{203}{66}$ **5.** 16 pen **6.** $130\frac{1}{2}$ kg **7.** ₹ $48\frac{3}{4}$ **8.** ₹946

Revision Exercise

1. (a) (iii) (b) (iii) (c) (iv) (d) (iii) (e) (ii) (f) (ii) (g) (ii) (h) (i)

2. 3 **3.** ₹ 26 **4.** 15 km **5.** ₹ 140 **6.** $\frac{4}{3}$ litres **7.** 7 girls **8.** 233 students **9.** 49 km

Ch-4 Decimals

Exercise 4.1

1. (a) 227.794 (b) 11.145 (c) 643.754 (d) 645.985 **2.** (a) 1.727 (b) 111.462 (c) 10.102 (d) 221.40

3. (a) $60+7+\frac{4}{10}+\frac{8}{100}$ (b) $900+20+5+\frac{3}{10}+\frac{7}{100}+\frac{9}{1000}$ (c) $200+40+9+\frac{0}{10}+\frac{0}{100}+\frac{7}{1000}$

(d) $200+80+7+\frac{2}{10}+\frac{3}{100}+\frac{9}{1000}$ **4.** (a) 7.432, 84.002, 72.500 (b) 5.390, 28.024, 987.700 (c) 25.730, 19.553, 6.759, 9.300

5. 49.71 km

Exercise 4.2

1. (a) 79.5 (b) 145 (c) 1530 (d) 675 (e) 5.5 (f) 3567 (g) 279010 (h) 2873.3

2. (a) 350.474 (b) 140 (c) 87.36 (d) 29.7 (e) 157.76 (f) 27572.4 (g) 26077.8 (h) 24.475

3. (a) 14.3715 (b) 17.358 (c) 3.9345 (d) 598.558 (e) 153.552 (f) 139.649 (g) 0.9375

(h) 0.16605 **4.** 164.01 km **5.** 320.41 m2

Exercise 4.3

1. (a) 0.2 (b) 0.1 (c) 1.8 (d) 0.63 (e) 3.99 (f) 2 **2.** (a) 0.1728 (b) 0.926

(c) 0.005685 (d) 0.075399 **3.** 13.63 **4.** 0.95 cm2 **5.** ₹ 15050

Exercise 4.4

1. (a) 7000 ml (b) 25000 m (c) 3500 gm (d) 2.900 kg (e) 5000 l (f) 2.945 kg (g) 0.00725 *l* (h) 2 m

2. 26 books **3.** 124.23 kg **4.** 4525 kg **5.** 106.8 cm and 1.068 m

Revision Exercise

1. (a) (ii) (b) (iii) (c) (iv) (d) (iii) (e) (iii) (f) (ii) (g) (iii)

2. ₹ 14400.00 **3.** 32 round **4.** ₹ 1213.10 extra money required **5.** ₹ 823.50

6. 48.267 m2 **7.** 725.5 kg **8.** 20.58 litres **9.** ₹ 8055.00 **10.** 29.25 km

Ch-5 Exponents and Powers

Exercise 5.1

1. (a) 64 (b) 2187 (c) 125 (d) 256 (e) 729 (f) -343

2. (a) base = 5, exponent = 3 (b) base = -5, exponent = 4 (c) base = -1, exponent = 11 (d) base = y, exponent = m

(e) base = m , exponent = y (f) base = -100, exponent= 5 **3.** (a) y3 (b) 95 (c) a2 (d) (7)3 (3)2 (e) (n)3 (m) 2

(f) (x) 3 (y)2 (z)2 **4.** (a) 91 is greater (b) 36 is greater (c) 9 10 is greater > (d) 52>(-2)5

(e) (10)1 >(-1)10 (f) (3)7 > (7)2 X3 **5.** (a) 3 × 5 × 5 = 3 × 52 (b) 23 × 32 × 53 (c) 54 (d) 34 × 5

(e) 24 × 32 × 52 (f) 33 × 52 **6.** (a) 3000 (b) 3072 (c) 6400 (d) 2304 (e) 225 (f) 500

7. (a) -50 (b) -64 (c) -8 (d) 40 (e) 30375 (f) 256

Exercise 5.2

1 (a) 22 × 32 (b) 5^{-4} (c) a2 b (d) 2^{1} (e) p12 (f) 2^{10} (g) $5^{2}X2^{-1}X3^{-1}$ (h) 3^{2} p2 = (3P)2

2. (a) 5 2+4+3 = 59 (b) -35+4 = (-3)9 (c) p4+5 = p9 (d) 5^{15} (e) 10^{x3} (f) 1 (g) (m×n)7 (h) 3 3+2 = 35

