

# **Factors and Multiples**

Divisor divides dividend completely is a factor of number, hence A factor is a number which divides another number without leaving a remainder.

For Example: 
$$12 \div 3 = 4$$
;

$$12 \div 4 = 3$$

We can find all the factors of a number in two ways:

### By multiplication

$$1 \times 12 = 12$$

$$2 \times 6 = 12$$

$$3 \times 4 = 12$$

### By division b.

$$12 \div 1 = 12$$

$$12 \div 2 = 6$$

$$12 \div 3 = 4$$

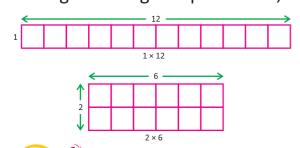
$$12 \div 4 = 3$$

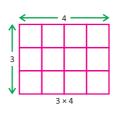
$$12 \div 6 = 2$$

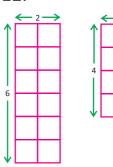
$$12 \div 12 = 1$$

1, 2, 3, 4, 6 and 12 are all factors of 12.

Rectangles can be used to find the factor pairs of a number. Let us draw all possible rectangles having 12 square units, to find the factors of 12.









### Facts to Know

- A number is divisible by another number, if it is also divisible by its co-prime factors. The co-prime factors of 15 are 3 and 5.
- O The two definite factors of a number are 1 and the number.

### **Properties of Factors**

- ❖ 1 is a factor of all number.
- Every non-zero number is a factor of itself.















- ❖ 1 is the smallest factor of a number.
- The largest factor of a number is the number itself.
- Every factor of a non-zero number is less than or equal to the number.



O The factor of a number is either less than or equal to the number.



## **Tests of Divisibility**

Divisibility tests help us to discover whether a number has a factor besides 1 and the number itself. Let us learn the divisibility rules of 2, 3, 5, 9, 10 and 11.

51 1 11 11 1	2.1	
Divisibility by	Rule	Examples
2	The digit in the ones place	Th H T O
	should be 2, 4, 6, 8 or 0.	6 5 3 <u>2</u>
		9 3 7 <u>4</u>
3	The sum of the digits of the	5943 = 5 + 9 + 4 + 3 = 21
	number should be divisible by 3.	4563 = 4 + 5 + 6 + 3 = 18
5	The digit in the ones place should	Th H T O
	be 5 or 0.	3 9 8 <u>5</u>
		4 3 5 <u>0</u>
9	The sum of the digit of the number	1359=1+3+5+9=18
	should be divisible by 9.	32688 = 3 + 2 + 6 + 8 + 8 = 27
10	The digit in the ones place	Th H T O
	should be 0.	7 6 3 <u>0</u>
		5 4 8 <u>0</u>
11	The difference of the sum of the	245762
	alternate digits of a number should	4+7+2=13
	be either 0 or a multiple of 11.	2+5+6=13
		13-13=0













- Which of the following numbers are divisible by 2? What kind of numbers are they?
  - a. 35169
- b. 59234
- c. 47893
- d. 72038 e. 69175 f. 70800

- Find the number divisible by 3 in the following. 2.
- b. 3852
- c. 72153
- d. 62961 e. 53281 f. 459231

- Is 9 a factor of the following numbers? 3.
  - a. 2043
- b. 7841
- c. 5683
- d. 9639
- e. 67008 f. 2331
- Check the divisibility by 2, 3, 5, 9, 10 and 11 of following numbers. Complete 4. the table by putting  $(\checkmark)$  for yes and (×) for no.

	Numbers	By 2	By 3	By 5	By 9	By 10	By 11
a.	1009						
b.	7094						
c.	8005						
d.	2657						
e.	8592						
f.	25,465						
g.	32,094						
h.	96,180						
I.	33,039						
j.	8,64,700						

# Prime and Composite Numbers

**Prime Number** 

Any number greater than 1, which has only two factors, 1

and the number itself, is called a prime number.

**For Examples** 

2, 3, 5, 7 etc., are prime numbers.

**Composite Numbers** 

A number which has more than two factors is called a

composite number.



For Examples: 4, 6, 8, 9, 10 etc., are composite numbers.

### Prime Numbers between 1 and 100

The steps to find prime numbers between 1 to 100 are given below.

