Lines, Ray and Angles

Point and line are the building blocks of geometry. These two concepts form the basis of geometrical concepts and theories. Let us start with point.

Point

A circle of zero radius is known as point. Point is simply a dot (.) marked with a sharp pencil, represents a point. A point has no length, no breadth or no height (thickness). Capital letters of the English alphabet are used to show different points represented by dots.

For Example : A, B and C are points.

$$A \qquad B \qquad C$$
$$\bullet \qquad \bullet \qquad \bullet$$

Line

A line is a set of infinite number of points join together. It can be produced infinitely in any direction. A line has neither end points nor a definite length. Take the line $\overleftrightarrow{AB}$. The both sided arrow '$\longleftrightarrow$' marked above AB shows that the line AB can be extended infinitely from both the sides.

$$\overset{A}{\bullet} \qquad\qquad \overset{B}{\bullet}$$

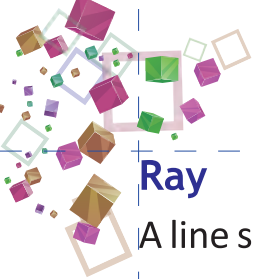AB is a line and it is denoted as $\overleftrightarrow{AB}$

A line cannot be drawn but it can be represented by a diagram on a piece of paper or any other flat surfaces.

Line Segment

A small part of a line with definite length is known as line segment. Line segment has two end points with definite length. Line segment can be measured.

$$\overset{C}{\bullet}\rule{4cm}{0.4pt}\overset{D}{\bullet}$$

CD is a line segment and it is denoted by $\overline{CD}$.

Ray

A line segment produced endlessly in one direction is called ray. A ray has no definite length. A ray has one end point called "initial point" or "starting point" or "end point". Take the ray AB. The arrow '→' marked above AB shows that point A is the initial point of ray $\overrightarrow{AB}$.

A Line Segment	A Line	A Ray
1. A line segment can be drawn on a piece paper.	1. A line cannot be drawn on a piece of paper but it can be represented by a diagram.	1. A ray cannot be drawn on a piece of paper but it can be represented by a diagram.
2. A line segment has definite length.	2. A line does not have a definite length.	2. A ray does not have a definite length.
3. A line segment has two end-points.	3. A line has no end point.	3. A ray has one end-point.

Relationship between Points and Lines

We can draw an infinite number of lines passing through a point. Lines that contain the same point are known as concurrent lines.

Collinear Points

If three or more points lie in a straight lines the points are said to be collinear points.
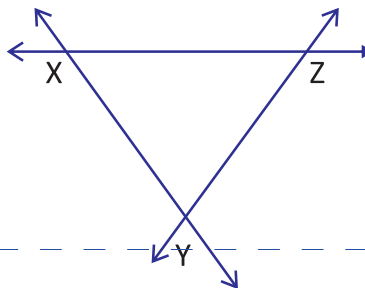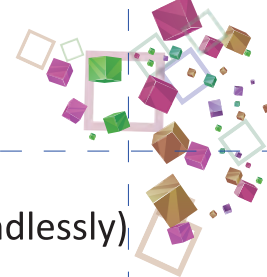

A, B and C are points on line $\overleftrightarrow{AC}$. So, A, B and C are collinear points.

Non-collinear Points

Three points not lying in the same straight line are called the non-collinear points.

X, Y and Z are non-collinear points.

Plane

A plane is a flat surface. A plane extends in all the four directions infinitely (endlessly) in length and breadth.

A plane can not be drawn on a piece of paper. It has no boundary and what we draw on a paper is a part of a plane only but not the plane itself. Table top, walls, roof of the room etc., are the examples of the part of plane.
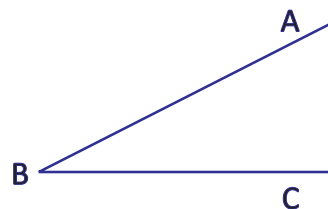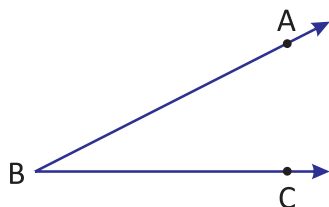
Angle

Two rays with a common end point form an angle or two line segments with the common end point also form an angle. The common end point of the two rays or line segments forming an angle is called the vertex of the angle. The two rays or line segments are called arms (or sides) of the angle. In each of the given figures, an angle is formed, where O is the vertex and rays or line segments OA and OB are the arms of the angles.
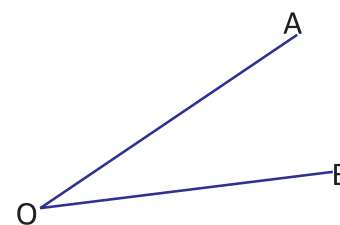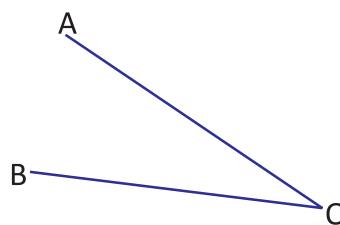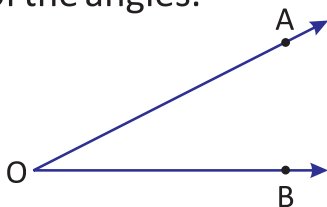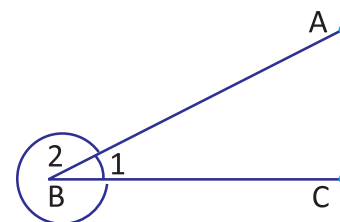
Naming an Angle

To name an angle, the capital letters of English alphabets are used. Letter denoting the vertex of an angle is always written in the middle. We name the angle ABC or angle CBA. Symbol '$\angle$' is used to denote the word 'angle'. Therefore, the angle ABC can be written as $\angle$ABC and angle CBA as $\angle$CBA.

