5 Fractions

A fraction is a part of a whole. If a whole thing is divided into equal parts, each part is a fraction. It is made up of two parts.

$\dfrac{1}{2}$ ← Numerator ← Denominator

"A fraction represents an equal part of a whole".

❖ A fraction can also represent part of a set.

For Example : $\dfrac{5}{8}$

❖ A proper fraction has a numerator less than the denominator.

For Example : $\dfrac{2}{3}$ → Numerator → Denominator

❖ An improper fraction has a numerator greater than the denominator.

For Example : $\dfrac{5}{4}$ → Numerator → Denominator

❖ Like fractions have the same denominators.

For Example : $\dfrac{2}{5}$, $\dfrac{1}{5}$ and $\dfrac{3}{5}$ are like fractions.

❖ Unlike fractions have different denominators.

For Example : $\dfrac{4}{7}$, $\dfrac{2}{5}$ and $\dfrac{1}{3}$ are unlike fractions.

❖ Unit fractions have only the digit 1 as the numerator.

For Example : $\dfrac{1}{3}$, $\dfrac{1}{4}$, $\dfrac{1}{7}$ and $\dfrac{1}{8}$ are unit fractions.

❖ Equivalent fractions have the same value even though the numerators and denominators are different.

For Example : $\dfrac{1}{2}$ and $\dfrac{4}{8}$ are equivalent fractions.

❖ A mixed fraction is a combination of a whole number and a proper fraction.

For Example : 2 and $\dfrac{2}{5}$ is $2\dfrac{2}{5}$; is a mixed fraction.
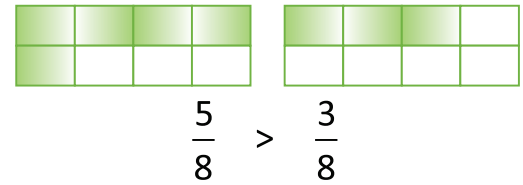
Comparing and Ordering Fractions

Comparing Like Fractions

It is easy to compare, when the denominators are same. We compare the numerators easily.

Example I : Compare the fraction $\dfrac{5}{8}$ and $\dfrac{3}{8}$.

Solution : Here, the denominators are same.

Numerator $5 > 3$, so $\dfrac{5}{8} > \dfrac{3}{8}$.

$$\dfrac{5}{8} \;>\; \dfrac{3}{8}$$

Comparing Unlike Fractions

When the denominators are different, we compare the fractions in two ways.
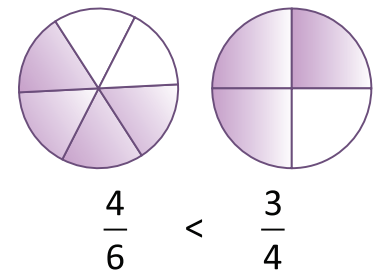
1. We can compare by cross multiplication. The side which has the bigger product has the bigger fraction.

Example II : Compare the fractions $\dfrac{4}{6}$ and $\dfrac{3}{4}$.

Solution : We cross multiply the two fractions,

Cross product

$$\dfrac{4}{6} \diagdown\!\!\!\!\diagup \dfrac{3}{4} \qquad 6 \times 3 = 18$$
$$4 \times 4 = 16$$

Since, $16 < 18$, so $\dfrac{4}{6} < \dfrac{3}{4}$.

$$\dfrac{4}{6} \;<\; \dfrac{3}{4}$$

2. We can compare unlike fractions by converting them into like fractions.

Example III : Compare the fractions $\dfrac{5}{8}$ and $\dfrac{2}{4}$.

Solution : Converting given fractions into like fractions, we get

$$\dfrac{5}{8} = \dfrac{5}{8} \times \dfrac{1}{1} = \dfrac{5}{8} \quad \text{and} \quad \dfrac{2}{4} = \dfrac{2}{4} \times \dfrac{2}{2} = \dfrac{4}{8} \quad \text{(LCM of 8 and 4 is 8).}$$

On comparing the numerators, we find $5 > 4$,

$$\therefore \quad \dfrac{5}{8} > \dfrac{4}{8} \quad \text{or} \quad \dfrac{5}{8} > \dfrac{2}{4}.$$

1. **Find the least common denominator for each part by multiple method.**

   a. $\dfrac{2}{3} \times \dfrac{3}{5}$

   b. $\dfrac{1}{2} \times \dfrac{7}{10}$

   c. $\dfrac{3}{4} \times \dfrac{4}{5}$

   d. $\dfrac{4}{5} \times \dfrac{6}{10}$

2. **Find the least common denominator for each part by division method.**

   a. $\dfrac{1}{2}, \dfrac{2}{3}, \dfrac{3}{4}$

   b. $\dfrac{3}{4}, \dfrac{5}{6}, \dfrac{1}{2}$

   c. $\dfrac{5}{6}, \dfrac{7}{8}, \dfrac{9}{10}$

   d. $\dfrac{5}{10}, \dfrac{11}{15}, \dfrac{13}{20}$

3. **Compare by finding the LCM of the denominators.**

   a. $\dfrac{2}{3} \square \dfrac{4}{5}$

   b. $\dfrac{3}{8} \square \dfrac{3}{4}$

   c. $\dfrac{7}{8} \square \dfrac{3}{4}$

   d. $\dfrac{3}{5} \square \dfrac{7}{10}$

4. **Fill in the blanks for equivalent fractions.**

   a. $\dfrac{3}{5} = \dfrac{12}{\square}$

   b. $\dfrac{6}{7} = \dfrac{\square}{28}$

   c. $\dfrac{2}{9} = \dfrac{16}{\square}$

   d. $\dfrac{7}{10} = \dfrac{\square}{20}$

5. **Write the fractions in ascending order.**

   a. $\dfrac{1}{4}, \dfrac{2}{5}, \dfrac{3}{8}$

   b. $\dfrac{1}{3}, \dfrac{2}{9}, \dfrac{1}{5}$

   c. $\dfrac{3}{4}, \dfrac{2}{9}, \dfrac{3}{5}$

   d. $\dfrac{4}{5}, \dfrac{9}{10}, \dfrac{7}{15}$

6. **Write the fractions in descending order.**

   a. $\dfrac{1}{2}, \dfrac{2}{3}, \dfrac{3}{4}$

   b. $\dfrac{2}{3}, \dfrac{4}{5}, \dfrac{5}{6}$

   c. $\dfrac{2}{3}, \dfrac{3}{5}, \dfrac{3}{4}$

   d. $\dfrac{2}{3}, \dfrac{1}{5}, \dfrac{5}{6}$

Addition and Subtraction of Fractions

Addition and Subtraction of Unlike Fractions

We know that, how to add and subtract like fractions (with a common denominator).