To find out prime and composite numbers, we use the method derived an ancient Greek mathematician named Eratosthenes.

**Step 1** : Write numbers from 1 to 100.

**Step 2** : Make a crown on 1 because it is a unique number.

**Step 3** : Encircle 2 and cross all the numbers divisible by 2.

**Step 4** : Encircle 3 and cross all the numbers divisible by 3.

**Step 5** : Encircle 5 and cross all the numbers divisible by 5.

**Step 6** : Encircle 7 and cross all the numbers divisible by 7.

**Step 7** : Encircle 11 and cross all the numbers divisible by 11.

**Step 8** : Continue this process till all the numbers are either crossed-out or

encircled.

All the encircled numbers are prime numbers and the crossed-out numbers except 1 are composite. The prime numbers between 1 and 100 are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89 and 97.

						_			
1	2	3	*	5	<b>6</b>	7	<b>%</b>	>9<	10
11	22	13	14	25	16	17	18	19	20
21	22	23)	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43)	44	45	46	47)	48	49	50
34	52	53	54	35	56	37	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	X	78	79	80
28	82	83)	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100







Twin Primes: Two prime numbers with a composite number in between are called twin primes. For example: 3 and 5, 11 and 13 are twin primes.

## Prime Factorisation

A composite number can be shown as the product of prime factors. This is called prime factorisation.

There are two methods of prime factorisation.

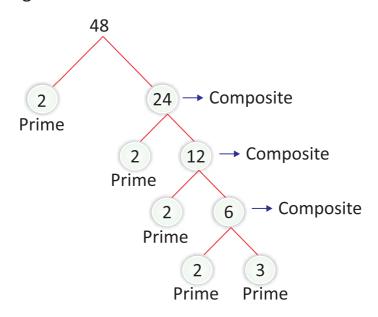
- a. Factor tree method
- b. Division method



- O Prime numbers have only 2 factors, 1 and the number itself.
- O Composite numbers have more than two factors.
- 1 is neither prime nor composite.
- The only even prime number is 2.

### **Factor Tree Method**

In this method, we factorise a composite number till we get all prime factors. Let us factorise 48 using the factor tree method.



The prime factorisation of 48 is  $2 \times 2 \times 2 \times 2 \times 3$ .

### **Division Method**

In this method, we start dividing the given number by the smallest prime number and continue division by prime numbers, till we reach 1.



Let us factorise 56 by the division method.

2	56
2	28
2	14
7	7
	1

(Divide by the smallest prime number.)

(Carry on dividing by prime numbers, till we reach 1.)

The prime factorisation of 56 is  $2 \times 2 \times 2 \times 7$ .



- 1. Write all twin primes between 1 and 100.
- 2. Pick out the composite numbers from the following numbers. 47, 49, 51, 53, 55, 57, 59, 61, 63, 65, 75, 79, 83, 86, 95
- 3. Pick out the prime numbers from the following numbers. 17, 8, 21, 13, 41, 27, 31, 51, 67, 72, 77, 83, 88, 91
- 4. Write True or False against the following statements.
  - a. Prime numbers have only 2 factors.
  - b. 1 is a prime number. .....
  - c. 2 is not a prime number.
  - d. There is no even prime numbers.
  - e. The largest 1-digit and 2-digit numbers are composite numbers. .....
- $5. \quad Use \ division \ method \ to \ find \ the \ prime \ factor is at ion \ of \ the \ following \ numbers.$ 
  - a. 128
- b.
- 176
- c. 108
- d. 288



# **Highest Common Factor (HCF)**

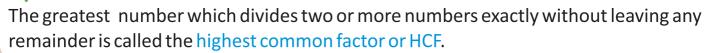
Let us consider two numbers 12 and 18.

Factors of 12 are 1, 2, 3, 4, 6 and 12.

Factors of 18 are 1, 2, 3, 6, 9 and 18.

We can see that 1, 2, 3 and 6 are the factors which are common to both 12 and 18.

Out of these common factors, 6 is the greatest factor which is common to both 12 and 18.



### **HCF of 3 Numbers**

To find the HCF of 3 numbers, first find the factors of the 3 numbers separately. Then take out the common factors. Next, identify the HCF.