Interior and Exterior Angle

An angle has two parts – interior and exterior. When two rays or line segments with a common end-point are drawn then two angles are formed, one angle is called interior angle i.e. $\angle$1 and the other exterior angle i.e. $\angle$2.

Measuring an Angle

To measure an angle, we define a unit angle. First two opposite rays having the same end point are drawn forming a straight angle ∠AOB. To obtain a unit angle, divide the straight angle AOB into 180 equal parts.

A   O   B
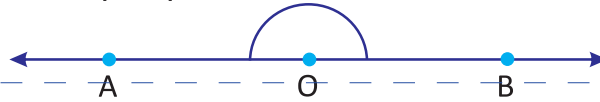
Then, each angle represents a unit angle. The measure of a unit angle is called a degree. ∠AOB is a straight angle.

For Example   :   Consider the ∠ABC.

Base line   Centre line
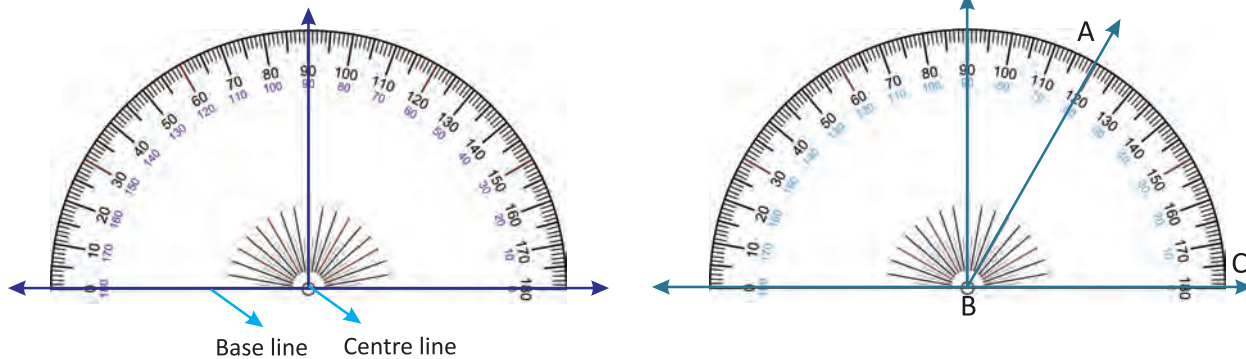
Place the centre of the protractor on the vertex of the angle ABC and adjust the protractor so that the base line falls on arm BC of the angle. Look for the scale inner or outer which begins with zero and read the mark on the protractor where the other arm BA concide the scale on the protractor. In the figure measurement of ∠ABC = 60°.

Types of Angles

According to their degree of measures, angles are of five types :

1. **Acute Angle**

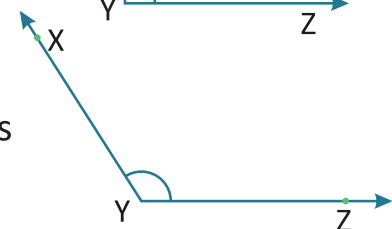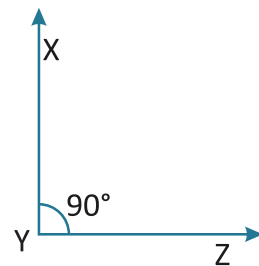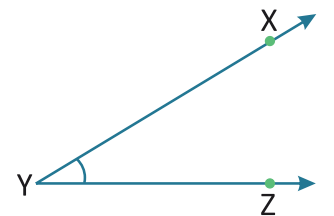   An angle whose measure is greater than 0° but less than 90° is called an acute angle. ∠XYZ is an acute angle.
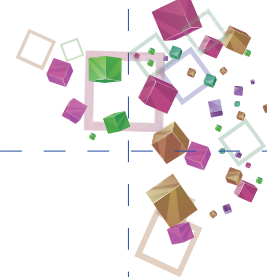
2. **Right Angle**

   An angle whose measure is 90° is called a right angle. ∠XYZ is a right angle.

3. **Obtuse Angle**

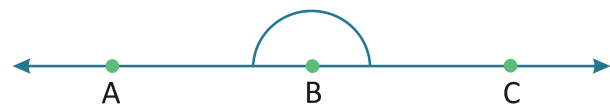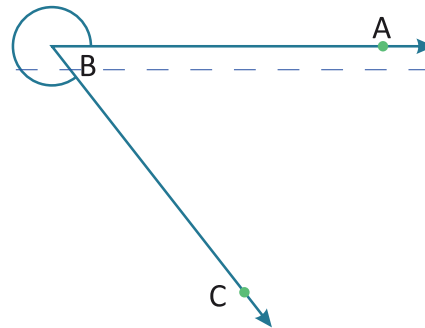   An angle whose measure is between 90° and 180° is called an obtuse angle. ∠XYZ is an obtuse angle.