(i) 8^{X}-3 **3.** (a) 2^{8} × 34 (b) 24 × 31 × 5 3 (c) 3^{10}

(d) 211 × 31 × 52 (e) (37) × (54) **4.** (a) 2^{14} X (5/7)^{3} (b) 2^{10} X 5^{3} (c) $\frac{3}{73}$

5. (a) False (b) False (c) False (d) False (e) True **6.** (a) x = 8 (b) x = 21 (c) x=19 (d) x=3

Exercise 5.3

1. (a) $9\times10^{5}+3\times10^{4}+4\times10^{3}+6\times10^{2}+5\times10^{1}-7\times10^{0}$ (b) $8\times10^{7}+8\times10^{-4}+8\times10^{2}+7\times10^{0}$

(c) $3\times10^{6}+2\times10^{5}+1\times10^{4}+3\times10^{2}+1\times10^{0}+2\times10^{0}$ (d) $7\times10^{4}+1\times10^{1}+8\times10^{0}$

2. (a) 7005030 (b) 57890

(c) 403050 (d) 76100

3. (a) 9×107 (b) 4.1569×109 (c) 5.9×105 (d) 5.9×105

4. (a) 3×108 m (b) 1.2756×106m (c) 1.2×10 10 Years (d) 1.027×109 (e) 3.84×108 m (f) 1.4×109 m

Revision Exercise

1. (a) (ii) (b) (iv) (c) (ii) (d) (iii) (e) (iii) (f) (iii) (g) (ii) (h) (iii) (i) (i)

2. (a) 256 (b) 1024 (c) 125 (d) 46656 (e) 243 (f) -343 **3.** (a) base = 7, power = 7 (b) base = -11, power = 10 (c) base = xy, power = a (d) base = m, power = 0 (e) base = 101, power = 1 (f) base = x, power = y

4. (a) m3 (b) (-1)4 (c) 35 (d) 23×m×n (e) 53×72 (f) m3×n2×p3

5. (a) 53 (b) equal (c) (-2)3 (d) 72 (e) 54 (f) (-1)3

6. (a) 5×22×33 (b) 53×72 (c) 25×52×3 (d) 23×3^4

7. (a) 500000 (b) 225 (c) 64 (d) (-4)120 (e) m9 (f) 0.25

8. (a) 26×35×57 (b) 215 (c) 54 (d) 517×310 **9.** (a) False (b) False (c) True (d) True

10. (a) y=5 (b) (c) y=6 (d) y= 14

11. (a) 5×105+7×103+3×102+4×10+5×100 (b) 5×106+7×102+8×10+5×100 (c) 4×105+9×104+5×103+7×102+8×10+1×100 (d) 6×105+7×104+9×103+1

12. (a) 50830201 (b) 460094 **13.** (a) 7×107 (b) 3.1678×109 (c) 1.353×109

Formative Assessment - 1

A. 1. (ii) 2. (i) 3. (i) 4. (iii) 5. (iii) 6. (iii) 7. (iv) 8. (iv) 9. (ii) 10. (i)

B. 1. $\frac{13}{3}$ 2. multiplication inverse 3. standard 4. litre 5. zero 6. mixed 7. decima 8. proper 9. irrational numbers 10. integers

C. 1. F 2. T 3. F 4. T 5. F 6. T 7. F 8. F 9. F 10. T

Ch-6 Algebraic Expression

Exercise 6.1

1. (a) x+250 (b) x- 670 (c) 100 - a×b (d) (x+x+1+x+2)2 (e) (x)2+(x+1)2

2.

Algebraic Expression	Terms	Numerical factor	Literal Factor
(a) 5x-6	5*x* , -6	5, -6	*x*
(b) -5*x*+8	-5*x* , 8	-5, 8	*x*
(c) 5x2 - 6^x	5x2, -6*x*	5, -6	*x* 2, *x*
(d) *x* 2y2-6*x* y	*x* 2y2, -6*x* y	1, -6	*x* 2y2, *x* y

3. (a) 15r5z (b) 15 p3z (c) 15 q4z (d) 15z **4.** (a) 4x2y, 4y (b) x2,1 (c) x2y2,y2

5. (a) like = ab2, 5b2a, unlike = 6a2b, 11 ab (b) like = (xyz, zyx), unlike= zy2x, zx2b (c) like = 17b, 15b , unlike = 14a, 18bc (d) like = 7x2y2z2, 5y2z2x2 unlike=3xyz, -5xy2z

6. (a) constant (b) Binomial (c) Trinomial (d) Monomial **7.** (a) –11 (b) No (c) No (d) 15