**Example I:** Find the HCF of 8, 12 and 16.

$$1 \times 8 = 8$$
  $1 \times 12 = 12$   $1 \times 16 = 16$ 

$$2 \times 4 = 8$$
  $2 \times 6 = 12$   $2 \times 8 = 16$   $3 \times 4 = 12$   $4 \times 4 = 16$ 

### **Alternative Method**

HCF by short division method

We can find the HCF by two methods.

- a. Prime factorisation method
- b. Division method



When two or more numbers have the same factor, that factor is called a common factor.

### **HCF by Prime Factorisation**

To find the HCF by prime factorisation, we find the prime factors of the given numbers and list the prime factors which are common to all. The product of the common prime factors is the required HCF.

**Example II**: Find the HCF of 18, 24 and 42.

**Solution**: Prime factorisation of  $18 = 2 \times 3 \times 3$ 

Prime factorisation of  $24 = 2 \times 2 \times 2 \times 3$ 

Prime factorisation of  $42 = 2 \times 3 \times 7$ 

Common prime factor = 2 and 3

$$HCF = 2 \times 3 = 6$$

		2	24			
		2	24		2	42
2	18	2	12	-   -		42
_			12	-1	3	21
3	9	2	6	-	J	<u> </u>
3	2		Ū	-	7	7
2	3	3	3	-	-	-
	1		1	-1		1
	_		Т			

### **HCF** by Division Method

To find the HCF of two numbers by division method, we follow the steps given below:

**Example III**: Let us consider two numbers 192 and 72.

**Solution**:

**Step 1** : Make the smaller number as divisor and

the larger number as dividend and start

dividing.

**Step 2** : The remainder left (if not zero) becomes the

new divisor and the last divisor becomes

the new dividend.

**Step 3** : Continue the process, till a zero remainder is obtained.

**Step 4** : The last divisor is the required HCF.

∴ The HCF of 72 and 192 is 24.

### Facts to Know

- O If two numbers have their HCF as 1, they are known as co-prime numbers.
- O Co-prime numbers need not be prime numbers.
- O HCF of numbers is smaller than or equal to the smallest of the given numbers.



- 1. Find the HCF of the following numbers. Explain what is common in all the answers.
  - a. 10 and 32
- b. 12 and 48
- c. 15 and 45
- d. 14 and 56











2 and 32

b. 10 and 15

c. 28 and 40

d. 24 and 56

e. 12 and 18

f. 20 and 50

16 and 84 g.

h. 25 and 75

#### Find the HCF of the following by listing the common factors. 3.

10, 15, 20 a.

b. 12, 15, 18

c. 28, 70, 84

d. 32,40,56

e. 18, 27, 36

20, 30, 40

g. 45, 60, 75

h. 18, 45, 63

#### Find the HCF of the following by prime factorisation. 4.

72, 108 a.

b. 16, 24

c. 90,405

d. 15,30

e. 52,78

f. 32,40

g. 48, 144

h. 98, 245

#### Find the HCF of the following by division method. 5.

a. 18, 24

b. 48,72

c. 28, 35

d. 56,63

e. 150,350

f. 189, 315

g. 144, 216

h. 288,540

# **Multiples**

### Observe the following:

$$8 \times 1 = 8$$

$$8 \times 2 = 16$$

$$8 \times 3 = 24$$

$$8 \times 4 = 32$$

Here, 8 has been multiplied by 1, 2, 3 and 4 consecutively to get the products as 8, 16, 24 and 32 respectively.

So, the multiples of 8 = 8, 16, 24, 32 and so on.

Similarly, multiples of 2 = 2, 4, 6, 8, ...

multiples of 
$$5 = 5, 10, 15, 20, ...$$

The multiples of a number are obtained by multiplying the number consecutively by each of the natural numbers.















- The first multiple of a number is the number itself.
- O There is no end to the multiples of a number.

**Example IV** : Write down the first ten multiples of 7.

**Solution**: The first ten multiples of 7 are 7, 14, 21, 28, 35, 42, 49, 56, 63 and 70.

### **Properties of Multiples**

- Every number is a multiple of itself.
- ❖ Every number is a multiple of 1.
- Every multiple of a number is greater than or equal to the number.
- 0 is a multiple of every number.
- ❖ The smallest multiple of a number is the number itself.
- ❖ The multiples of an even number are always even number.
- The multiples of an odd numbers are alternatively odd and even numbers.