4. Straight Angle

An angle whose measure is 180° is called a straight angle. ∠ABC is a straight angle.

5. Reflex Angle

An angle whose measure is more than 180° but less than 360° is called a reflex angle. The out side ∠ABC is a reflex angle.

Comparison of Angles

An angle whose degree of measure is greater than the degree of measure of another angle is a greater angle. Then we can say that :

a.   A right angle is greater than an acute angle.

b.   An obtuse angle is greater than an acute angle and it is also greater than a right angle.

c.   A straight angle is always greater than an acute angle, a right angle and an obtuse angle.

d.   A reflex angle is always greater than an acute angle, a right angle, an obtuse angle and a straight angle.

Facts to Know

○   **Two angles whose sum is 90° known as complementary angles.**

Complementary Angles

Two angles are said to be complementary to each other if the sum of their measure is 90°.

In the given figures ∠ABC = 50°, ∠XYZ = 40° and ∠ABC + ∠XYZ = 50° + 40° = 90°. Thus, ∠ABC and ∠XYZ are complementary to each other.
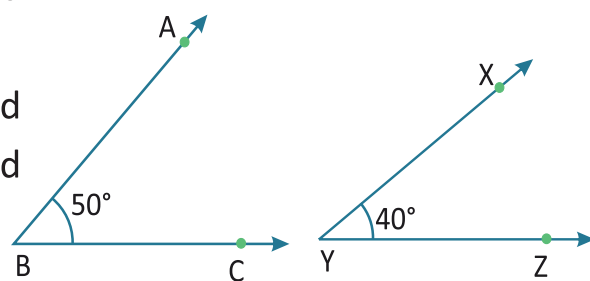
Supplementary Angles

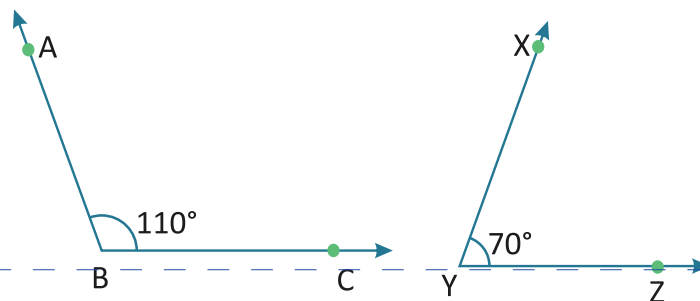Two angles are said to be supplementary to each other if the sum of their measure is 180°.

In the given figure, $\angle ABC = 110°$ and $\angle XYZ = 70°$. Then,

$\angle ABC + \angle XYZ = 110° + 70° = 180°$.

Thus, $\angle ABC$ and $\angle XYZ$ are supplementary to each other.

Adjacent Angles

Two angles are said to be adjacent to each other if :

a.   they have the same vertex.

b.   they have a common arm and the other arms of the two angles are on the apposite side of the common arm.

In the figure, the angles $\angle ABD$ and $\angle DBC$ have common vertex B and a common arm BD. The other arms BA and BC of the angles $\angle ABD$ and $\angle DBC$ are on opposite sides of the common arm BD. Hence, the $\angle ABD$ and $\angle DBC$ form a pair of adjacent angles.
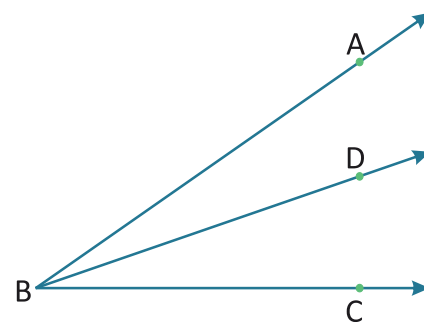
Vertically Opposite Angles

Two angles formed by two intersecting lines without common arm are called vertically opposite angles.

In the given figure, lines AB and CD intersect each other at point O. The four angles $\angle AOC$, $\angle COB$, $\angle BOD$ and $\angle AOD$, in which $\angle AOC$ and $\angle BOD$ are vertically opposite angles and $\angle COB$ and $\angle AOD$ are vertically opposite angles. $\angle AOC$ and $\angle COB$, $\angle COB$ and $\angle BOD$, $\angle BOD$ and $\angle DOA$, $\angle DOA$ and $\angle AOC$ are pairs of adjacent angles.

Facts to Know

○   **Two angles whole sum is 180° known as supplementary angles.**
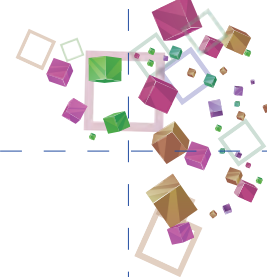
1. **Fill in the blanks.**

    a. An angle measuring 180° is called a .... angle.

    b. A line segment has a .... length.

    c. A line can be produced .... in any direction.

    d. A ray has no .... length.

    e. An angle which measures more than 90° but less than 180° is called an ....

    f. A reflex angle is always .... than 180° but .... than 360°.

    g. An angle whose measure is 45° is an .... angle.

    h. An angle has .... vertex and .... arms.

    i. A straight angle measures ....