8. (a) xy (b) mn2 (c) xyz2 (d) xyz2

Exercise 6.2

1. (a) 11x+10y (b) 6x+5y+14 (c) 10x+5y (d) 8x - 3y - 5z **2.** (a) 4x3+3x2y+xy2-2y3 (b) -3m4 +6m3n- 3mn3+m2n2+11n2 **3.** (a) 6x2-6y2 (b) m2- 4mn **4.** 2x+11y+16z **5.** -m2+3mn

Exercise 6.3

1. (a) 10 (b) 1 (c) 3 (d) 21 **2.** (a) 8 (b) -5 (c) -1 (d) -5

3. (a) 32 (b) 0 (c) 0 (d) -16 **4.** 179 **5.** 64 **6.** 31

Exercise 6.4

1. (a) 10,22,31,61 (b) 9,25,37,77 (c) 20,40,55,105 (d) 14,54,105,405 **2.** 4x **3.** m×n = mn **4.** 6 cm

Revision Exercise

1. (a) (ii) (b) (iv) (c) (iii) (d) (i) (e) (ii) (f) (iv) (g) (ii) (h) (i) (i) (ii) (j) (iii)

2. (a) 2x-2+2x+2x+2=111 (b) x+1000 (c) x-90 (d) 676- m×n **3.** (a) -10 (b) 9,11 (c) 8,-5 (d) -7

4. (a) pq2,7q2p (b) abc, cab (c) 111b, 11b (d) 5×2y2z2, 9y2z2×2 **5.** (a) xy2z (b) a2b (c) x2a2m2 (d) ×2y5

6. (a) 5p3-12p2q+16pq2+q3 (b) -4x4-4xy3+7x3y2 -2x2y2+16y^2 **7.** (a) 8p2-11q2 (b) 4x2-3xy

8. -2m3–7m2+7m+2 **9.** (a) 11 (b) 7 (c) 198 (d) 4 **10.** 329

11. Area = m × n and Perimeter= (m+n) × 2

Ch-7 Simple Linear Equation

Exercise : 7.1

1. (a) 32 added to 6 times m gives 62 (b) 31 added to 7/8 times x gives 120 (c) 10 subtracted from half of t gives 103 (d) 5 times l gives 125 (e) 30 subtracted from d gives –20 (f) 35 added to 5 times a gives 0 (g) 4 subtracted from one-third x gives 4 (h)15/16times q gives 225

2. (a) $\angle p + \angle p + \angle 2p = 180^\circ$ (b) $80x + 60 = 365$, $80x - 305 = 0$ (c) $x/200 + 40 = 1000$ (d) $3x - 100 = 330$
(e) $-2^{m3} - 7^{m2} + 7m + 2$ **3.** (a) ✓ (b) ✓ (c) × (d) × **4.** (a) No (b) No (c) No

Exercise : 7.2

1. 67 **2.** 64.5°, 24.7° and 90.8° **3.** Sunita's age = 18 years ; Kamal's age = 12 years **4.** ₹125

5. $\frac{42}{52}$ **6.** (a) $\frac{1}{5}t + 10 = 14t$ (b) $\frac{y}{88} = 4$ (c) $19r - 10 = 180$ (d) $7r + 11 = 81$

(e) $\frac{(x \times 7) + 110}{100} = \frac{12}{13}$ (f) $\frac{(200 - m)}{17} = \frac{1}{6}$ **7.** (a) No (b) No (c) Yes (d) Yes

8. (a) Six times of x gives 139.
(b) A number x is multiplied by 17 and 21 is added to it. This entire term is divided by 121, we get 87.
(c) 108 subtracted from five times of gives 12.
(d) 25 is subtracted from of a number l gives 75.
(e) A number p is multiplied by 87 and 100 is added to it, the result is 13.
(f) Seven is added to one Sixth of a number. This entire term is multiplied by 3, we get 3.

9. 8 **10.** 1 **11.** 13 **12.** 3 **13.** (a) 70 (b) 10 (c) -7 (d) 20 **14.** 17, 18, 19
15. (a) simple (b) changed (c) 53 (d) 63 (e) 3 **16.** p = ₹816 **17.** 150 km **18.** 42 and 24
19. Dolly earns ₹ 18000, Sally earns ₹ 15000. **20.** 10

Revision Exercise

1. (a) (iv) (b) (iii) (c) (iv) (d) (iii) (e) (iii) (f) (iv) (g) (ii) (h) (iii) (i) (iii) (j) (iv)
2. (a) $\frac{k}{17} = 10$ (b) $7x + 10 = 310$ (c) $\frac{7}{6}x + 21 = 121$ (d) $7b - 2 = 112$