# Facts to Know

- Multiples of even numbers are even numbers, e.g.: 2, 4, 6, 8....
- Multiples of odd numbers are alternatively odd and even numbers, e.g. :3, 6, 9, 12....



- 1. Write the next four multiples of each of the following.
  - a. 6  $\rightarrow$  6, 12, 18, 24, ....., ...., ....
  - b. 12  $\rightarrow$  12, 24, 36, 48, ....., ...., ....,
  - c.  $16 \rightarrow 16,32,48,64$  ....., ....,
  - d.  $18 \rightarrow 18, 36, 54, 72$  ....., ...., ....
  - e.  $20 \rightarrow 20,40,60,80$  ....., ...., ....,
  - f.  $25 \rightarrow 25,50,75,100$  ....., ...., ....,
- 2. Write the first five multiples of 13.
- 3. Find the fifth multiple of 17.
- 4. Is 144 a multiple of 9? Give reasons.
- 5. 15 × 12 = 180 means is a multiple of and
- 6. Choose any odd number between 1 and 20 and write its first ten multiples. Check if the numbers are alternatively even and odd or not.









**Lowest Common Multiple (LCM)** 

Let us consider the numbers 4 and 6 and find the multiples of both the numbers.

Multiples of 4 are 4, 8, 12, 16, 20, 24, 28, 32, 36, ...

Multiples of 6 are 6, 12, 18, 24, 30, 36, ...

The common multiples are 12, 24, 36, ...

Out of these common multiples, 12 is the multiple which is the lowest (least).

The smallest number that can be divided by the given numbers without leaving any remainder is called the lowest common multiple.

Lowest common multiple is also called the least common multiple and is written as LCM.

### **LCM of 3 Numbers**

Write down a few multiples of 3 numbers. Take out the common multiples and pick up the LCM.

**Example V**: Find the LCM of 2, 3 and 5.

**Solution** : Multiples of 2 = 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, ...

Multiples of 3 = 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, ...

Multiples of 5 = 5, 10, 15, 20, 25, 30, ...

LCM = 30

### LCM by Prime Factorisation

In this method, we first list the prime factors of the numbers and then multiply the common factors and the remaining prime factors.

**Example VI**: Find the LCM of 18 and 24.

**Solution**: Prime factorisation of  $18 = 2 \times 3 \times 3$ 

Prime factorisation of  $24 = 2 \times 2 \times 2 \times 3$ 

Multiplying the common factors 2 and 3 and the remaining prime

factors, we get LCM =  $2 \times 3 \times 3 \times 2 \times 2 = 72$ .

### LCM by Common Division Method

In this method, we follow the steps given below.

Let us consider the numbers 12, 16 and 20.

2	12,	16,	20
2	6,	8,	10
2	3,	4,	5
	3,	2,	5













**Step 1** : Divide by the smallest prime number, which can divide at least one of the numbers and bring down the numbers that cannot be divided further.

**Step 2** : Continue division by the smallest possible prime numbers, till the last row contains prime numbers or co-prime numbers.

Step 3 : Multiply all the factors and the numbers in the last row to get the LCM. LCM of 12, 16, and  $20 = 2 \times 2 \times 2 \times 3 \times 2 \times 5 = 240$ .