To add or subtract unlike fractions, first we convert the fractions into equivalent fractions with a common denominator.

Facts to Know

○ To find equivalent fractions with a common denominator, we first find the LCM of the denominators.

○ Fractions must be simplified wherever possible.

Example IV : Add $\dfrac{2}{9}$ and $\dfrac{5}{6}$.

Solution :

Step 1 : Find the LCM of the denominators.

$$\begin{array}{c|cc} 3 & 9, & 6 \\ \hline 3 & 3, & 2 \\ \hline 2 & 1, & 2 \\ \hline & 1, & 1 \end{array}$$

LCM $= 3 \times 3 \times 2 = 18$, LCM of 9 and 6 is 18.

Step 2 : Write equivalent fractions with LCM as denominator.

$$\dfrac{2}{9} \times \dfrac{2}{2} = \dfrac{4}{18} \quad \text{and} \quad \dfrac{5}{6} \times \dfrac{3}{3} = \dfrac{15}{18}$$

Step 3 : Add numerators and change to mixed number.

$$\therefore \dfrac{4}{18} + \dfrac{15}{18} = \dfrac{19}{18} = 1\dfrac{1}{18}$$

Example V : Subtract $\dfrac{1}{4}$ from $\dfrac{7}{8}$.

Solution :

Step 1 : Find the LCM of the denominators.

LCM of 4 and 8 is 8.

Step 2 : Write equivalent fractions with the LCM as denominator.

$$\dfrac{7}{8} \times \dfrac{1}{1} = \dfrac{7}{8} \quad \text{and} \quad \dfrac{1}{4} \times \dfrac{2}{2} = \dfrac{2}{8}$$

Step 3 : Subtract numerators.

$$\dfrac{7}{8} - \dfrac{2}{8} = \dfrac{5}{8}$$

Addition and Subtraction of Mixed Numbers

The steps are the same whether we are adding or subtracting mixed numbers.

Example VI : Add $2\dfrac{3}{2}$ and $1\dfrac{5}{8}$.

Solution :

Method 1 :

Step 1 : Add the whole numbers.

$2 + 1 = 3$

Step 2 : Find the LCM of the denominators.

LCM of 2 and 8 is 8.

A Gateway to Mathematics-5

Step 3 : Find equivalent fractions with LCM as denominator.

$$\frac{3}{2} \times \frac{4}{4} = \frac{12}{8} \text{ and } \frac{5}{8} \times \frac{1}{1} = \frac{5}{8}$$

Step 4 : Add the numerators.

$$\frac{12}{8} + \frac{5}{8} = \frac{17}{8}$$

Step 5 : Simplify.

$$\frac{17}{8} = 2\frac{1}{8}$$

Step 6 : Add the whole number and mixed number.

$$3 + 2\frac{1}{8} = 5\frac{1}{8}$$

Method 2 :

Step 1 : Change to improper fractions.

$$2\frac{3}{2} = \frac{7}{2}, \qquad 1\frac{5}{8} = \frac{13}{8}$$

Step 2 : Find the LCM of the denominator.

LCM of 2 and 8 is 8.

Step 3 : Find equivalent fractions with LCM as denominator.

$$\frac{7}{2} \times \frac{4}{4} = \frac{28}{8} \text{ and } \frac{13}{8} \times \frac{1}{1} = \frac{13}{8}$$

Step 4 : Add the numerators.

$$\frac{28}{8} + \frac{13}{8} = \frac{41}{8}$$

Step 5 : Simplify $\frac{41}{8} = 5\frac{1}{8}$.

Example VII : Subtract $2\frac{3}{4}$ from $4\frac{1}{6}$.

Solution :

Step 1 : Change to improper fractions.

$$4\frac{1}{6} - 2\frac{3}{4} = \frac{25}{6} - \frac{11}{4}$$

Step 2 : Find the LCM of the denominators.

LCM of 6 and 4 is 12.

Facts to Know

- To add mixed numbers change the fractions to improper fractions.
- If the sum of fractions is an improper fraction, change the improper fraction to a mixed number.

Step 3 : Find equivalent fractions.

$$\frac{25}{6} \times \frac{2}{2} = \frac{50}{12} \quad \text{and} \quad \frac{11}{4} \times \frac{3}{3} = \frac{33}{12}$$

Step 4 : Subtract the numerators.

$$\frac{50}{12} - \frac{33}{12} = \frac{17}{12}$$

Step 5 : Simplify as a mixed number.

$$\frac{17}{12} = 1\frac{5}{12}$$

Properties for addition and Subtraction of Fractions

1. **Zero property** : The sum and difference of zero and a fraction is the number itself.
   For Example : $\frac{4}{7} + 0 = \frac{4}{7}, \frac{4}{7} - 0 = \frac{4}{7}$

2. **Commutative property** : The sum stays the same when the order of addends is changed in addition, but not in subtraction.