2. **Which of the following pairs of angles are complementary 'or' supplementary?**

    a. 160°, 30°   b. 45°, 45°   c. 30°, 60°   d. 135°, 45°

    e. 10°, 170°   f. 20°, 70°   g. 70°, 20°   h. 20°, 170°

3. **Which of the following pairs of angles are supplementary?**

    a. 90°, 90°   b. 130°, 50°   c. 180°, 0°   d. 80°, 100°

    e. 45°, 145°   f. 135°, 45°   g. 30°, 170°   h. 60°, 135°

4. **Find the measure of the complement of each of the following angles.**

    a. 15°   b. 22°   c. 38°   d. 65°

    e. 90°   f. 50°   g. 42°   h. 71°

5. **Find the measure of the supplement of each of the following angles.**

    a. 137°   b. 128°   c. 112°   d. 65°

    e. 105°   f. 47°   g. 135°   h. 85°

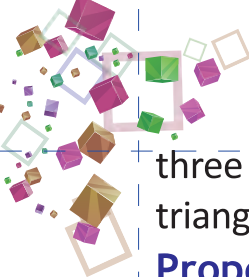# Triangle

A closed figure bounded by three line segments is called a triangle. The symbol for a triangle is △ .

If we join three non-collinear points A, B and C lying on the plane of paper, then we will get a closed figure bounded by three line segments AB, BC and CA. This closed figure is called a **triangle**. Therefore, a triangle has three line segments, three vertices and
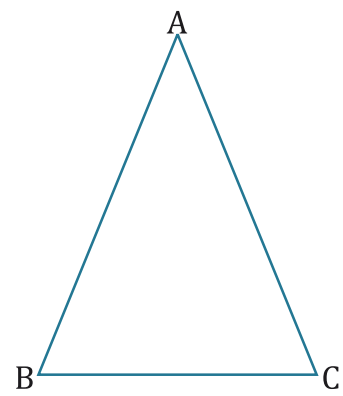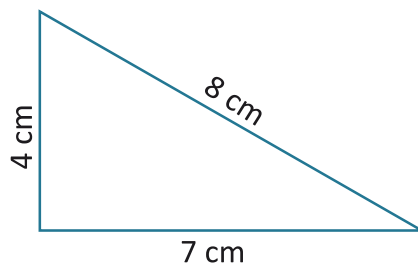
three angles. The three line segments of a triangle are also known as sides of the triangle. A triangle has six elements, three sides and three angles.

Properties of a Triangle

1. If we measure the length of all three sides of the given triangle, we find that the sum of the lengths of any two sides of a triangle is greater than the length of the third side.

4 cm + 7 cm = 11 cm > 8 cm

7 cm + 8 cm = 15 cm > 4 cm

4 cm + 8 cm = 12 cm > 7 cm

2. If we measure all three angles of a triangle and add them, we find that the sum of the measures of three angles of a triangle is 180°.
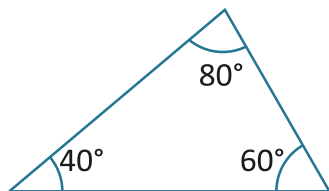
80° + 40° + 60° = 180°

Classification of Triangles

Triangles are classified in two ways :

1. according to their sides
2. according to their angles

1. Triangle according to their sides are of three types :

a. **Scalene Triangle**

A triangle whose all three sides are of different lengths is called a scalene triangle. All the angles of a scalene triangle are also different.
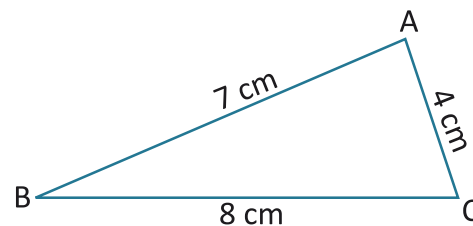
b. **Isosceles Triangle**

A triangle whose any two sides are equal in length is called an isosceles triangle. Two angles of an isosceles triangle are also equal.

c. Equilateral Triangle

(i)   A triangle whose all three sides are of equal in length is called an equilateral triangle.