3. (a) One hundred eleven added to seven times p gives two hundred one. (b) A number of by subtracted twelve gives twenty.
(c) Ten added to five times of x gives fifteen. (d) 7 is added to x then divided by 3 and 2 more is added equal to the 10.
4. (a) $x = -2$ (b) $x = 48$ (c) $x = 11$ (d) $x = 3$ (e) $x = 1$ **5.** Present age of Rupa is = 25 years **6.** 6
7. (a) $2p + 11 = 0$, $4p + 22 = 0$, $6p + 33 = 0$ (b) $7x - 18 = 0$, $14x - 36 = 0$, $21x - 54 = 0$ (c) $14y - 84 = 0$, $7y - 42 = 0$, $2y - 12 = 0$
(d) $72l - 91 = 0$, $144l - 182 = 0$, $216l - 273 = 0$ **8.** 2 **9.** 90 and 72 **10.** 17 **11.** 19

Ch-8 Ratio and Proportion

Exercise 8.1

1. Raja = 105000, John = 45000, Supriya = 75000 Rs. **2.** 8 cm, 12 cm, 16 cm
3. (a) 5 : 7 < 4 : 5 (b) 3 : 2, 11 : 13, 5 : 7 (c) 100 : 1 (d) 8 : 5 **4.** 333 : 969, 444 : 1292, 555 : 1615 **5.** 7 : 2

Exercise 8.2

1. 13 : 22 : **2.** A = ₹240, B = ₹360, C = ₹504 **3.** 54 **4.** $\frac{12}{13}$ **5.** 132, 88 and 66. **6.** 44

7.

No. of orange	(a) 12 oranges	(b) 48 oranges	(c) 200 oranges
Weight	1800 gm 1.800 kg	7200 gm 7.200 kg	30000 gm 30 kg

8. 630 km
9. 3 days
10. 1000 km

Exercise 8.3

1. 4 days **2.** 2.5 days **3.** ₹2310 **4.** Sonu = ₹67.5 and Monu = ₹157.5 **5.** ₹ 14000 **6.** 3 days **7.** 3 hours
8. 3 days **9.** 40 minutes **10.** 19 km

Revision Exercise

1. (a) (iii) (b) (iii) (c) (iii) (d) (i) (e) (iv) (f) (i) (g) (iv)
2. A = 5,10,000 B = 11,90,000 C= 8,50,000 Rs. **3.** $\frac{228}{8} > \frac{121}{7}$ **4.** 40 days **5.** 1050 km

6. (a) 576 = 576 direct proportion (b) 1532.16 = 1532.16 in inverse proportion
7. (a) 27 (b) $\frac{5}{7}$ **8.** $\frac{2}{5} < \frac{1}{2} < \frac{3}{5} < \frac{9}{13} < \frac{5}{7}$ **9.** 17 **10.** $\frac{1}{7}$ 11. 450 workers

Ch-9 Percentage and its Applications

Exercise 9.1

1. (a) 60 % (b) 37.5 % (c) 33.33 % (d) 75 % **2.** (a) 12.5 % (b) 24 % (c) 275 % (d) 240 %
3. (a) 25 % (b) 340 % (c) 125 % (d) 220 % **4.** (a) 16 % (b) 125 % (c) 62.5 % (d) 26525 %
5. (a) $\frac{9}{25}$ (b) $\frac{5}{4}$ (c) $\frac{1}{8}$ (d) $\frac{2}{25}$ **6.** (a) 2:125 (b) 13 :20 (c) 3 : 8 (d) 21: 200

7. (a) 0.18 (b) 0.225 (c) 2.25 (d) 0.01123
8. (a) 10 *l* (b) ₹6375 (c) 300 kg (d) 75 km (e) 3 minutes 36 seconds (f) 13.5
9. (a) 500 (b) 480 (c) 240 (d) 280 **10.** 33.33% **11.** 20,000 **12.** Mukesh **13.** 800 marks
14. 400 candidates **15.** ₹25714 **16.** ₹2,50,000 **17.** 8000 students

Exercise 9.2

1. (a) ₹2175 (b) ₹1300 (c) ₹2510.75 (d) ₹114.75 **2.** (a) ₹800 (b) ₹2237 (c) ₹1395.5 (d) ₹746
3. (a) Profit = ₹90 and Profit% = 20% (b) Selling Price = ₹2640 and Profit = ₹440
(c) Cost Price = ₹600 and Profit% = 10% (d) Cost Price = ₹150 and Profit% = 20% (e) Cost Price = ₹720 and Selling Price = ₹828
4. SP. = ₹15,600 **5.** ₹48 per dozen **6.** ₹30000 **7.** ₹26000 **8.** 1.5% profit **9.** 25%
10. ₹5312.5 **11.** 58.33% **12.** ₹1200 **13.** 3.57% **14.** ₹675 **15.** (a) ₹60 per kg (b) ₹80 per kg