   For Example : $\frac{3}{5} + \frac{5}{9} = \frac{5}{9} + \frac{3}{5}$ but $\frac{3}{5} - \frac{5}{9} \neq \frac{5}{9} - \frac{3}{5}$

3. **Associative property** : The sum stays the same when the grouping of addends is changed, but not in subtraction.

$$\left(\frac{5}{8} + \frac{3}{5}\right) + \frac{2}{3} = \frac{5}{8} + \left(\frac{3}{5} + \frac{2}{3}\right)$$

Exercise 5.2

1. **Solve and write the answer in the simplest form.**

   a. $\frac{5}{6} + \frac{2}{6}$ b. $\frac{3}{4} + \frac{1}{4}$ c. $\frac{5}{6} - \frac{2}{6}$ d. $\frac{8}{6} - \frac{7}{6}$

2. **Add by finding the common denominator. Simplify if needed.**

   a. $\frac{2}{3} + \frac{3}{4}$ b. $\frac{7}{8} + \frac{1}{2}$ c. $\frac{3}{5} + \frac{5}{6}$ d. $\frac{4}{5} + \frac{7}{10}$

3. **Subtract by finding the common denominator. Simplify if needed.**

   a. $\frac{7}{8} - \frac{3}{4}$ b. $\frac{3}{4} - \frac{1}{2}$ c. $\frac{5}{6} - \frac{1}{2}$ d. $\frac{9}{10} - \frac{2}{5}$

4. Find the sum or the difference. Simplify if needed.

a. $\dfrac{8}{9} + 2\dfrac{1}{3}$　　　b. $2\dfrac{1}{4} + 3\dfrac{1}{2}$　　c. $4\dfrac{3}{5} - 2\dfrac{7}{10}$　　d. $8\dfrac{3}{5} - 6\dfrac{7}{10}$

5. Solve and simplify the following.

a. $2\dfrac{3}{4} + 4\dfrac{1}{2} - 3\dfrac{5}{6}$　　　　　　　　b. $1\dfrac{7}{8} + 2\dfrac{11}{16} - 1\dfrac{3}{4}$

c. $3\dfrac{4}{5} + 2\dfrac{7}{10} - 4\dfrac{1}{2}$　　　　　　　　d. $5\dfrac{1}{2} + 2\dfrac{2}{3} - 4\dfrac{5}{6}$

Multiplication of a Whole Number by a Fraction

Let us learn the multiplication of fractional numbers.

Let us suppose 6 is multiplied by $\dfrac{1}{3}$.

☐ represent 1　and　☐☐☐☐☐☐ represent 6.

$\dfrac{1}{3}$ of 6 means 2.

☐☐☐☐☐☐

or $\dfrac{1}{3}$ of 6 $= \dfrac{1}{3} + \dfrac{1}{3} + \dfrac{1}{3} + \dfrac{1}{3} + \dfrac{1}{3} + \dfrac{1}{3} = \dfrac{1+1+1+1+1+1}{3} = \dfrac{6}{3} = 2$

☐☐☐☐☐☐　$\dfrac{1}{3}$ of 6 $= \dfrac{1}{3} \times 6 = 2$, the shaded portion. Therefore, $\dfrac{1}{3} \times 6 = 2$.

Fraction × Whole Number =

$$\dfrac{\text{Numerator}}{\text{Denominator}} \times \dfrac{\text{Whole Number}}{1} = \dfrac{\text{Numerator} \times \text{Whole Number}}{\text{Denominator}}$$

Example VIII :　Find the product :

a. $\dfrac{2}{3} \times 4$　　　　　b. $\dfrac{2}{9} \times 3$　　　　　c. $1\dfrac{2}{9} \times 5$

Solution :

a. $\dfrac{2}{3} \times 4 = \dfrac{2 \times 4}{3} = \dfrac{8}{3} = 2\dfrac{2}{3}$

b. $\dfrac{2}{9} \times 3 = \dfrac{2 \times 3}{9} = \dfrac{\cancel{6}^2}{\cancel{9}^3} = \dfrac{2}{3}$ (reducing to the lowest form)

Another method : $\dfrac{2}{9} \times 3 = \dfrac{2 \times \cancel{3}^1}{\cancel{9}^3} = \dfrac{2 \times 1}{3} = \dfrac{2}{3}$

c. $1\dfrac{2}{9} \times 5 = \dfrac{11}{9} \times 5 = \dfrac{11 \times 5}{9} = \dfrac{55}{9} = 6\dfrac{1}{9}$

Another method : $1\dfrac{2}{9} \times 5 = (1 + \dfrac{2}{9}) \times 5$

$$= 1 \times 5 + \dfrac{2}{9} \times 5 = 5 + \dfrac{10}{9}$$

$$= \dfrac{5 \times 9}{9} + \dfrac{10}{9} = \dfrac{45 + 10}{9}$$

$$= \dfrac{55}{9} = 6\dfrac{1}{9}$$

Exercise 5.3

1. **Find the product of the following.**

a. $\dfrac{5}{6} \times 11$ b. $\dfrac{4}{9} \times 6$ c. $\dfrac{3}{11} \times 12$ d. $\dfrac{4}{6} \times 2$

2. **Multiply the following.**

a. $\dfrac{17}{19}$ by 38   b. $\dfrac{5}{13}$ by 39   c. $\dfrac{15}{18}$ by 54   d. $\dfrac{13}{15}$ by 60

3. **Multiply the following.**

a. $17\dfrac{1}{5}$ by 5   b. $4\dfrac{1}{2}$ by 16   c. $8\dfrac{3}{15} \times 30$ d. $4\dfrac{5}{12}$ by 24

4. **Fill in the blanks.**

a. $\dfrac{5}{8} \times 4 = \square$ b. $\dfrac{3}{5} \times \square = \dfrac{12}{5}$ c. $\dfrac{7}{9} \times \square = \dfrac{35}{9}$ d. $\dfrac{9}{11} \times 2 = \dfrac{\square}{11}$

Multiplication of a Fractional Number by a Fractional Number

$\frac{1}{2}$ is multiplied by $\frac{1}{3}$ $\Rightarrow$ $\frac{1}{3}$ of $\frac{1}{2}$.

Suppose this is whole (1).

The whole figure has been divided into two halves ($\frac{1}{2}$).

For showing $\frac{1}{3}$ of $\frac{1}{2}$, it is further sub divided half of the figure into 3 equal parts.

Whole figure is divided into 6 equal parts.

Here, the double shaded portion is $\frac{1}{3}$ of $\frac{1}{2}$ part.

Now, $\frac{1}{3}$ of $\frac{1}{2}$ is $\frac{1}{6}$ of the whole figure.

Therefore, $\frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$.

or $\frac{1}{3} \times \frac{1}{2} = \frac{1 \times 1}{2 \times 3} = \frac{1}{6}$

Example IX : Multiply $\frac{1}{5}$ by $\frac{1}{6}$.

Solution : $\frac{1}{5} \times \frac{1}{6} = \frac{1 \times 1}{5 \times 6} = \frac{1}{30}$

Therefore, multiplication of $\frac{1}{5}$ by $\frac{1}{6}$ is $\frac{1}{30}$.