### 1. Find the LCM of the following numbers by listing the multiples.

a. 3 and 8 b. 4 and 14 c. 6 and 15 d. 4 and 24

e. 6 and 18 f. 8 and 20 g. 9 and 27 h. 8 and 12

### 2. Find the LCM of the following numbers by listing the multiples.

a. 2, 3 and 4 b. 4, 6 and 8 c. 6, 12 and 18 d. 7, 14 and 28

e. 2,3 and 5 f. 6,9 and 18 g. 2,4 and 5 h. 4,8 and 12

### 3. Find the LCM of the following numbers by prime factorisation method.

a. 12 and 18 b. 42 and 56 c. 24 and 32 d. 26 and 39

e. 24 and 36 f. 21 and 28 g. 30 and 45 h. 12 and 30

### 4. Find the LCM of the following numbers by common division method.

a. 16 and 48 b. 15 and 45 c. 30 and 40 d. 36 and 54

e. 18,60 and 54 f. 20,25 and 30 g. 21,63 and 105 h. 12,30 and 32

### 5. Find the HCF and LCM of the following numbers by division method.

a. 12 and 48 b. 32 and 40 c. 24 and 36 d. 14 and 21

e. 25, 30 and 35 f. 42, 56 and 70 g. 24, 48 and 60 h. 112, 168 and 252

### 6. Solve the following.

a. Two containers have 16 / and 24 / of oil respectively. Which is the largest measuring can that can exactly measure oil from both the containers?

b. What is the smallest length of a room in which an exact number of tables of length 12 metres and 9 metres can fit?

### 7. Express the following numbers as the sum of 2 primes.

a. 16 b. 24 c. 30



The product of HCF and LCM of two natural numbers is equal to the product of the two numbers.

**Example VII**: Take any two numbers 24 and 36. Find their HCF and LCM.

**Solution**:

Common factors

HCF

2 | 24, 36

2 | 12, 18

3 | 6, 9

2, 3

LCM

2	24, 36
2	12, 18
3	6, 9
	2. 3

There is no other common factor.

 $HCF = 2 \times 2 \times 3 = 12$ 

Product

The remaining factors 2 and 3 are co-prime numbers so they have only 1 as the common factor. Find the product of HCF and LCM, i.e. 12 and 72.

 $LCM = 2 \times 2 \times 3 \times (2 \times 3) = 72$ 

Find the product of the 2 numbers, i.e. 24 and 36.

Remaining factors

Product of HCF and LCM = Product of the two numbers

**Example VIII**: Find the HCF, if the product of the two numbers is 720 and LCM is 36.

**Solution** : HCF × LCM = Product of two numbers

HCF = 
$$\frac{\text{Product of two numbers}}{\text{LCM}}$$
$$= \frac{720}{36} = 20$$
HCF = 20

**Example IX** : The HCF of the two numbers is 15 and the LCM is 45. If one number is 25, find the other number.





$$=$$
 HCF $\times$ LCM

So, one number 
$$\times$$
 2nd number = HCF $\times$ LCM

$$=\frac{15\times45}{25}=27$$

Hence, the other number is 27.



- 1. Show the relationship of the following numbers with their HCF and LCM.
  - a. 20 and 30
- b. 40 and 64
- c. 48 and 80
- 21 and 56

- e. 42 and 70
- f. 60 and 75
- g. 65 and 91
- h. 84 and 14
- 2. If the product of two numbers is 1296 and the HCF is 9, find the LCM.
- 3. The product of the HCF and LCM of the two numbers is 1575. If one number is 25, find the other number.
- 4. Find the greatest number that divides 14,27 and 40 leaving the remainder 2,3 and 4 respectively.
- 5. Find the largest number which is a factor of 180 and 336.
- 6. Three baskets of fruits contain 42, 98 and 70 pieces respectively. What is the highest common number of fruits that can be taken from all three baskets at one given time?
- 7. Find the greatest number that can divide 510 and 425 exactly.
- 8. Find the greatest number that divides 38 and 53, leaving 8 as remainder in each case.

## Points to Remember 🖫

- Every number (except 1) has at least two factors.
- Prime numbers have only 2 factors, i.e. 1 and the number itself.
- Composite numbers have more than two factors.
- The number 1 is neither prime nor composite.
- The number 2 is the only even prime number.
- A pair of prime numbers with a composite number in between are called twin primes.
- Numbers which have 1 as their HCF are called co-prime numbers
- The product of LCM and HCF of two natural numbers is equal to the product of the two numbers.















(CCE Pattern)

#### Multiple Choice Questions (MCQs) 1.

Tick ( 🏑	the correct ontion	ď.