(ii) All the three angles of equilateral triangle are equal and each angle is of 60°.
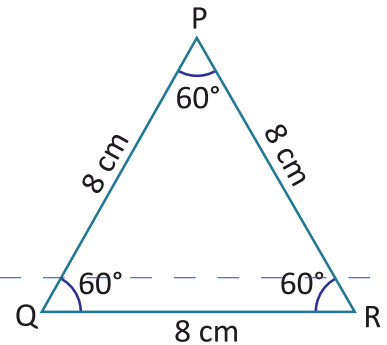
2.   **Triangle according to their angles are also of three types :**

a.   **Acute-angled Triangle (or Acute Triangle)**

A triangle whose all angles are acute (i.e. less than 90°) is called an acute-angled triangle or acute triangle.
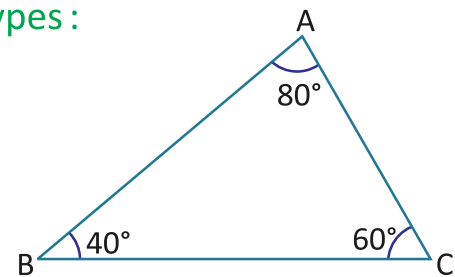
b.   **Right-angled Triangle (or Right Triangle)**

A triangle whose one angle is a right angle (i.e. 90°) is called a right-angled triangle or right triangle.
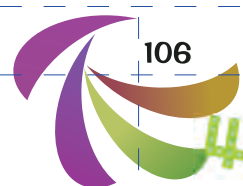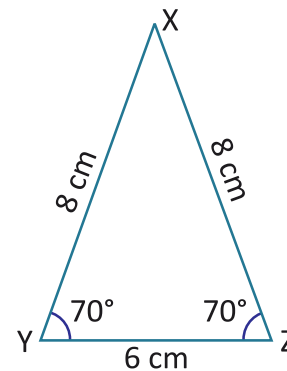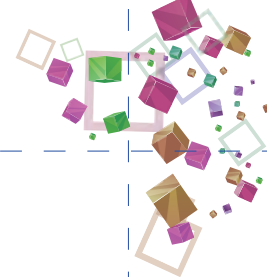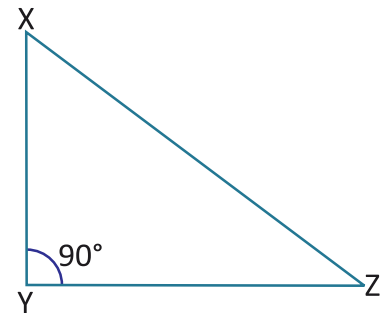
c.   **Obtuse-angled Triangle (or Obtuse Triangle)**

A triangle whose one angle is an obtuse (i.e. more than 90°) is called an obtuse-angled triangle or an obtuse triangle.
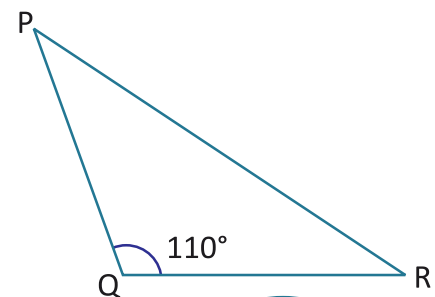
Circle

Circle is a closed curve. The outer edges of a rupee coin, a motor cycle wheel etc., are the examples of a circle. A circle can be drawn by putting a one rupee coin or a bangle on a plane of the paper and moving a pencil around their boundaries.

We can also say that circle is the set of points that are at the same distance from a fixed point O. This fixed point is called the centre of the circle.

The set of points that lie on the circle form the circumference of the circle.
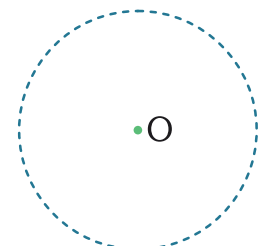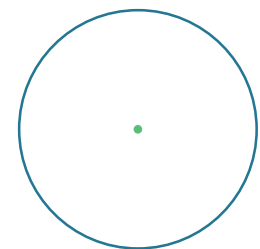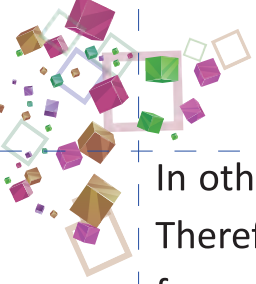
In other words, the boundary or perimeter of a circle is called its circumference. Therefore, a circle is a simple closed curve, every point of which is equidistance from a fixed point, called the centre of the circle.

Radius

The distance between centre of the circle and any point on the circle is called its radius. OA and OB are the two radii of given circle. There are many radii of a circle. The length of all radii of a circle is equal.

Diameter

The line segment passing through the centre of the circle and whose end points lie on the circle, is called the diameter of the circle. XY is the diameter of the given circle. The diameter of a circle is twice the radius. Therefore, in any circle, length of a diameter = 2 × length of a radius. The centre of a circle always lies on its diameter. There are many diameters of a circle.
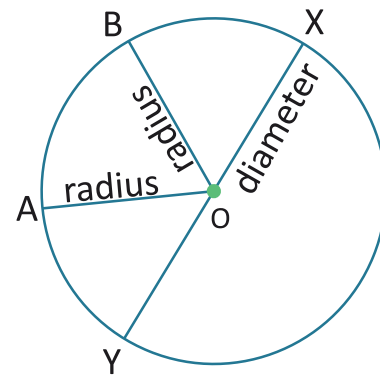
Chord

A line segment whose end points lie on the circle is called chord. A large number of chords can be drawn from a point A on the circumference of the circle. One chord passes through the centre O. It is the diameter of the circle. Therefore, the diameter is the longest chord of the circle.
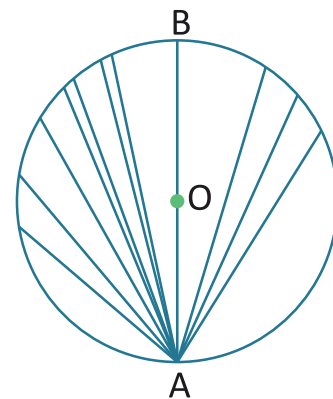
Arc and Semicircle

Any part of a circle is called an arc of the circle. ACB represents an arc of the circle.