Exercise 9.3

1. (a) Amount = ₹2016 and Time = 2 years (b) Amount = ₹6200 and Rate = 6% (c) Principal = ₹562.50 sand Interest = ₹337.5
(d) Principal = ₹4000 and Time = 2.5 years **2.** P = ₹4000 **3.** P = ₹21484.6 **4.** 16.67% **5.** ₹312.5 **6.** 4 years
7. 12.5% **8.** P = ₹4000, R = 5% **9.** R = 12.5%, A = ₹13500 **10.** P = ₹6000

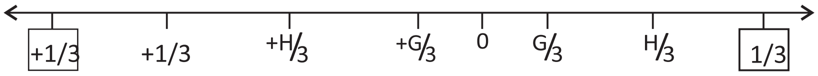
Revision Exercise

1. (a) (iii) (b) (iii) (c) (i) (d) (iv) (e) (ii) (f) (iii) (g) (iv) (h) (iii) **2.** (a) 75% (b) 50%
(c) 62.5% (d) 25% **3.** (a) ₹37.5 (b) 62*l* (c) ₹128 km (d) 0.24 kg **4.** Sehwag – 266.66% **5.** 11.1%
6. 500 marks **7.** 33.3% **8.** Loss 4% **9.** ₹1500 **10.** ₹200 **11.** Amount = ₹8000 **12.** 7 years
13. Rate = 8% **14.** P = ₹6250

Formative Assessment-II

A. 1. (iv) 2. (iii) 3. (iv) 4. (ii) 5. (iii) 6. (iii) 7. (iii) 8. (iv) 9. (i) 10. (iv)
B. 1. trinomial 2. literal 3. balancing 4. continued 5. amount 6. factor 7. constant term
8. consequents 9. percentage 10. interest
C. 1. T 2. F 3. T 4. F 5. F 6. F 7. T 8. F 9. F 10. T

Summative Assessment- I

A. 1. =64 x (100+5) =(64 x 100) + (64 x 5) = 6720 2. $\frac{1}{4}$ 3. $\frac{5}{4}$ 4. 333.315 5. $\left(\frac{-3}{2}\right)^5$ 6. -11
7. 14,15,16 8. ₹1000 9. 16000 10. 20% Profit 11. ₹1437.5
B. 12. Difference between odd and even number between 20 and 30 = 25
13.


14. 15. Do it yourself 16. $9x^4 - 7x^3 + 6x^2 - 12$ 17. $\frac{2}{7}$
18. (a) 11:9 (b) 9:20 (c) 11:20 19. Profit 4.3% 20. 8 years 4 months 21. 300
C. 22. (a) 240 (b) No 23. $\frac{-41}{63}$ 24. 41.6% profit
25. Value of first prize=1050, value of second prize 875 and value of third prize =700 26. 30, 120 27. Paid interest= ₹36000

Ch-10 Triangle and Its Properties

Exercise 10.1

1. 20°, 60°, 100° **2.** 360° **3.** (a) P = 65°, P = 65° (b) P = 70° (c) P = 30°, 2P = 60° (d) P = 50°, Y = 80°
(e) y = 60°, P = 70° **4.** (a) a = 112°, b = 147°
5. (a) The sum of all angles of a triangle is equal to 180°. A triangle can not have two angles of 90° each. (b) Yes
(c) Scalene Triangle (d) Equilateral Triangle **6.** (a) $58^0, 58^0$ and 64^0 **7.** (a) Vertically opposites
(b) Alternate interior angles (c) Corresponding angles (d) Exterior angle (e) Interior angles (f) Right angles
8. (a) An equilateral triangle have all three sides equal but an isosceles triangle have only 2 equal sides.
(b) An equilateral triangle have all three angles equal but an isosceles triangle have two angles equal.
9. (a) x = 66° (b) y = 92° (c) y = 22° **10.** 2 cm and 14 cm

Exercise 10.2

1. 25 km **2.** 25 cm **3.** Perimeter = 68 cm. **4.** Length of diagonal = 13 cm.
5. Since $6^2 + 4.5^2 = 7.5^2$ (Pythagoras theorem) **6.** x = 115°, y = 65°, z = 25°

Revision Exercise

1. (a) Three angles (b) 90° (c) AB2+BC2=AC2 (d) Longest side (e) No (f) None of these (g) Less than 90°
2. y = 62°, x = 148° 3. (a) Possible Scalene (b) Possible, Scalene Triangle (c) Possible, Scalene Triangle
(d) Impossible (e) Possible, Scalene Triangle 4. (a) The sum of all angles of a triangle is equal to 180°.
(b) Do it yourself 5. third angle = 43°, acute angle, 6. 19°
7. (a) A scalene triangle have all sides unequal but an isosceles triangle have two equal sides and one different.
(b) Do it yourself (c) X = 60° Y = 70° 8. (a) 40°, 60°, 80° (b) 107° (c) 152° (d) equilateral (e) 90° (f) 98m
(g) longest (h) greater 9. Do it yourself 10. $x = 70°$, $y = 50°$, $z = 60°$ 11. Do it yourself
12. 135° 13. ∠P+∠Q+∠R+∠S+∠X+∠T=360° 14. 6 cm and 16 cm 15. Do it yourself 16. $x = 120°$, $y = 60°$, $z = 70°$