Exercise 5.4

1. **Find the following product.**

a. $7\frac{1}{4} \times \frac{4}{5}$

b. $\frac{2}{9} \times \frac{3}{7}$

c. $6\frac{2}{3} \times 1\frac{1}{3}$

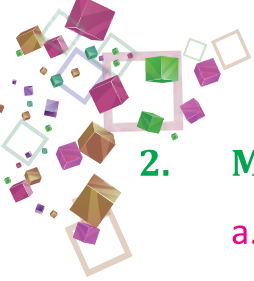d. $\frac{4}{3} \times \frac{3}{12}$

e. $4\frac{4}{6} \times \frac{2}{8}$

f. $4\frac{2}{5} \times \frac{3}{4}$

g. $\frac{3}{10} \times \frac{15}{21}$

h. $\frac{4}{5} \times \frac{25}{28}$

2. **Multiply the following.**

a. $2\dfrac{1}{3}$ x $3\dfrac{1}{2}$ x $1\dfrac{1}{7}$ b. $\dfrac{1}{2}\times\dfrac{5}{7}\times\dfrac{4}{5}$ c. $\dfrac{2}{3}\times\dfrac{12}{3}\times\dfrac{1}{6}$ d. $\dfrac{7}{6}\times\dfrac{6}{14}\times\dfrac{24}{49}$

e. $\dfrac{2}{5}$ x $\dfrac{3}{7}$ x $\dfrac{7}{9}$ f. $\dfrac{4}{5}\times\dfrac{6}{10}\times\dfrac{3}{4}$ g. $\dfrac{3}{7}\times\dfrac{5}{9}\times\dfrac{14}{15}$ h. $\dfrac{2}{5}\times\dfrac{1}{3}\times\dfrac{10}{16}$

3. **Find the following product.**

a. $12\times\dfrac{3}{16}$ b. $21\times\dfrac{5}{14}$ c. $60\times\dfrac{3}{10}$ d. $128\times\dfrac{5}{6}$

e. $126\times2\dfrac{2}{4}$ f. $32\times3\dfrac{1}{8}$ g. $50\times9\dfrac{1}{2}$ h. $140\times\dfrac{3}{8}$

Properties of Multiplication of Fractional Numbers

Property 1 : The product of two fractional numbers remains the same even if their order of multiplication is changed.

For Example :
$$18\times\dfrac{5}{12}=\dfrac{\cancel{18}^{3}\times5}{\cancel{12}^{2}}=\dfrac{15}{2}=7\dfrac{1}{2}$$

$$\text{and }\dfrac{5}{12}\times18=\dfrac{5\times\cancel{18}^{3}}{\cancel{12}^{2}}=\dfrac{15}{2}=7\dfrac{1}{2}$$

Therefore, $18\times\dfrac{5}{12}=\dfrac{5}{12}\times18.$

Property 2 : The product of three fractional numbers remains the same even if their grouping of multiplication is changed.

For Example :
$$\dfrac{1}{3}\times\dfrac{2}{5}\times\dfrac{3}{4}=\dfrac{1}{3}\times\left(\dfrac{\cancel{2}^{1}}{5}\times\dfrac{3}{\cancel{4}^{2}}\right)$$

$$=\dfrac{1}{3}\times\dfrac{3}{10}=\dfrac{1}{10}$$

$$\text{and }\dfrac{1}{3}\times\dfrac{2}{5}\times\dfrac{3}{4}=\left(\dfrac{1}{3}\times\dfrac{2}{5}\right)\times\dfrac{3}{4}$$

$$=\dfrac{\cancel{2}^{1}}{\cancel{15}^{5}}\times\dfrac{\cancel{3}^{1}}{\cancel{4}^{2}}=\dfrac{1}{10}$$

Therefore, $\dfrac{1}{3}\times\left(\dfrac{2}{5}\times\dfrac{3}{4}\right)=\left(\dfrac{1}{3}\times\dfrac{2}{5}\right)\times\dfrac{3}{4}$.

Property 3 : The product of 1 and a fractional number is the fractional number itself.

For Example :

a. $\dfrac{4}{5} \times 1 = \dfrac{4 \times 1}{5} = \dfrac{4}{5}$

b. $4\dfrac{6}{5} \times 1 = \dfrac{26}{5} \times 1 = \dfrac{26 \times 1}{5} = \dfrac{26}{5} = 4\dfrac{6}{5}$

Property 4 : The product of '0' and a fractional number is always zero.

For Example :

a. $\dfrac{4}{9} \times 0 = \dfrac{4 \times 0}{9} = \dfrac{0}{9} = 0$

b. $3\dfrac{5}{7} \times 0 = \dfrac{26}{7} \times 0 = \dfrac{26 \times 0}{7} = \dfrac{0}{7} = 0$

Multiplicative Inverse of a Fractional Number

If the product of two numbers is 1, then each number is known as multiplication inverse or the reciprocal of one another.

For Example : $3 \times \dfrac{1}{3} = \dfrac{3 \times 1}{1 \times 3} = 1$

Therefore, the multiplicative inverse of 3 is $\dfrac{1}{3}$

and the multiplicative inverse of $\dfrac{1}{3}$ is 3.

Property 1 : If the product of a fractional number and a whole number is 1, then each of the factors is the multiplicative inverse of the other.

In $\dfrac{1}{5} \times 5 = 1$, 5 is the multiplicative inverse of $\dfrac{1}{5}$ and $\dfrac{1}{5}$ is the multiplicative inverse of 5.

Similarly,

$\dfrac{1}{7}$ is the multiplicative inverse of 7 and 7 is the multiplicative inverse of $\dfrac{1}{7}$.

$\dfrac{1}{11}$ is the multiplicative inverse of 11 and 11 is the multiplicative inverse of $\dfrac{1}{11}$.

Property 2 : If the product of two fractional numbers is 1, then each of the fractional numbers is the multiplicative inverse of the other.

In $\dfrac{2}{3} \times \dfrac{3}{2} = 1$, then $\dfrac{2}{3}$ is the multiplicative inverse of $\dfrac{3}{2}$ and $\dfrac{3}{2}$ is the multiplicative inverse of $\dfrac{2}{3}$.