	(-	) the correct operan.				
a.	The	ninth multiple of 19 is				
	(i)	171		(ii)	150	
	(iii)	135		(iv)	none of these	
b.	The	largest number which is a	factor of 72 a	nd 120	) is	
	(i)	24		(ii)	42	
	(iii)	62		(iv)	12	
c.	The	first two multiples of 14 ar	·e	• • • • • • • • • • • • • • • • • • • •		
	(i)	14 and 70		(ii)	7 and 14	
	(iii)	14 and 28		(iv)	13 and 26	
d.	The	prime factorisation of 18 i	S	······································		
	(i)	2×3×3×4		(ii)	2×2×2×2	
	(iii)	2×3×2×2		(iv)	2×3x3	
e.		product of the HCF and LCI e number is 36.	M of the two	numbe	ers is 1728. Find the other nu	mber
	(i)	38		(ii)	48	
	(iii)	24		(iv)	20	
Fin	d the	HCF of 128, 136 and 152.				

- 2.
- 3. Find the HCF of 24 and 32 using prime factorisation.
- 4. Which of these numbers are divisible by 2?
  - a. 28,564
- **b**. 70,963
- 90,760 C.
- d. 38,462
- Which of these numbers are divisible by both 2 and 3? **5.** 
  - a. 52,084
- **b**. 36,744
- 19,647
- d. 54,324
- 6. What will be the 8th multiple of the following?
  - a. 7
- b. 9

- 13
- d. 8
- e. 23

7. Find the HCF of 36 and 54 using division method.









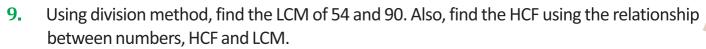














Mehak, Reeta and Shagun began walking together. Mehak stopped after 42 m, Reeta after 56 m and Shagun after 112 m. They continued their walk after a brief halt. After how many metres would they meet again?



**Objective** 

- : To reinforce the concept of factorisation by drawing rectangles of different measurements.
- Materials Required: Sheet of paper, scale, pencil, a pair of scissors and adhesive like fevicol

### **Activities:**

- On the sheet of paper, draw all possible rectangles that have 32 square units.
- Mark the square units used for lengths and breadths.
- Write the factor pairs below each rectangle as shown.







 $4 \times 8$ 

- Now cut all the rectangles and paste them in the order as shown.
- Notice that each rectangle has two factors starting with the factor 1 and the number itself.
- Write all the factors of 32 in ascending order with the help of the rectangular strips. For example : 1, 2, 4, ...
- Now, choose another number and draw all possible rectangles that have square units equal to the chosen number.
- Discuss with the teacher which factors are common for all the numbers discussed.



## Formative assessment-i

### (Based on Chapters 1 to 4)

<b>A.</b>	Multiple Choice Questions (MCQs)
	Tick (✓) the correct option:
	1. The face value of 3 in 43960 is

I ICI	K (* ) the correct option:		
1.	The face value of 3 in 43960 is		
	(a) 9000 🔀	(b) 3000	
	(c) 3	(d) none of these	
2.	The place value of 9 in numeral 38609	91 is	
	(a) 9 🚫 (b) 90 🚫 (c	c) 900 🚫(d) 91	
3.	In International place value system, 10	O lakhs is written as	
	(a) 1 million	(b) 10 millions	
	(c) 1 billions	(d) none of these	
4.	When a number is multiplied by 0, the	e answer is	
	(a) 1	(b) 100	
	(c) 0	(d) none of these	
5.	The number that comes just before 36 -		
	(a) 56 🚫 (b) 58 🚫 (c		
6.	The multiplicative identity of a whole		
	(a) 0 $\bigcirc$ (b) 3 $\bigcirc$ (c		
7.	Which of the following numbers are d	• •	
	(a) $36,746$ (b) $19,647$ (c)		
8.	The highest common factor of 4 and 1		
		c) 4 🚫(d) 6	
9.	The H.C.F. of 600, 1500, 2500 and 3000	- A-	
	(a) 50 (b) 100 (c)		
10.	Which of the following is divisible by 4		
		c) 3629 (d) 4287	
Mat	tch the columns:		

### В.

### Column A

- ₹7.15 1.
- 10 lakhs 2.
- $3. 2 \times 2 \times 2 \times 3$
- 4. HCF×LCM

### **Column B**

- ١. 24
- 1<sup>st</sup> number × 2<sup>nd</sup> number II.
- 715 paise III.
  - IV. 1 million