Ch-11 Congruence

Exercise 11.1

1. TM ≅ XY (given)
XY ≅ MN (given)
∴ MN ≅ TM
2. Do it yourself
3. (a) One side of a square is congruent to any one side of the other square.
(b) They have a equal length and breath. (c) Their length are equal.
(d) The radius of two circle are equal. (e) Their measures are equal.
4. (a) ∠P (b) $\overline{QR}$ (c) ∠R (d) $\overline{PQ}$ 5. 60° 6. Do it yourself 7. Do it yourself

Exercise 11.2

1. Do it yourself 2. Do it yourself 3. ∠6 = ∠3, ∠4 = ∠2, ∠5 = ∠1, TQ = DM, TB = DR, QB = MR
4. △QTN ≅ △△RSN (by SAS) 5. Because, AAA is rarely seen in congruence of triangles. 6. △AxM ≅ △BMY (by SSS)
7. (a) T (b) T (c) T (d) T 8. Yes, because their angles are congruent. 9. Yes 10. Do it yourself

Revision Exercise

1. (a) (iv) (b) (iv) (c) (iii) (d) (ii) (e) (ii) (f) (iii) (g) (ii) (h) (iv) 2. Do it yourself 3. No, because RT[1] DB
4. In △XYZ and △YPZ
XY = YP (given)
∠XYZ = ∠LPZ (given)
YZ = YZ (common)
∴ △XYZ ≅ △YPZ (by SAS)
5. Yes
6. △DEF ≅ △PEF
7. $p = 22^\circ$, $q = 40^\circ$
8. Do it yourself
9. Yes
10. (a) 4.5 cm (b) PQ (c) areas (d) congruent (e) AAA (f) 70° (g) RHS

Ch-12 Perimeter and Area

Exercise 12.1

1. 21.6 m 2. (a) 32 cm (b) 18 cm 3. (a) 18 cm (b) 27 cm (c) 36.9 cm 4. 37 m 5. 16.6 cm
6. Length=36m, Breath=12 m 7. Length = 196cm, Breath = 147cm 8. 24m, 36m and 48m 9. Side = 21cm
10. (a) 20cm (b) 38m (c) 30.4cm (d) 56m

Exercise 12.2

1. (a) $120m^2$ (b) $45m^2$ (c) $90m^2$ (d) $58m^2$ 2. (a) $56.25m^2$ (b) $169m^2$ (c) $51.84m^2$ (d) $144m^2$ 3. $80 cm^2$
4. 16m 5. (a) $84m^2$ (b) $19.2m^2$ 6. $432m^2$ 7. $180m^2$ 8. 75m and 50m 9. $31cm^2$
10. F = $7cm^2$, L = $5cm^2$, M = $9cm^2$, H = $7cm^2$ and J = $6cm^{2T}$, T = $5cm^2$ 11. 32m 12. 72m

Exercise 12.3

1. (a) $111m^2$ (b) $225m^2$ 2. $445m^2$ 3. ₹1462.5 4. $4875m^2$ 5. ₹4608 6. ₹90900 7. ₹1472.5 8. $824m^2$
9. (a) $200cm^2$ (b) $56m^2$

Exercise 12.4

1. (a) 176cm (b) 22cm (c) 66m (d) 26.4cm 2. (a) 22m (b) 13.2cm (c) 264m (d) 44m 3. $7546m^2$
4. $616cm^2$ 5. $3758.86m^2$ 6. $182.5m^2$ 7. $638m^2$ 8. $1386m^2$ 9. $990cm^2$ 10. (a) $119m^2$ (b) $192.5m^2$

Revision Exercise

1. (a) (iii) (b) (ii) (c) (i) (d) (iv) (e) (ii) (f) (ii) (g) (iv) (h) (iii) 2. Length = 42.5m, Breath = 17m
3. 36m, 24m and 24m 4. 56 m 5. Rahul, 270m 6. $12500m^2$ 7. 98m 8. $4608m^2$
9. Base 91m, altitude = 52m 10. ₹1120 11. $411m^2$ 12. $3500m^2$ 13. 5280m 14. (a) $70.56m^2$ (b) $154cm^2$ (c) $81.44cm^2$