Half of a circle is called a semicircle.

PRQ is a semicircle. PSQ is also a semicircle. Semicircle is also an arc of a circle. There are two semicircles of a circle.

Drawing a Circle of a Given Radius

Example I : Draw a circle of radius 4 cm.
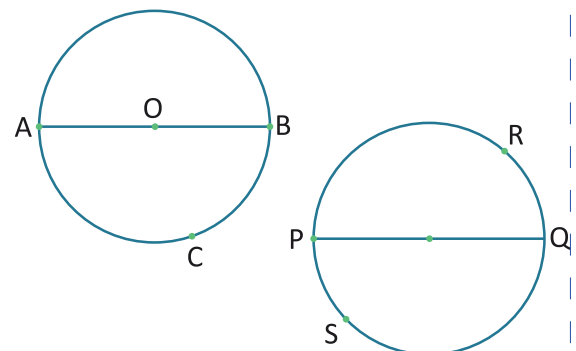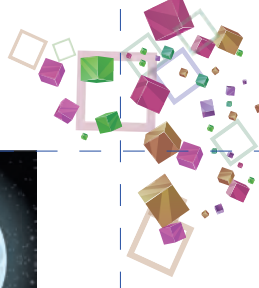
Solution :

> "The full moon reminds us a circle"

Step 1 : Take a sharpened pencil and a compass. Fix the pencil in the compass.

Step 2 : Stretch the arms of the compass and adjust the opening between the pointed ends as shown in figure.

P 4cm O

4cm

S

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
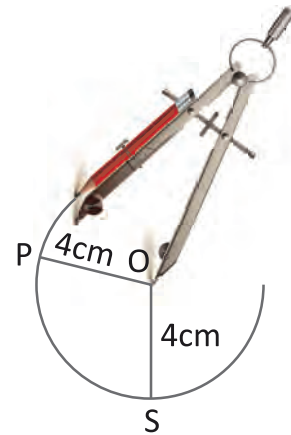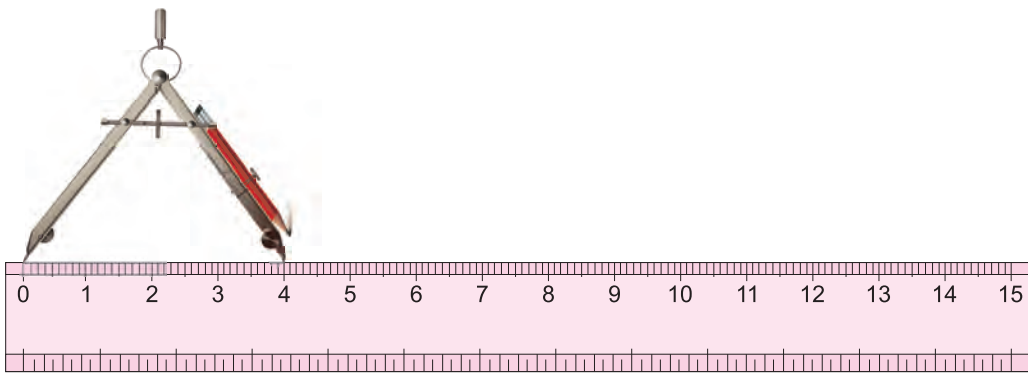
Step 3 : Keep the metal point fixed on the paper and move the pencil arm around it.

Exercise 8.2

1. Fill in the blanks.

a. An angle means dividing it into two equal angles the ray bisects is known as ....

b. A .... triangle has one angle of 90°.

c. Triangle is formed by joining three .... points.

d. A point where two sides of a triangle meet is known as .... of a triangle.

e. A triangle has .... vertices and .... sides.

f. A triangle has .... elements.

g. The sum of any two sides of a triangle is always .... than the third side of the triangle.

h. In an obtuse triangle, the remaining two angles are ....
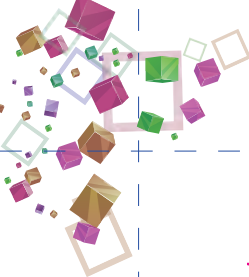
i. In a/an .... triangle, all the angles are equal to 60° each.

j. Points lying on a same line are called .... points.

k. The sum of the measures of three angles of a triangle is ....

l. A triangle whose any two sides are of equal length is known as .... triangle.