Property 3 : The multiplicative inverse of 1 is 1 itself because $1 \times 1 = 1$, then 1 is the only number whose multiplicative inverse is the number itself.

Property 4 : '0' multiplied by any number is 0 and never equal to 1. Therefore, the multiplicative inverse of 0 does not exit.

Property 5 : The multiplicative inverse of a number greater than 1 is always less than 1.

For Example :

Number	Multiplicative inverse
5	$\frac{1}{5}$, Since $5 \times \frac{1}{5} = 1$
8	$\frac{1}{8}$, Since $8 \times \frac{1}{8} = 1$

Property 6 : The multiplicative inverse of a number less than 1 is always greater than 1.

For Example :

Number	Multiplicative inverse
$\frac{1}{4}$	4, Since $\frac{1}{4} \times 4 = 1$
$\frac{1}{6}$	6, Since $\frac{1}{6} \times 6 = 1$

Example X : What is the multiplicative inverse of the following mixed number.

a. $7\frac{1}{3}$　　b. $5\frac{1}{4}$

Solution : a. $7\frac{1}{3} = \frac{22}{3}$

Since, $\frac{22}{3} \times \frac{3}{22} = 1$, therefore, $\frac{3}{22}$ is the multiplicative inverse of $7\frac{1}{3}$.

b. $5\frac{1}{4} = \frac{21}{4}$

Since, $\frac{21}{4} \times \frac{4}{21} = 1$, therefore, $\frac{4}{21}$ is the multiplicative inverse of $5\frac{1}{4}$.

1. **Fill in the blanks.**

a. $\dfrac{5}{7} \times 4 = 4 \times$ ....

b. $5 \times \dfrac{7}{8} =$ .... $\times$ 5

c. $3\dfrac{1}{4} \times$ .... $= 7 \times 3\dfrac{1}{4}$

d. $\dfrac{3}{5} \times \dfrac{4}{7} =$ .... $\times \dfrac{3}{5}$

2. **Fill in the blanks.**

a. $\left(\dfrac{1}{6} \times \dfrac{4}{5}\right) \times \dfrac{2}{7} =$ .... $\times \left(\dfrac{4}{5} \times \dfrac{2}{7}\right)$

b. $\dfrac{1}{4} \times \left(\dfrac{2}{5} \times \dfrac{5}{9}\right) = \left(\dfrac{1}{4} \times \dfrac{2}{5}\right) \times$ ......

c. ....$\times \left(3\dfrac{1}{3} \times 2\dfrac{1}{4}\right) = \left(8\dfrac{3}{4} \times 3\dfrac{1}{3}\right) \times 2\dfrac{1}{4}$

d. $\left(5\dfrac{4}{7} \times 2\dfrac{3}{4}\right) \times$ ...... $= 5\dfrac{4}{7} \times \left(2\dfrac{3}{4} \times 1\dfrac{1}{9}\right)$

3. **Fill in the blanks.**

a. $\dfrac{1}{12} \times 12 =$ ....

b. $\dfrac{3}{5} \times$ .... $= 1$

c. .... $\times \dfrac{5}{13} = 1$

d. $\dfrac{7}{19} \times$ .... $= 1$

e. $\dfrac{13}{17} \times \dfrac{17}{13} =$ ....

f. $5\dfrac{1}{3} \times \dfrac{3}{16} =$ ....

4. **Find the multiplicative inverse of the following.**

a. $\dfrac{1}{13}$

b. $\dfrac{9}{16}$

c. $\dfrac{1}{20}$

d. 17

e. $5\dfrac{3}{4}$

f. $10\dfrac{1}{3}$

Word Problems

Example XI : Express 20 cm as a fraction of a metre.

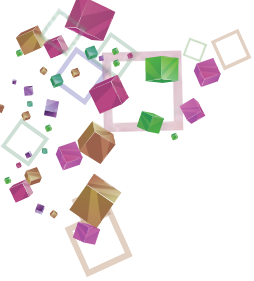**Solution** : Required fraction $= \dfrac{20\ cm}{1\ metre}$

or required fraction $= \dfrac{\cancel{20}^{1}\ cm}{\cancel{100}^{5}\ cm}$. Since, 1 m = 100 cm,

therefore, 20 cm is $\dfrac{1}{5}$ fraction of a metre.

Example XII : If I sleep 8 hours daily, then what part of a day I spend in sleeping?

Solution : Time spent in sleeping a day = 8 hours

Then the part of a day I spend in sleeping $= \dfrac{8 \text{ hours}}{1 \text{ day}}$

Therefore, required fraction $= \dfrac{\cancel{8}^{1}}{\cancel{24}^{3}} = \dfrac{1}{3}$, since 1 day = 24 hours.

Thus, I spend $\dfrac{1}{3}$ part of a day in sleeping.

Example XIII : One-half of the students in a school are boys. $\dfrac{5}{6}$ of these boys are studying in lower classes. What fraction of boys are studying in lower classes ?

Solution : Fraction of boys studying in school $= \dfrac{1}{2}$

Fraction of boys studying in lower classes $= \dfrac{5}{6}$ of $\dfrac{1}{2}$

$= \dfrac{5}{6} \times \dfrac{1}{2}$

$= \dfrac{5 \times 1}{6 \times 2} = \dfrac{5}{12}$

Thus, $\dfrac{5}{12}$ of boys are studying in lower classes.

Exercise 5.6

1. A book has 80 pages. Ponty read $\dfrac{1}{8}$ of it. How many pages Ponty has read?

2. At a party, Rani found that $\dfrac{4}{5}$ of a cake was left. She ate $\dfrac{1}{2}$ of it . What part of the cake did she eat ?

3. The cost of 1 kg of sweets is ₹ $50\dfrac{2}{3}$. What is the cost of $1\dfrac{1}{2}$ kg of sweets?

4. A can holds $8\dfrac{1}{4}$ litre of oil. How many litre of oil 12 such cans can hold?

5. A bag contains $96\dfrac{1}{3}$ kg of sugar. How much sugar do 9 such bags contain?

6. A Cobbler works $8\dfrac{1}{2}$ hours a day . How long does he work in a month ?

7. The cost of one kg of apples is ₹ $30\dfrac{1}{2}$. What is the cost of $2\dfrac{2}{3}$ kg of apples?

8. Rajesh gives $\dfrac{3}{5}$ of his income to his wife. If his total income is ₹ 5000, how much does he give to his wife?