Ch-13 Construction Geometry

Exercise 13.1

Do it yourself

Exercise 13.2

1. Do it yourself 2. (a) Yes (b) Yes (c) No (d) Yes 3. Do it yourself 4. Do it yourself
5. Do it yourself 6. Do it yourself 7. Do it yourself 8. Do it yourself 9. Do it yourself 10. Do it yourself

Revision Exercise

1. (a) 1 (b) 180° (c) 2.5 cm, 8.5 cm, 3.5 cm (d) 3.5 cm (e) (f) 55° (g) hypotenuse and a side is given.
(h) compasses (i) parallel to each other. (j) 2 **2.** Do it yourself **3.** Do it yourself
4. Do it yourself **5.** Do it yourself **6.** Do it yourself **7.** Do it yourself **8.** Do it yourself
9. Do it yourself **10.** Do it yourself **11.** (a) Yes (b) No (c) Yes (d) Yes
12. Do it yourself **13.** Do it yourself

Formative Assessment-III

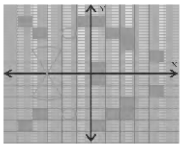
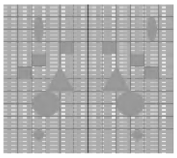
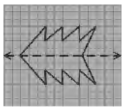
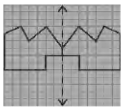
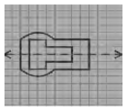
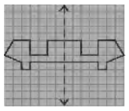
A. 1 (ii) 2. (i) 3. (iii) 4. (ii) 5. (iii) 6. (ii) 7. (iv) 8. (ii) 9. (iv) 10. (iv)
B. 1. one angle 2. line of 3. infinite 4. sum 5. 17 6. isosceles triangle 7. 60°
8. greater 9. 0 10. symmetric
C. 1. T 2. T 3. F 4. F 5. T 6. T 7. T 8. T 9. F 10. T

Ch-14 Symmetry

Exercise 14.1

1. (a) 2 (b) Infinite (c) Four (4) (d) 4 (e) 3 (f) 5 (g) No (h) 2 (i) 6 (j) 1
(k) 1 (l) 1 **2.** Do it yourself
3. A B C D E H M, O T U V W X Y **4.** (a) F (b) F (c) T (d) T (e) T

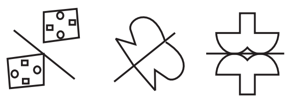
Exercise 14.2

1. Do it yourself **2.**  **3.**  **4.** (a)  (b)  (o)  (d) 

Exercise 14.3

1. Do it yourself **2.** (a) 3 (b) 4 (c) 2 **3.** (a) Octagon (b) Hexagon (c) Square (d) Equilateral triangle
4. A (4, -10), B (0, 0) **5.** (a) (-2, -3) (b) (5, -4) (c) (-4, 2) (d) (5, 3) **6.** (a) (2, -3) (b) (5, 4) (c) (-2, -4) (d) (-4, 6) **7.** Do it yourself

Revision Exercise

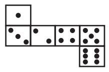
1. (a) P' ↔ (–3, –8) (b) eight (c) One point of rotation (d) rhombus (e) get inverted (f) reflection
(g) 2 (h) None of these **2.** V W X Y M U B A T **3.** 
4. (a) (7, 6) (b) (18, 12) (c) (-6, -20) (d) (-7, -21) (e) (0, 4) (f) (11, -8)
5. (a) (-6, -18) (b) (4, 9) (c) (7, -2) (d) (-10, 5) (e) (7, 0) (f) (0, -11)
6. (a) F (b) T (c) T (d) T (e) F (f) T (g) T (h) F (i) T (j) T **7.** Do it yourself

Ch-15 Representing 3-D in 2-D

Exercise 15.1

1. (a) Cylinder (b) Sphere (c) Sphere (d) Cylinder (e) Cuboid (f) Cuboid (g) Prism (h) Pyramid
2. Do it yourself **3.** Do it yourself **4.** (a) 0 (b) 0 (c) 8 (d) 1 (e) 5 vertices

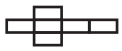
Exercise 15.2

1. Do it yourself **2.** (a) blank (b) 0 (c) $ (d) * (e) * and 0 (f) $ (g) M and E
(h) M→E, O→*, $ blank **3.** Do it yourself **4.** (c) **5.** 