2. The measures of two angles of a triangle are 60° and 100°. What is the measure of third angle of the triangle?

3. Which of the following cannot be the measures of three angles of a triangle?

a. m∠A = 60°, m∠B = 50° and m∠C = 50°

b. m∠A = 60°, m∠B = 90° and m∠C = 30°

c. m∠A = 90°, m∠B = 90° and m∠C = 10°

d. m∠A = 100°, m∠B = 50° and m∠C = 30°

e. m∠A = 120°, m∠B = 40° and m∠C = 35°

4. If the measure of three angles are given below then in which of the following cases it is possible to draw a triangle.

a. 50°, 70°, 40°   b. 80°, 80°, 60°   c. 70°, 80°, 100°

d. 35°, 55°, 110°   e. 50°, 80°, 50°

5. Fill in the blanks.

a. There are .... semicircles of a circle.

b. Any part of a circle is called an .... of the circle.

c. The centre of a circle always lies on its ....

d. The length of all radii of a circle is ....

e. The distance around the circle is known as ....

f. A diameter is the .... chord of a circle.

g. Perimeter of a circle is called the .... of the circle.
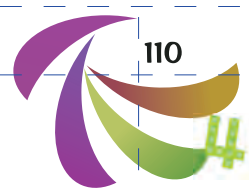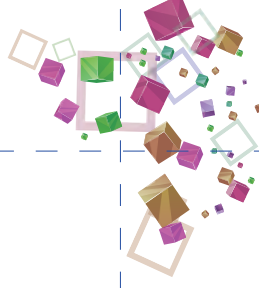
Circumference

The length of the boundary of the circle known as is its Circumference. Let we can say that this is the perimeter of the circles. Circumference of the circle = πd=$2\pi r$ where 'd' represents diameter of the circle, 'r' its radius and $\pi = \dfrac{22}{7}$. It is also found that the circumference of a circle is about 3 times the diameter of the circle, i.e. circumference = 3 × diameter (approx).

Example II : Find the diameter of the circle whose radius is 6.25 cm.

Solution : We have,

Diameter = 2 × radius = 2 × 6.25 cm = 12.50 cm

Example III : Find the circumference (approx.) of the circle whose diameter is 12 cm.

Solution : We have,

Circumference = 3 × diameter (approx.)

= 3 × 12 cm = 36 cm (approx.)

Exercise 8.3

1. Find the lengths of the diameters of circles whose radii are.
   a. 4.5 cm
   b. 3.5 cm
   c. 2.25 cm

2. Find the radius of circles whose diameters are.
   a. 26 cm
   b. 14 cm
   c. 9 cm

3. Find the circumferences of the circles whose diameters are.
   a. 7 cm
   b. 9 cm
   c. 14 cm

4. Find the circumferences of the circles whose radii are.
   a. 13 cm
   b. 6.5 cm
   c. 9 cm

Points to Remember

❖ The circle lies in its interior the centre.
❖ Circumference = 3 x diameter (approx).
❖ An angle measuring more than 0° but less than 90° is called and acute angle.
❖ An angle measuring 90° is called a right angle.
❖ An angle measuring more than 90° but less than 180° is called an obtuse angle.
❖ Diameter is a longest chord of the circle.
❖ Distance around the circle is called the circumference.
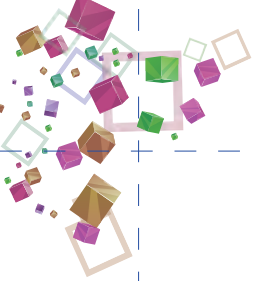
1. **Multiple Choice Questions (MCQs)**
   Tick (✓) the correct option:

   a. A ray has .... end point.
      (i) one ☐ (ii) two ☐ (iii) three ☐ (iv) four ☐

   b. A line has .... end point.
      (i) one ☐ (ii) two ☐ (iii) three ☐ (iv) no ☐

   c. An angle of 90° is called .... angle.
      (i) acute ☐ (ii) right ☐ (iii) obtuse ☐ (iv) none of these ☐

   d. An angle whose measure is 45° is called .... angle.
      (i) acute ☐ (ii) right ☐ (iii) obtuse ☐ (iv) none of these ☐

   e. Triangle has .... vertices.
      (i) one ☐ (ii) two ☐ (iii) three ☐ (iv) four ☐

   f. A triangle whose all three sides are equal is called .... triangle.
      (i) isosceles ☐ (ii) scalene ☐
      (iii) equilateral ☐ (iv) none of these ☐

   g. The circumference of a circle is equal to ....
      (i) πd ☐ (ii) $2\pi r$ ☐ (iii) $3 \times d$ ☐ (iv) all of these ☐

2. Write the complementary angles of the following.
   a. 45°   b. 30°   c. 60°   d. 80°

3. Write the supplementary angles of the following.
   a. 90°   b. 120°   c. 150°   d. 30°

4. From which of the following sides you can draw the triangle?
   a. 7, 8, 9   b. 4, 6, 8   c. 3, 4, 8   d. 4, 6, 12

5. From which of the following angles you can draw the triangle?
   a. 60°, 30°, 90°   b. 100°, 40°, 40°   c. 70°, 80°, 90°   d. 90°, 100°, 110°

6. Draw the circles of the following radius.
   a. 4 cm   b. 6 cm   c. 8 cm   d. 10 cm
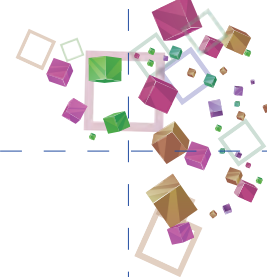
7. Find circumferences whose radii are...

   a. 14 cm   b. 7 cm   c. 24 cm   d. 12 cm

8. In a right-angled triangle, one angle is 60°, what are the other two angles?

9. If diameter of a circle is 8 cm, what is its circumference?

HOTS

How many intersecting lines can be drawn from a single point and from a double point?