A Gateway to Mathematics-5

Division of Fractional Number

Let us now consider that Shiva has a full chocolate bar.

Shiva cuts it into two equal parts and gives one part to Pavan. Pavan cuts his part of chocolate bar into four equal parts and gives one of the parts to his sister Richa. What fraction of the full chocolate bar does Richa get?

The full chocolate bar represents 1.

(full chocolate bar with Shiva)

The half part of chocolate bar represents $1 \div 2$ or $\frac{1}{2}$ of 1 or $1 \times \frac{1}{2} = \frac{1}{2}$.

One part of the chocolate bar represents $\frac{1}{2} \div 4$ or $\frac{1}{2}$ of $\frac{1}{4}$ or $\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$.

Thus, Richa gets $\frac{1}{8}$ of the full chocolate bar.

Example XIV : Find $8\frac{1}{2} \div 4$.

Solution : $8\frac{1}{2} \div 4 = \frac{17}{2} \div 4 = \frac{17}{2} \times \frac{1}{4}$

$$= \frac{17 \times 1}{2 \times 4} = \frac{17}{8} \text{ or } 2\frac{1}{8}$$

Division of a Whole Number by a Fractional Number

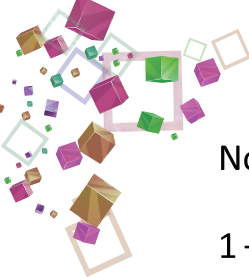There are 3 apples. If we cut each apple into 2 equal pieces, then each piece will be $\frac{1}{2}$ of an apple. We will get 6 pieces from 3 apples .

We can also solve it by dividing 3 by $\frac{1}{2}$.

$$3 \div \frac{1}{2} = 3 \times \frac{2}{1} = \frac{3 \times 2}{1} = 6$$

Therefore, 6 pieces, each equal to $\frac{1}{2}$ of an apple, can be obtained from 3 apples.

Now, study the following:

$$1 \div \frac{1}{5} = 5 \qquad\qquad 3 \div \frac{1}{2} = 6$$

$$1 \div \frac{1}{5} = 1 \times 5 \qquad\qquad 3 \div \frac{1}{2} = 3 \times 2$$

$$\downarrow \qquad\qquad\qquad\qquad \downarrow$$

Multiplicative inverse of $\frac{1}{5}$ Multiplicative inverse of $\frac{1}{2}$

Similarly,

$$6 \div \frac{1}{2} = 6 \times 2 = 12 \qquad\qquad 6 \div \frac{1}{5} = 6 \times 5 = 30$$

$$5 \div \frac{1}{3} = 5 \times 3 = 15 \qquad\qquad 7 \div \frac{1}{8} = 7 \times 8 = 56$$

Now, study some more examples:

(i) $4 \div \dfrac{2}{5} = 4 \times \dfrac{5}{2} = \dfrac{4 \times 5}{2} = \dfrac{\cancel{20}^{10}}{\cancel{2}^{1}} = 10$

$$\downarrow$$

= Multiplicative inverse of $\dfrac{2}{5}$

(ii) $6 \div \dfrac{6}{11} = 6 \times \dfrac{11}{6} = \dfrac{6 \times 11}{6} = \dfrac{\cancel{66}^{11}}{\cancel{6}^{1}} = 11$

$$\downarrow$$

= Multiplicative inverse of $\dfrac{6}{11}$

Therefore, to divide whole number by a fractional number, we multiply the whole number by the multiplicative inverse of the fractional number.

Example XV : Divide 15 by $\dfrac{5}{6}$.

Solution : $15 \div \dfrac{5}{6} = 15 \times \dfrac{6}{5} = \dfrac{\cancel{15}^{3} \times 6}{\cancel{5}^{1}} = \dfrac{18}{1} = 18$

Example XVI : Divide 35 by $3\dfrac{8}{9}$.

Solution : $35 \div 3\dfrac{8}{9} = 35 \div \dfrac{35}{9} = 35 \times \dfrac{9}{35} = \dfrac{\cancel{35}^{1} \times 9}{\cancel{35}^{1}} = 9$

Division of a Fractional Number by a Fractional Number

Now, consider the division $\frac{2}{3} \div \frac{1}{3}$. We know that division can be represented as repeated subtraction.

Thus, $\frac{2}{3} \div \frac{1}{3}$ can be written as $\frac{2}{3} - \frac{1}{3} = \frac{1}{3}$, $\frac{1}{3} - \frac{1}{3} = 0$. We have to subtract $\frac{1}{3}$ two times to get the remainder 0.

There are 2 one-thirds in $\frac{2}{3}$ or $\frac{2}{3} \div \frac{1}{3} = \frac{2}{\cancel{3}^1} \times \frac{\cancel{3}^1}{1} = \frac{2}{1} = 2$.

We can also divide $\frac{2}{3}$ by $\frac{1}{3}$ as shown below.

$$\frac{2}{3} \div \frac{1}{3} = \frac{\frac{2}{3}}{\frac{1}{3}} = \frac{\frac{2}{\cancel{3}} \times \cancel{3}}{\frac{1}{\cancel{3}} \times \cancel{3}} = \frac{2}{1} = 2$$

Therefore, $\frac{2}{3} \div \frac{1}{3} = \frac{2}{3} \times 3 = 2$.

Multiplicative inverse of $\frac{1}{3}$

Facts to know

○ Whole number ÷ fraction = whole number x reciprocal of fraction

Therefore, to divide a fractional number by another fractional number, we multiply the first fractional number by the multiplicative inverse of the second fractional number.

Example XVII : Divide $\frac{1}{8}$ by $\frac{5}{12}$.

Solution : $\frac{1}{8} \div \frac{5}{12} = \frac{1}{8} \times \frac{12}{5} = \frac{1 \times 12}{8 \times 5} = \frac{\cancel{12}^{3}}{\cancel{40}^{10}} = \frac{3}{10}$

Therefore, $\frac{1}{8} \div \frac{5}{12} = \frac{3}{10}$.