Exercise 15.3

1. Do it yourself **2.** Do it yourself **3.** Do it yourself **4.** Do it yourself

Revision Exercise

1. (a) pyramid (b) none of these (c) on a squared paper (d) circle (e) V + F - E = 2
(f) polyhedrons (g) rectangles (h) edge (i) tetrahedron
2. (a) Vertices – A, B, C, D, P, Q, R, S
Faces – ABCD, PQRS, ABQP, SRCD, ADSP, BCRQ
Edges – AB, BC, CD, AD, PQ, QR, RS, SP, AP, DS, BQ, CR
(b) Curve face
Plane faces No edge
(c) Vertices – L, M, N, R, S, T
Faces – LMN, RST, LMRS, LNST, MNTR
Edges – LM, LN, MN, LS, SR, MR, NT, ST, RT
3. Do it yourself
4. (a) Horizontally → Circle
Vertically → Rectangle
(b) Horizontally → Square
Vertically → Square
(c) Horizontally → Triangle
Vertically → Circle
(d) Horizontally → Triangle
Vertically → Quadrilateral
5. (a) 4 and 6 (b) 1 (c) 6 (d) 5 **6.** (a) cone (b) tetrahedron (c) cube (d) volleyblall
7. (a) 
(b) 
(c) 
(d) 

8. 2 cylinders, height 16 cm and 12 cm **9.** Do it yourself **10.** (a) F (b) T (c) F (d) T (e) T (f) F (g) T (h) T (i) F (j) T

Ch-16 Data Handling and Probability

Exercise 16.1

1. Frequency distribution table :

Marks	Tally marks	Frequency
15	IIII	4
18	II	2
19	~~IIII~~	5
20	IIII	4
25	IIII	4
30	I	1
35	III	3
36	I	1
38	I	1

2. Pictograph of absentees :

(• represents 1 day)

Monday	• • • • •
Tuesday	• • •
Wednesday	• • •
Thursday	• •
Friday	• • • • • •
Saturday	• • • • •

3. (a) 2 laptops
(b) 22 laptops
(c) Wednesday

Exercise 16.2

1. $\text{Mean} = \frac{80+150+16+46}{4} = \frac{292}{4} = 73$ Readers

2. Median = 45.5, Mode = 46, Range = 23

3. Range = 32 Mean = 98.9 Median = 96 Mode = 90 & 96 **4.** Mean of first five odd number = 5 **5.** The value of x = 62

Exercise 16.3

1. (a) April (b) 400 (c) June (d) May **2.** Do it yourself **3.** (a) 4000 biscuits (b) 2003-04 (c) 2006-07 4. Do it yourself

Exercise 16.4

1. (a) $\frac{1}{4}$ (b) $\frac{7}{12}$ (c) $\frac{1}{6}$ **2.** (a) $\frac{1}{12}$ (b) $\frac{1}{12}$ **3.** (a) $\frac{1}{26}$ (b) $\frac{1}{13}$ (c) $\frac{1}{52}$ (d) $\frac{1}{4}$

4. (a) $\frac{1}{6}$ (b) $\frac{1}{3}$ (c) $\frac{11}{12}$ **5.** (a) $\frac{1}{4}$ (b) $\frac{1}{4}$ (c) $\frac{3}{4}$

Revision Exercise

1. (a) (ii) (b) (iii) (c) (iv) (d) (ii) (e) (iii) (f) (iii) (g) (ii) **2.** 6 hours **3.** 294 **4.** 3

5. Do it yourself **6.** (a) $\frac{1}{26}$ (b) $\frac{4}{13}$ (c) $\frac{3}{13}$ **7.** (a) $\frac{1}{5}$ (b) $\frac{2}{5}$

Formative Assessment–IV

A. 1. (iv) 2. (iii) 3. (ii) 4. (i) 5. (iii) 6.(iii) 7. (i) 8. (i) 9. (iv) 10. (ii)

B. 1. 4 2. Statistics 3. concentric 4. congruent 5. 3 6. Perimeter 7. Radius 8. 4 x side 9. Area 10. π $(R^2 – r^2)$

C. 1. F 2. F 3. T 4. T 5. T 6. F 7. T 8. F 9. F 10. T

Summative Assessment–II

A. 1. 21.7cm 2. 65° 3. Do it yourself 4. Edges =12; Faces =6; Vertices= 8
5. AB; PO; BC=OR ; AC=PR, ∠A = ∠P; ∠B= ∠O, ∠C = ∠R 6. Do it yourself
7. 128m 8. 101.4 9. 3872 m 10. 8 cm^2 11. COY =70°

B. 12. 40°, 35°, 105° 13. 17 m 14. Do it yourself 15. Do it yourself 16. Do it yourself 17. Do it yourself 18. 280 m^2
19. Mean = 113.8, Median = 117, Mode = 117 20. (a) 1/7 (b) 2/7 (c) 2/7 21. Base =44 m, height = 33 m

C. 22. Do it yourself 23. 180°, 360°, order =2, 2 24. 41 cm 25. Do it yourself 26. Do it yourself 27. ₹ 61563.25
28. Do it yourself