Lab Activity

Objective : To discover one-sided shapes.

Materials Required : 3 long strips of paper of any length but of 6 cm width, a pair of scissors, pencil, adhesive like fevicol and cello tape

Activities :

❖ Take one strip of paper and paste the ends together. Draw a line down the middle. Cut along the line and discover what happens.

❖ Now take another strip, twist it once and paste the ends together. Draw a line down the middle and cut. Discover what happens.

❖ Take the third strip. Make two twists and tape the ends. Draw a line down the middle and cut. What happens ? Would you like to try a strip with three twists ?

These one-sided shapes are known as *Mobius Strips* !

REVISION TEST PAPER-II

(Based on Chapters 5 to 8)

A. **Multiple Choice Questions (MCQs)**

Tick ($\checkmark$) the correct option:

1. The equivalent fraction of $\frac{7}{9}$ is ....
   (a) $\frac{14}{18}$ ☐   (b) $\frac{3}{6}$ ☐   (c) $\frac{2}{3}$ ☐ (d) none of these ☐

2. The fractions having the same denominators are called ....
   (a) like fractions   (b)   unit fractions
   (c) proper fractions   (d)   mixed fraction

3. Rays have common end point from ....
   (a) line ☐   (b) vector ☐   (c) angle ☐ (d) none of these ☐

4. Decimal showing the same number of decimal place are called ....
   (a) unlike decimal   (b)   like decimals
   (c) mixed   (d)   none of these

5. $129.4 \times 1000 =$ ....
   (a) 12940 ☐   (b) 129400 ☐   (c) 12.940 ☐ (d) None of these ☐

6. Vertically opposite angles are ....
   (a) supplementary   (b)   unequal
   (c) ruler   (d)   none of these

7. Triangle is a figure formed by ....
   (a) 2 lines ☐   (b) 3 lines ☐   (c) 4 lines ☐ d) none of these ☐

8. Scalene triangle has all the sides ....
   (a) equal   (b)   unequal
   (c) proportional   (d)   none of these

9. What is 125 % of 160?
   (a) 50 ☐   (b) 100 ☐   (c) 200 ☐ (d) none of these ☐

10. To change per cent into decimal we .... by 100.
    (a) add ☐   (b) multiply ☐   (c) divide ☐ (d) none of these ☐

B. **Match the columns:**

Column A		Column B
1. The reciprocal of $\frac{7}{8}$	I.	$22\frac{31}{100}$
2. 22.31	II.	one end point
3. Per cent	III.	$\frac{8}{7}$
4. A ray has	IV.	three vertices
5. A Triangle has	V.	Out of hundred

(Based on Chapters 1 to 8)

A. Fill in the blanks.

1. The first five places in the Indian and International systems of numeration are ....

2. 1 is neither prime nor ....

3. The smallest factor of a number is ....

4. The product of 0 and a fractional number is ....

5. The fractional and decimal form of one hundredth is $\frac{1}{100}$ and .... respectively.

6. The symbol for percent is ....

7. The sum of 4.5321 + 2.823 = ...

8. A circle of zero radius is known as ....

9. A percentage is a way of expressing a number as a fraction of ....

10. A closed figure bounded by three line segments is called a ....

B. Tick (✓) the true and cross (✗) the false statements:

1. All the odd numbers are prime. ☐

2. V and L can not be subracted in Roman Numerals. ☐

3. There is no even prime number. ☐

4. Like fractions have same denominator. ☐

5. To change the percentage into fraction, we divide by 100. ☐

6. Measurement of a complete angle is 180°. ☐

7. Diameter is a longest chord of the circle. ☐
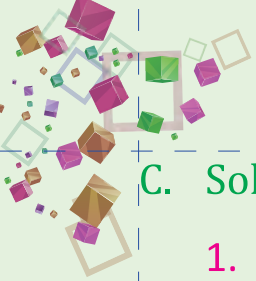
C. Solve.

1. Find the seventh multiple of 19.

2. Write the short form of $50 + 4 + \dfrac{3}{10} + \dfrac{1}{100} + \dfrac{5}{1000}$.

3. Find the circumference of the following radii :

   (a) 8 cm   (b) 9 cm   (c) 11 cm

4. A right angled triangle has an angle of 30°. What will be its other two angles?

D. Solve the following.

1. A factory produces 12340 chairs a day. If the factory has 328 working days in a year then how many chairs will be produced by the factory in a year?

2. The product of the HCF and LCM of the two numbers is 1280. If one number is 32, find the other number.

3. Abhishek gives $\dfrac{2}{5}$ of his income to his wife. If his total income is ₹ 20000. How much does he give to his wife?

4. In a college library there were 10000 books. Out of these books 1000 books were discarded. What percentage of books was discarded?

5. The measures of two angles of a triangle are 70° and 80°. What is the measure of third angle of the triangle?