Example XVIII : Divide $\frac{3}{5}$ by $\frac{9}{10}$.

Solution : $\frac{3}{5} \div \frac{9}{10} = \frac{\cancel{3}^{1}}{\cancel{5}^{1}} \times \frac{\cancel{10}^{2}}{\cancel{9}^{3}} = \frac{1 \times 2}{1 \times 3} = \frac{2}{3}$
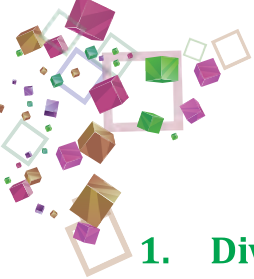
Therefore, $\frac{3}{5} \div \frac{9}{10} = \frac{2}{3}$.

1. **Divide the following.**

a. $\dfrac{3}{4}$ by 3

b. $\dfrac{7}{10}$ by 4

c. $\dfrac{12}{17}$ by 6

d. $\dfrac{7}{8}$ by 7

2. **Find the quotient.**

a. $5\dfrac{1}{3} \div 2$

b. $4\dfrac{3}{8} \div 7$

c. $12\dfrac{1}{2} \div 5$

d. $11\dfrac{1}{4} \div 3$

3. **Divide the following.**

a. $21 \div \dfrac{7}{11}$

b. $42 \div \dfrac{6}{7}$

c. $48 \div \dfrac{4}{9}$

d. $39 \div 6\dfrac{1}{2}$

e. $21 \div 5\dfrac{1}{4}$

f. $56 \div 3\dfrac{1}{2}$

4. **Find the quotient of the following.**

a. $\dfrac{2}{3} \div \dfrac{2}{3}$

b. $\dfrac{2}{5} \div \dfrac{7}{5}$

c. $\dfrac{4}{9} \div \dfrac{1}{12}$

d. $\dfrac{21}{25} \div \dfrac{7}{10}$

e. $8\dfrac{1}{2} \div \dfrac{7}{2}$

5. **Which of the following are true?**

a. $\dfrac{1}{2} \div \dfrac{1}{5} = \dfrac{1}{2} \times \dfrac{1}{5}$

b. $\dfrac{2}{5} \div 3 = \dfrac{2}{5} \times \dfrac{1}{3}$

c. $3\dfrac{1}{2} \div 3 = \dfrac{7}{2} \times 3$

d. $5\dfrac{1}{3} \div 5 = \dfrac{3}{16} \times 5$

e. $3\dfrac{1}{4} \div 6 = \dfrac{13}{7} \times \dfrac{1}{12}$

Properties of Division of Fractional Numbers :

Property 1 : If a fractional number is divided by 1, then the quotient is the fractional number itself.

For Example : $\dfrac{2}{5} \div 1 = \dfrac{2}{5} \times \dfrac{1}{1} = \dfrac{2 \times 1}{5 \times 1} = \dfrac{2}{5}$

Property 2 : If a non-zero fractional number is divided by itself, then the quotient is 1.

For Example : $\dfrac{4}{5} \div \dfrac{4}{5} = \dfrac{4}{5} \times \dfrac{5}{4} = \dfrac{20}{20} = 1$

Property 3 : If zero (0) is divided by non-zero fractional number, then the quotient is always zero.

For Example : $0 \div \dfrac{1}{3} = 0 \times 3 = 0$

Property 4 : The reciprocal of zero or multiplicative inverse does not exist. So, a fractional number can not be divided by zero (0).

Exercise 5.8

1. **Fill in the blanks.**

   a. $\dfrac{3}{4} \div 1 =$ ....

   b. $\dfrac{4}{5} \div \dfrac{4}{4} =$ ....

   c. .... $\div 3\dfrac{2}{5} = 0$

   d. $0 \div 8\dfrac{3}{5} =$ ....

   e. $7\dfrac{3}{4} \div$ .... $= 7\dfrac{3}{4}$

   f. $5\dfrac{1}{8} \div$ .... $= 1$

2. **Which of the following are true?**

   a. $0 \div 3\dfrac{1}{2} = 0$

   b. $4\dfrac{1}{2} \div 1 = 1$

   c. $5\dfrac{3}{7} \div 5\dfrac{3}{7} = 1$

   d. $1 \div 6\dfrac{1}{3} = 1$

   e. $\dfrac{2}{7} \div \dfrac{2}{7} = \dfrac{49}{4}$

   f. $3\dfrac{1}{5} \div 3\dfrac{1}{5} = 1$

Word Problems

Example XIX : Farukh sheikh has 8 kg of apples. He distributed these apples among his friends. If each friend got $\dfrac{1}{4}$ kg of apples, then how many friends Farukh sheikh has?
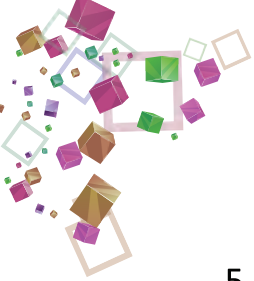
Solution : Quantity of apples Farukh sheikh has = 8 kg

Quantity of apples that each friend got = $\dfrac{1}{4}$ kg

Then, number of friends to get the apples = $8 \div \dfrac{1}{4}$

$$= \dfrac{8}{1} \times \dfrac{4}{1} = 32.$$

Therefore, number of friends is 32.

Example XX : How many quarters are there in $2\dfrac{3}{4}$?

Solution : One quarter = $\dfrac{1}{4}$

Number of quarters in $2\dfrac{3}{4} = 2\dfrac{3}{4} \div \dfrac{1}{4}$

$$\dfrac{11}{4} \div \dfrac{1}{4} = \dfrac{11}{4} \times \dfrac{4}{1} = 11.$$

Therefore, number of quarters is 11.

1. $15\dfrac{5}{2}$ litre of curd is poured equally into 25 peoples. How much curd will be there in each equally to peoples ?

2. The weight of 15 packets of sweets is $7\dfrac{1}{2}$ kg. Find the weight of each packet.

3. A man walks $5\dfrac{5}{8}$ km in $2\dfrac{1}{2}$ hours. What distance does he cover in 1 hour?

4. The product of two number is $5\dfrac{1}{4}$. If one number is $2\dfrac{1}{3}$, then find the other number.

5. Rita has $\dfrac{3}{4}$ kilogram of oranges. She wants to distribute them equally among 3 of her friends. How much of oranges will each of her friends get?

6. The weight of 8 bags together is $14\dfrac{8}{4}$ kg. Find the weight of each bags.

7. A drum contains $38\dfrac{1}{4}$ litre of oil. If oil is poured into small cans of capacity $4\dfrac{1}{4}$ litre each, then how many cans are required to hold entire oil of the drum?

Points to Remember

- ❖ Fraction having same value.
- ❖ When the numerator and the denominator are interchanged in a faction, the result is called multiplicate inverse / reciprocal of the fraction.
- ❖ A fraction represents an equal part of a whole.
- ❖ A proper fraction has a numerator less than the denominator.
- ❖ To add the mixed fractions we change the fraction to improper fraction.
- ❖ The product of 1 and a fractional number is the fractional number itself.
- ❖ The product of 0 and a fractional number is always 0.
- ❖ If the fractional number is divided by 1, then quotient is the fractional number itself.
- ❖ If the fractional number is divided by it self, then the quotient is 1.
- ❖ If zero (0) is divided by a non–zero fractional number, the quotient is always zero (0).

EXERCISE
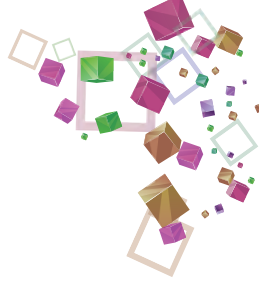
1. **Multiple Choice Questions (MCQs)**

   Tick ( ) the correct option:

   a. $5 \times \dfrac{3}{5} = \dfrac{3}{5} \times$ ....

   (i) 1 ☐   (ii) 3 ☐   (iii) 5 ☐   (iv) 8 ☐

   b. $\dfrac{2}{3} \times \left(\dfrac{3}{5} \times \dfrac{4}{5} \right) = \left(\dfrac{2}{3} \times \dfrac{3}{5} \right) \times$ ....

   (i) $\dfrac{4}{5}$ ☐   (ii) $\dfrac{6}{5}$ ☐   (iii) $\dfrac{6}{15}$ ☐   (iv) $\dfrac{12}{25}$ ☐

   c. $0 \times \dfrac{4}{5} =$ ....

   (i) 1 ☐   (ii) 0 ☐   (iii) $\dfrac{4}{5}$ ☐   (iv) $\dfrac{5}{4}$ ☐

   d. The reciprocal of $\dfrac{7}{8}$ is ....

   (i) $\dfrac{7}{8}$ ☐   (ii) 7 ☐   (iii) 8 ☐   (iv) $\dfrac{8}{7}$ ☐

   e. The multiplicative inverse of 11 is ....

   (i) 11 ☐   (ii) 1 ☐   (iii) 0 ☐   (iv) $\dfrac{1}{11}$ ☐

2. **Fill in the blanks with suitable symbols <,> or =.**

   a. $\dfrac{3}{5}$ ☐ $\dfrac{4}{5}$ b. $\dfrac{7}{9}$ ☐ $\dfrac{5}{8}$ c. $\dfrac{1}{5}$ ☐ $\dfrac{5}{25}$ d. $\dfrac{11}{12}$ ☐ $\dfrac{11}{13}$

3. **Find the product of the following.**

   a. $3 \times \dfrac{3}{5}$ b. $\dfrac{2}{5} \times \dfrac{5}{2}$ c. $\dfrac{5}{7} \times \dfrac{8}{9}$ d. $\dfrac{7}{5} \times 25$

4. **Find the product of the mixed fractions.**

   a. $2\dfrac{1}{5} \times 3\dfrac{2}{5}$ b. $5\dfrac{5}{11} \times 7\dfrac{2}{12}$ c. $1\dfrac{1}{8} \times \dfrac{8}{9}$ d. $4\dfrac{5}{7} \times 5\dfrac{5}{13}$

5. **Divide the following.**

   a. $\dfrac{7}{5} \div \dfrac{7}{5}$ b. $\dfrac{1}{8} \div 8$ c. $\dfrac{4}{10} \div \dfrac{8}{5}$ d. $1\dfrac{6}{9} \div 2\dfrac{12}{15}$

6. **Find the reciprocal or multiplicative inverse of the following.**

   a. 25   b. $\dfrac{1}{8}$ c. $\dfrac{15}{19}$ d. $2\dfrac{5}{7}$ e. $1\dfrac{8}{9}$

7. A man distributed his property among his three sons. He gave $\frac{2}{7}$ part to his first son and $\frac{3}{10}$ part to his second son. How much property did the third one get?

8. There are 840 students in a school. On a rainy day $\frac{1}{12}$ of the students were absent. How many students were present?

9. A rope of $16\frac{2}{3}$ m long is cut into 2 equal pieces. What is the length of each piece?

HOTS

Ravi told to Hari, "I have $\frac{1}{2}$ of $\frac{1}{2}$ of $\frac{1}{2}$ of $\frac{1}{2}$ toffees of your toffees". If Hari has 16 toffees then how much toffees does Ravi have?
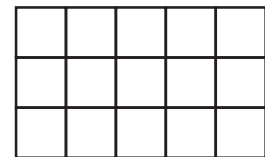
Lab Activity

Objective : To build an understanding of multiplication of fractions.

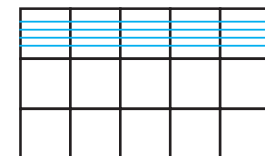**Materials Required** : Square-lined paper and crayon of two colours

Activities :

To solve $\frac{1}{3} \times \frac{2}{5}$

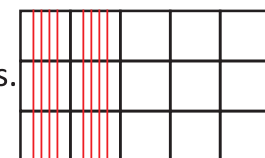❖ Since the two denominators are 3 and 5, the students draw a rectangle of sides by 3 x 5 square-lined paper.

Step 1

❖ Then one student colours $\frac{1}{3}$ of the rectangle with blue horizontal lines.

Step 2

❖ The second student colours $\frac{2}{5}$ of the rectangle with red vertical lines.

Step 3

What fraction of the sheet does have both vertical and horizontal lines?

This shows that

$$\frac{1}{3} \times \frac{2}{5} = \frac{2}{15}.$$